HIGHER SURVEYING

BREED-HOSMER-BONE

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AND
GEORGE L. HOSMER

THE PRINCIPLES AND PRACTICE OF SURVEYING

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HIGHER SURVEYING

PRINCIPLES AND PRACTICE OF SURVEYING

VOLUME II

BY

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PREFACE TO THE FIRST EDITION

THIS second volume on surveying is chiefly devoted to a consideration of the various methods of conducting topographic and hydrographic surveys. While certain subjects taken up in this book might properly be included in an elementary treatise, the arrangement adopted has the decided advantage of grouping together and treating in a connected manner the different topographic and hydrographic methods.

The subject matter is divided into four parts, namely: The Control of the Survey, Filling in Topographic Details, Hydrographic Surveying and Stream Gauging, and Constructing and Finishing Maps.

In the first part some consideration has been given to Geodetic Surveying, not only to show the methods of controlling the accuracy of the survey, but also to connect the different methods taken up in Parts II and III and to show their relation to the survey as a whole. It is not the intention in any sense to present a treatise on Geodesy, but rather to discuss the subject from the standpoint of the practical surveyor. The matter is therefore limited to what any surveyor without unusual equipment might be expected to apply.

In Part II the methods of making topographic surveys are considered, including the stadia, plane-table, and photographic methods. In the chapter on The Plane Table an attempt has been made to show that this method has advantages which apparently are not appreciated by many surveyors and that its use might be much more general than it is at present. In the description of field methods a comparison has been made of the methods required for maps of different scales. In regard to

Photographic Surveying nothing has been attempted beyond an explanation of the fundamental principles involved in this method. In the chapter on the Relation of Geology to Topography, written by Professor D. W. Johnson, the subject of topography is treated from a point of view that is coming to be more and more appreciated by expert topographers. The importance of geological study to the topographer is now much emphasized by those recognized as authorities. The illustrations for this chapter were drawn by F. E. Matthes, Topographic Inspector, United States Geological Survey, to whom the authors are especially indebted.

The chapter on Hydrographic Surveying treats of the common methods of conducting harbor and river surveys. Some of the up-to-date methods have been explained in detail and are illustrated by several sets of field notes. For valuable suggestions on this subject the authors express their thanks to John R. Burke, formerly Assistant Engineer, Massachusetts Harbor and Land Commission, and to A. J. Ober, Assistant Engineer, United States Engineer Office. Chapter IX, on Stream Gauging, was written by H. K. Barrows, Engineer United States Geological Survey, who for several years has been in charge of the hydrographic work in New York and in New England. Thanks are due also to Professor W. E. Mott of the Massachusetts Institute of Technology for his criticisms of the manuscript of this chapter.

In the last two chapters the common methods of constructing and finishing topographic and hydrographic maps have been described. The details of making conventional signs have been described rather fully and some consideration has been given to the use of symbols on landscape plans. Several illustrations of topographic maps on different scales have been introduced.

The authors desire to acknowledge their indebtedness to all who have aided in the preparation of this book, especially to Professors C. Frank Allen, A. G. Robbins, C. W. Doten, and A. E. Burton of the Massachusetts Institute of Technology for criticisms and valuable suggestions.

The authors wish to express their appreciation of the excellent

work of W. L. Vennard, who prepared the illustrations, and of Miss Edith T. Hosmer, who read all of the proof.

For the cuts and electrotypes which have been loaned for use in this volume acknowledgment has been made in each case under the illustration.

The authors will be grateful for notification of any errors which may be found.

C. B. B.
G. L. H.

Boston, Mass., July, 1908.

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CHAPTER 1

HORIZONTAL CONTROL — TRAVERSES — TRIANGULATION

1-1. Introduction. This text is devoted to advanced and specialized phases of surveying. It deals first with the establishment of networks of points of known position which form the frame work or " control " upon which topographic mapping or engineering surveys can be based. Such networks also supply the ground control points needed to convert data from aerial photographs into maps of uniform scale.

Astronomical observations and computations for time, longitude, latitude and azimuth are explained as they pertain to establishing position and direction.

The use of photogrammetry for obtaining survey data and the production of maps together with its application to engineering projects is covered in some detail. Hydrographic surveying is presented with emphasis on subaqueous surveys and measurement of flow of water in open channels. Consideration is given to map projections and map production, and the principles of error analysis examined as a tool for planning, checking and adjusting surveys.

The details of property surveys, construction surveys and topographic surveys of limited extent employing transit and tape, stadia or plane-table methods are covered in " Elementary Surveying," Vol. I by Breed and Hosmer and in " Surveying " by C. B. Breed.

1-2. Control. In making surveys, especially those of large extent, it is necessary that they be based on adequate horizontal and vertical control to insure the preservation of the desired accuracy of the survey as a whole. This control should consist of a network of a relatively few stations whose positions and elevations have been determined with a high degree of precision, and which are distributed at strategic points throughout the area. These points can be established horizontally by precise

(high order) triangulation or traverses, and vertically by precise leveling. The established points can be used as a tie-in or junction with adjoining or more distant surveys. It is extremely desirable to have the control network connected with and expressed in terms of the local State Coordinate system whenever possible. Traverses run by the conventional transit and tape method are usually more time consuming and likely to be less accurate than triangulation because of the short sights and the need to tape all distances. However, the traverse points are usually more accessible than triangulation stations, and are, therefore, useful for tying in local surveys. The use of long-range electronic distance measuring equipment such as the Tellurometer and Geodimeter (Arts. 1-35 to 39) have largely overcome the disadvantages of traversing with a notable gain in speed and accuracy and a resulting saving in cost.

Electronic techniques are being increasingly used for measuring the sides of triangles in geometric patterns as well as for traversing. This process of measuring distances of a triangulation net rather than angles is called *trilateration.* In general, the same standards of accuracy are required of trilateration as for triangulation or traverse.

1-3. Traverses. Strip surveys for route location or state boundary surveys require control points to be determined with great accuracy in places where their location directly by triangulation is impractical. In such cases the desired points must be located by precise traverses tied into a triangulation system.*

When traverses are run by conventional methods with transit and tape, care must be taken with angle and distance measurements to obtain the required accuracy (Art. 1-7). Since traverse lines are usually shorter than those in triangulation systems, signals must be carefully centered over points and the targets designed to offer a well defined sighting point (Art. 1-14). Generally traverse targets are smaller and less elaborate than those used in triangulation.

* For information on first order traverses see U. S. Coast and Geodetic Survey, Special Publication No. 137. Also "Technical Procedure for City Surveys," American Society of Civil Engineers, Manual No. 10, 1957.

Electronic distance measuring methods are well adapted to measuring long lines, especially over rough terrain where taping would be difficult and slow. Where slope distances are measured, as with electronic instruments, they must be corrected to the horizontal. For this purpose vertical angles should be taken or the elevations of the ends of the lines obtained by leveling.

For measuring short lines, other types of equipment are available that do not require taping. These employ the device of measuring an angular intercept on a rod, staff or bar from which the distance is computed. This procedure, known as stadia, tacheometry, telemetry or subtence,* yields results of a lower order of accuracy than needed for establishing basic control. The method is useful for establishing local control for survey details or for locating or setting identification points for the control of aerial surveys.

1-4. Connecting Traverses with Fixed Control. In order that traverses may be adjusted, they should either close upon themselves or be connected to previously established stations. A loop traverse produces a closure error which, combined with the length of the loop, gives the degree of precision of the traverse, and provides the data needed to distribute the error and so adjust the traverse.† A non-loop or spur traverse, to be adjustable, must be connected, preferably at the ends, with established stations.

After the traverses have been connected with the triangulation the accuracy of each one may be tested by computing its error of closure. This may be done in any traverse joining two triangulation points, by computing the difference in latitude and departure for each course of the traverse, the line joining the triangulation points being regarded as the closing line of the traverse (see Fig. 1-1). If the azimuths of the traverse lines are referred to the meridian through $\triangle A$, then the azimuth A–B should be used in computing the difference in latitude and departure for the triangulation line. The sums of the latitudes and departures are found and the error of closure distributed in

* See Elementary Survey by Breed and Hosmer, Vol. I, 9th Ed., Art. 16 and Chapter VII.

† See Appendix A for traverse adjustments.

the usual way, except that the corrections are made only on the traverse lines, the triangulation line remaining unchanged. If desired, the latitudes and departures can be computed by taking the triangulation line as a meridian instead of using the true meridian. The triangulation lines in such surveys are usually

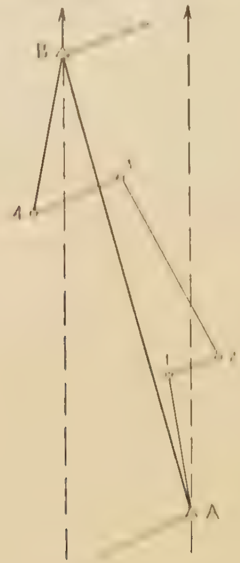

FIG. 1-1.

short, so that the effect of the earth's curvature on the traverse lines is negligible.

Fig. 1-1 illustrates a short traverse connecting two established triangulation points *A* and *B*. Assuming that these points are on a plane coordinate grid system, the computations for obtaining adjusted coordinates of traverse points are demonstrated below. Latitudes and departures are obtained from survey data by applying the cosine and sine of the azimuth, respectively, to the distances. Starting with the known coordinates of station *A* the unadjusted coordinates are first obtained for each survey point and station *B*. At this station a discrepancy, or error of closure, of +0.40 ft in *Y* and −0.06 in *X* is found between

computed and established coordinate values. These errors are distributed among the survey point coordinates using the compass rule; i.e., in the proportion that the accumulated length to each station is to the total length of the traverse. Note that the line AB is not included in the adjustment since it is already part of an adjusted system. The corrections are then applied to obtain adjusted coordinates, as shown in the computations. The adjusted coordinates establish the accepted relations between survey points and all other points in the coordinate system.

For another method of adjusting traverses see Appendix A.

Notes of Traverse

Station	Grid Azimuth	Grid Distance * Feet
△ *A*		
	171° 26′ 20″	1321.20
1		
	243 25 50	524.84 (1846.04)
2		
	152 06 10	1974.50 (3820.54)
3		
	66 15 30	901.08 (4721.62)
4		
	190 44 00	1570.71 (6292.33)
△ *B*		

* Figures in brackets are cumulative traverse distances used for adjusting error of closure.

Computation of Adjusted Grid Coordinates

Station	Latitude Feet (N+)(S−)	Departure Feet (E+)(W−)	Grid Coordinates — Feet	
			Y (North)	*X* (East)
△ *A*			161,287.49	602,355.02
	+1306.48	−196.68		
1			162,593.97	602,158.34
			+.10	−.01
			162,594.07	602,158.33
	+234.75	+469.41		
2			162,828.72	602,627.75
			+.14	−.02
			162,828.86	602,627.73
	+1745.04	−923.84		
3			164,573.76	601,703.91
			+.30	−.04
			164,574.06	601,703.87
	−362.79	−824.82		
4			164,210.97	600,879.09
			+.37	−.05
			164,211.34	600,879.04
	+1543.23	+292.53		
△ *B*			165,754.20	601,171.62
			+.49	−.06
			165,754.69	601,171.56

$$\text{Error of Closure} = \sqrt{.49^2 + .06^2} = .494 \qquad \text{Precision} = \frac{.494}{6292} = \frac{1}{12{,}740}$$

1-5. Triangulation Systems. In a triangulation system one line (the base line) is measured directly; all the other distances are derived by measuring the angles of the triangles and calculating the sides by trigonometry. The intervening ground does not have to be traversed, so that the accuracy with which a distant station may be located is nearly independent of the character of the intervening country.

When the triangulation consists of a narrow belt extending from one region to another it is usually composed of (1) a chain of triangles, (2) a chain of quadrilaterals, or (3) a chain of polygons each having an interior station. The first (Fig. 1-2, simple

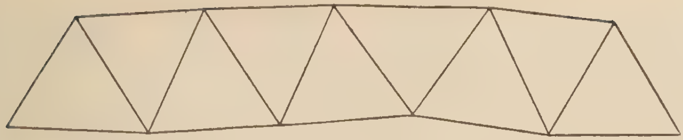

FIG. 1-2. SIMPLE CHAIN OF TRIANGLES.

triangles) gives so few checks that it is not used on important work. The chain of quadrilaterals (Fig. 1-3) is the strongest

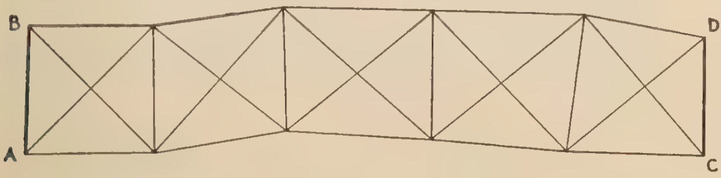

FIG. 1-3. CHAIN OF QUADRILATERALS.

system, all things considered. The third (Fig. 1-4, lowest view) is but little inferior to the second; in unsymmetrical figures it may be stronger.

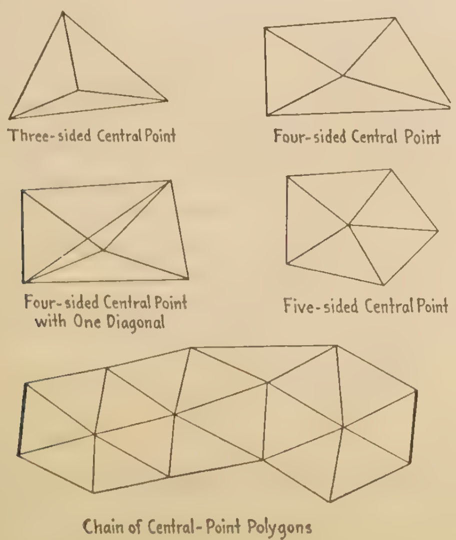

FIG. 1-4. FIGURES COMMONLY USED IN TRIANGULATION.

In the quadrilateral there are eight independent angles to be measured, and there are four rigid geometric conditions which must hold true; three angle conditions and one side condition. An angle condition is that the sum of the angles of a triangle must total 180° plus spherical excess. A side condition is that the length of one side as computed from the opposite side should be the same regardless of the route of computation. In any quadrilateral there are four possible ways to compute the length of one side from that of the opposite side (through the two diagonals and the two sides). Using unadjusted measurements this would result in four different computed lengths for the same line. In order that these four lengths may coincide and the geometric conditions be exactly fulfilled, it is essential to adjust the system by the " method of least squares " (Art. 10-6). This adjustment removes all inconsistencies and gives the most probable values of the angles. There are, however, approximate adjustment methods which are normally adequate (See Appendix B).

1-6. Strength of Figure. In the extension of triangulation from a measured base through chains of triangles the accuracy of the computed triangle sides deteriorates progressively until eventually the measurement of another base becomes necessary to maintain the desired tolerance. It is obvious that the configuration and inter-relationships of the various triangles bears directly on the rate of such deterioration or loss of " strength." The U. S. Coast and Geodetic Survey employs a method of testing the strength of any chain of triangles based on the computed " probable error " of a triangle side.

The factor used for comparing alternative chains of triangles is

$$R = \frac{D - C}{D} \times \text{Sum of } [\delta_A{}^2 + \delta_A\delta_B + \delta_B{}^2]$$

D = the number of new * directions observed (Art. 1-47).

* New directions in any figure include all directions except those defining the starting side.

C * = the number of geometric conditions in the net of triangles.

δ_A is the tabular difference for 1″ in log sin A in the 6th decimal place.

δ_B is the same for log sin B.

A and B here represent the angles opposite the side to be computed and the known side, respectively, in any triangle. Values of the quantity in brackets will be found in Table XVIII, p. 510.

Values of the factor $\frac{D-C}{D}$ for common figures (see Fig. 1-3) are given in the following table:

Figure	D	C	$\frac{D-C}{D}$
Single triangle	4	1	0.75
Completed quadrilateral	10	4	0.60
Three-sided, central-point figure	10	4	0.60
Four-sided, central-point figure	14	5	0.64
Four-sided, central-point figure with one diagonal also observed	16	7	0.56
Five-sided, central-point figure	18	6	0.67

In the completed quadrilateral, for example, there are 6 lines, and consequently 12 directions; but two of these directions, on the base or known side, are already fixed. Hence there are but 10 new directions to be established in the adjustment. In any quadrilateral in which all stations have been occupied and all directions observed there are four geometric conditions. So we have

$$(D - C) \div D = (10 - 4) \div 10 = 0.60$$

---

* The number of geometric conditions in any figure may be computed by the formula

$$C = (n' - S' + 1)_A + (n - 2S + 3)_S$$

where

$(\ \ )_A$ = angle conditions.
$(\ \ )_S$ = side conditions.
n' = the total number of lines observed in both directions.
S' = the total number of stations occupied.
n = the total number of lines.
S = the total number of stations.
(n' and n include the base line).

This factor (0.60) is the same for all completed quadrilaterals, regardless of shape.

For any one triangle, starting from the base, we find approximate values of the angles occurring opposite the base and opposite the advance side. These may be taken from a sketch map. From a table of log sines we may find the corresponding δ_A and δ_B and compute the term in brackets. Similarly for the next triangle, starting from the advance side of the preceding triangle as a base, and ending on the side of the quadrilateral that is opposite the base, we compute the quantity in brackets. Adding these two numbers and multiplying by 0.60 we have a value for R for one route through the quadrilateral.

Table XVI has been computed for different values of the angles so it is unnecessary to look up δ_A and δ_B separately. In using this table, the angle A may be read along the top and the angle B along the side, or vice versa.

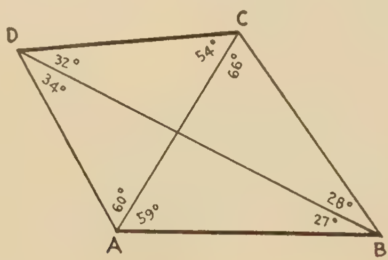

FIG. 1-5.

In the quadrilateral $ABCD$, in Fig. 1-5, AB is the known side and CD the side to be determined. Although there are four different routes by which the length CD may be found from AB, we shall examine in detail only the two routes using the diagonals as intermediate lengths. For the shape of the quadrilateral in Fig. 1-5, the two strongest routes are through the diagonals, the shorter diagonal giving the better strength. In some quadrilaterals, however, the strongest route is through one of the sides. Hence it is sometimes necessary to examine all four routes.

In Fig. 1-6, the quadrilateral $ABCD$ (Fig. 1-5) is shown divided

into the triangles used to compute CD from AB via AC and BD, respectively.

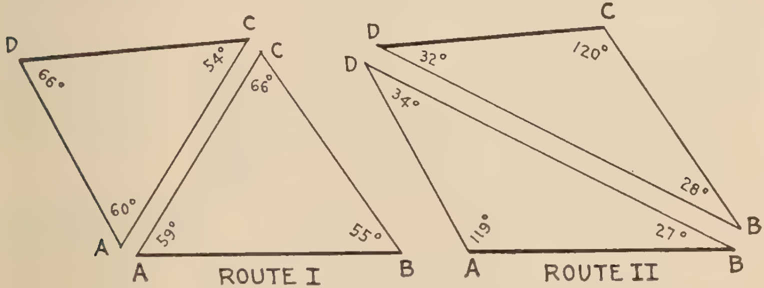

FIG. 1-6.

The following tabulation shows the application of the strength of the figure formula to the quadrilateral $ABCD$ for Route I (via AC) and Route II (via BD). The letters " A " and " B " in the table refer to terms in the formula and not to the letters in Fig. 1-6.

Route	Triangle *	" A "	" B "	$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$ †	$\frac{D-C}{D}$	R
I	BAC	55°	66°	5		
	CAD	60	66	4		
				Sum 9	0.60	5.4
II	BAD	119	34	8		
	BDC	28	120	12		
				Sum 20	0.60	12.0

* The sequence of triangle lettering is clockwise, starting with the known side.
† From Table XVI.

The values of R are numbered R_1 for the smallest figure (strongest route), R_2 for the next, and so on.* For this quadrilateral we have $R_1 = 5.4$ and $R_2 = 12.0$. An investigation of

* The number of the routes should not be confused with the R's which are assigned after the tests are made.

the other two routes via the sides yields values of $R_3 = 27.0$ and $R_4 = 27.0$. The R_1 value is the one depended upon mainly to express the strength of the quadrilateral. For any other figure, such as a polygon with an interior station, we would proceed in a similar manner and obtain R_1, R_2, etc. For example, a regular pentagon having an interior station (angles 54° and 72°) has 18 new directions and 6 geometric conditions, giving 0.67 for the first factor. In this figure $R_1 = 10$, and $R_2 = 15$.

The two R_1 values in the quadrilateral and the pentagon would then express mathematically the relative strength with which the final line would be determined by the two figures. The figure having the smaller R_1 is the stronger figure. This of course assumes that the error in measuring the angles would be the same for both figures.

Strength factors are used to compare different chains of figures between two base-lines. For example, in Fig. 1-3, if there was an alternate chain of quadrilaterals available using different stations from those shown in Fig. 1-3, the two systems could be compared by computing the R_1 and R_2 factor for each chain. These factors for each chain are found by adding progressively the R_1 and R_2 factors for each of the individual quadrilaterals making up the chain, the sums being designated as ΣR_1 and ΣR_2.

For further information see U. S. Coast and Geodetic Survey Special Publication No. 247.

1-7. Classification of Control and Standards of Accuracy. Classification and standards of accuracy for geodetic control surveys have been prepared by the Bureau of the Budget in co-operation with the Federal agencies concerned in making control surveys or utilizing their results. They also have the approval of the American Society of Civil Engineers and the American Congress on Surveying and Mapping.

The requirements for first, second and third order triangulation, traverse and leveling are summarized in "Classification and Standards of Accuracy," Tables I, II and III,* respectively, pp. 13 and 14. A full description of these standards is given in Appendix D.

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* Tables I, II and III reproduced by permission of the American Congress on Surveying and Mapping.

CLASSIFICATION AND STANDARDS OF ACCURACY

TABLE I

TRIANGULATION

	First Order			Second Order		Third Order
	Class I (Special)	*Class II* (Optimum)	*Class III* (Standard)	*Class I*	*Class II*	
Principal uses	Urban surveys, scientific studies	Basic network	All other	Area networks and supplemental cross arcs in National net	Coastal areas, inland waterways and engineering surveys	Topographic mapping
Spacing of arcs or principal station*	Stations: 1–5 miles or greater as required	Arcs: 60 miles. Stations: 10–15 miles	Stations: 10–15 miles	Stations: 4–10 miles	As required	As required
Strength of figure						
ΣR_1 between bases						
Desirable limit	25	60	80	80	100	125
Maximum limit	30	80	110	120	130	175
Single figure						
Desirable limit						
R_1	5	10	15	15	25	25
R_2	10	30	50	70	80	120
Maximum limit						
R_1	10	25	25	25	40	50
R_2	15	60	80	100	120	170
Base measurement						
Actual error not to exceed	1 part in 300,000	1 part in 300,000	1 part in 300,000	1 part in 300,000	1 part in 150,000	1 part in 75,000
Probable error not to exceed	1 part in 1,000,000	1 part in 1,000,000	1 part in 1,000,000	1 part in 1,000,000	1 part in 500,000	1 part in 250,000
Triangle closure						
Average not to exceed	1″	1″	1″	1.5″	3″	5″
Maximum seldom to exceed	3″	3″	3″	5″	5″	10″
Side checks						
Ratio of maximum difference of logs of sides to tab. diff. for 1″ of log sine of smallest angle	1.5	1.5–2	2	2–4	4	10–12
OR in side equation test, average corr. to direction not to exceed	0.3″	0.4″	0.4″	0.6″	0.8″	2″
Astro. azimuths						
Spacing-figures	6–8	6–10	8–10	8–10	10–12	12–15
Probable error	0.3″	0.3″	0.3″	0.3″	0.5″	2.0″
Closure in length (also position when applicable) after side and angle conditions have been satisfied, should not exceed	1 part in 100,000	1 part in 50,000	1 part in 25,000	1 part in 20,000	1 part in 10,000	1 part in 5,000

* Additional stations of same accuracy may be interspersed among principal stations.

TABLE II
TRAVERSE

	First-order	*Second-order*	*Third-order*
Number of azimuth courses between azimuth checks not to exceed	15	25	50
Astronomical azimuth: Probable error of result	0.5″	2.0″	5.0″
Azimuth closure at azimuth check points not to exceed*	2 sec. $\sqrt{N}$ or 1.0 sec. per sta.	10 sec. $\sqrt{N}$ or 3.0 sec. per sta.	30 sec. $\sqrt{N}$ or 8.0 sec. per sta.
Distance measurements accurate within	1 in 35,000	1 in 15,000	1 in 7,500
After azimuth adjustment, closing error in position not to exceed*	0.66 ft. $\sqrt{M}$ or 1 in 25,000	1.67 ft. $\sqrt{M}$ or 1 in 10,000	3.34 ft. $\sqrt{M}$ or 1 in 5,000

N is the number of stations for carrying azimuth.
M is the distance in miles.
* The expressions for closing errors in traverse surveys are given in two forms. The expression containing the square root is designed for longer lines where higher proportional accuracy is required. The formula which gives the smaller permissible closure should be used.

TABLE III
LEVELING

	First-order	*Second-order*		*Third-order*
		Class I	*Class II*	
Spacing of lines and cross lines	60 miles	25–35 miles	6 miles	Not specified
Average spacing of permanently marked bench marks along lines, not to exceed	1 mile	1 mile	1 mile	3 miles
Length of sections	½–1 mile	½–1 mile	½–1 mile	Not specified
Check between forward and backward running, between fixed elevations, or loop closures, not to exceed	4mm $\sqrt{K}$ or 0.017 ft. $\sqrt{M}$	8.4mm $\sqrt{K}$ or 0.035 ft. $\sqrt{M}$	8.4mm $\sqrt{K}$ or 0.035 ft. $\sqrt{M}$	12mm $\sqrt{K}$ or 0.050 ft. $\sqrt{M}$

K is the distance in kilometers.
M is the distance in miles.

1-8. Base-Line. In any system there must be at least one line whose length is known, called the *base-line*. In first and second order triangulation the base is measured with the invar tape apparatus (see Arts. 1-24, 25) with a probable error of 1 part in 1,000,000. High order measurements may also be made directly by the use of the tellurometer or geodimeter (Arts. 1-38, 39). For other classes of work the precision of the base may be correspondingly less. When a system extends over a long distance the errors of observation gradually accumulate, and it becomes necessary to measure additional base-lines, not only to test the accuracy of the results but also to strengthen the system and to keep the total error within the required limits.

Many surveys consist in extending a new system of triangulation from some triangle side which has already been established by the government surveys. In such a case this line becomes the base for the new system. The triangulation may be checked by closing on any other previously established line or on a measured base.

1-9. Triangulation Reconnaissance. The first step in reconnaissance is to secure the best maps available. The topographic maps of the U. S. Geological Survey, for example, are well suited for the purpose. A " paper reconnaissance " should be made on these maps by drawing lines between likely triangulation points and then studying the strength of the proposed triangulation net. The topography should also be studied for intervisibility of stations.

When suitable maps or aerial photographs are not available, a rough but accurate triangulation should be required in the field. Angles should be measured with a small transit or a sextant, elevations with a barometer, and directions with a compass. The importance of knowing the relative elevation of the prospective stations cannot be too greatly stressed, because points which are not intervisible from the ground can often be made intervisible by the erection of towers at one or both ends of the line. The heights of the required towers are calculated from the relative elevations of the stations and of the intervening terrain. (See Arts. 1-20 and 1-21.) In level country, stations may be selected where they will give the most desirable triangulation net; topography is then not a factor in their selection. In districts which are difficult of access, the reconnaissance must be accurately done in order to avoid errors that will prove costly to correct in the final triangulation.

In built-up areas of municipalities where intervisibility cannot be readily obtained, but where suitable maps are usually available, a reconnaissance may be made using the plane-table. All prospective stations and the various landmarks in the area (church steeples, tall chimneys, radio towers, etc.) should be plotted on the plane-table sheet. The plane-table is then taken to the proposed triangulation stations and the intervisibility of the proposed stations is determined. Those lines that pass close to obstructions are noted on the plane-table sheet and also how

close to these obstructions the line passes. After determining intervisibility, the completed plane-table sheet is brought into the office and the triangulation scheme is then laid out.

It is important that all lines shall clear obstructions by a suitable margin (a minimum of 10 feet is suggested) in order to avoid distortion of the line of sight due to refraction. Lines passing either to the side or above stacks and chimneys, even with the required clearance, should be rejected whenever possible. If such lines are definitely essential to the scheme, then these lines should be observed only when no smoke is issuing from the stack or when the wind direction is such as to deflect the heated fumes away from the line.* Lines should also adequately clear the terrain in the immediate vicinity of the observing station. In precise work, it is advisable to erect low observing towers so as to obtain greater clearance above the ground even though the distant stations are visible from the normal height of the instrument.

1-10. **Selecting Triangulation Stations.** In selecting stations the most elevated points will naturally be chosen, provided their location is such as to give well-shaped triangles; the selection of low points frequently makes necessary the construction of towers or high signals.

On the principal triangulation the points should be selected chiefly with reference to the shape of the triangles and the accuracy of results; the positions should also be suitable for connection with triangulation of the next lower order. On triangulation of a lower order the points may be chosen with regard to the distribution of stations and convenience in prosecuting the dependent topographic or hydrographic survey, or establishing control for an aerial survey.

1-11. **Reconnaissance for Base Site.** When the positions of the principal stations have been decided upon, the location of the base-line should be chosen. In case of the extension of a system already in existence, the base may be a side of one of the triangles. If a base is to be directly measured, the location must

---

* Even with these precautions, lines near stacks may cause considerable error. In city areas the observer should be constantly on the watch for lateral refraction.

be carefully selected with reference to convenience and accuracy, both in the measurement of the base itself and in its connection with the main triangulation. For convenience it should be on comparatively level ground, although so far as accuracy is concerned it is possible to do good work on steep grades. If better shaped triangles can be obtained by placing the base on rough ground it may be advisable to do this. Narrow gulleys will not be serious obstacles to the measurement, provided they can be spanned with the tape. Where the terrain is rough and there are gulleys, electronic distance measuring instruments may be used to good advantage.

The length of the base is usually from one-sixth to one-fourth of that of the sides of the principal triangles. The ends should be located so that the base can be connected with the main scheme by a few well-shaped triangles. This system of triangles connecting the base with the triangle sides is called the *base net*, or the *expansion*. If the triangulation covers a small area, such as a town, the base may be long in comparison with the length of the triangle sides, and no elaborate base net is necessary (see plan of Baltimore triangulation, p. 383, Vol. I).

The ideal expansion is one in which the base-line crosses the triangle side as shown in Fig. 1-7. By this arrangement well-

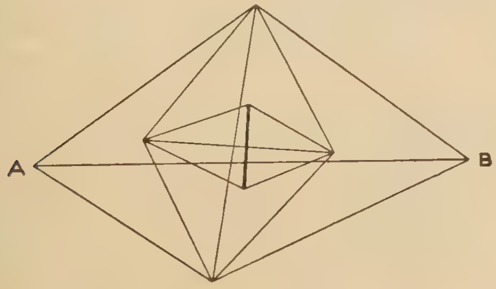

FIG. 1-7. BASE NET.

shaped triangles can be more easily secured. In practice, however, the arrangement of triangles is largely determined by the topography, and the ideal expansion can seldom be realized, as illustrated by the Massachusetts Base and the Fire Island Base

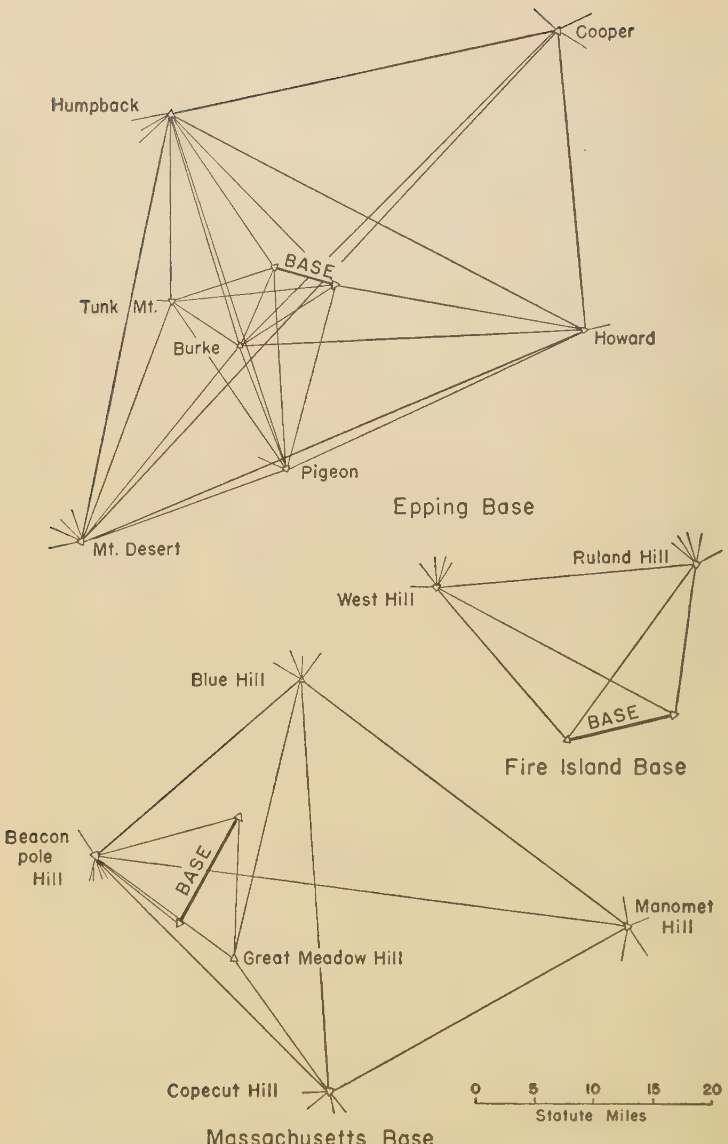

Fig. 1-8. Base Nets of the Triangulation of New England.

(Fig. 1-8), in the U. S. Coast Survey triangulation of New England. Fig. 1-9 shows triangulation net, Rochester, N. Y.

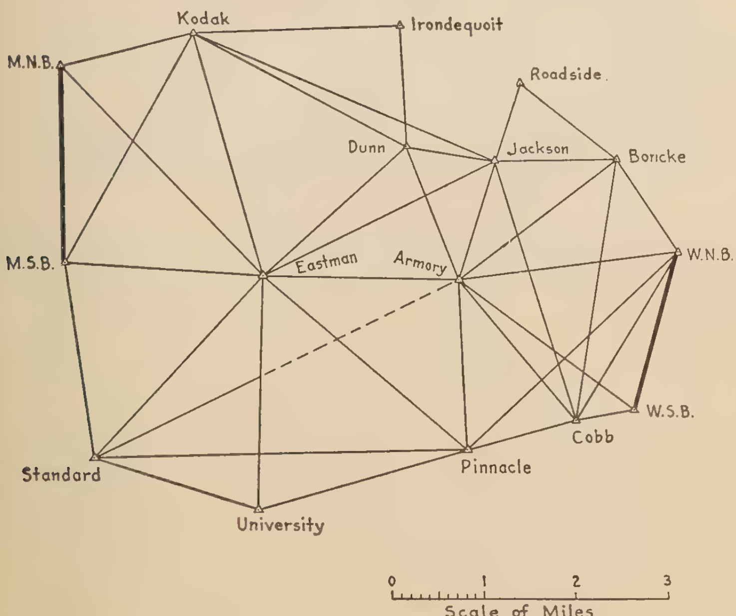

FIG. 1-9. EXAMPLE OF BASE NET FOR CITY TRIANGULATION. (ROCHESTER, N. Y. AND VICINITY)

1-12. **Marking the Stations.** The establishment of survey stations is an expensive operation, and the investment should be adequately protected. All stations should therefore be marked in such a manner that the points can subsequently be recovered and identified with certainty, and so that they are not likely to be disturbed.

Triangulation stations were formerly marked chiefly by drill-holes, with copper bolts in the case of important stations. It was considered best not to use conspicuous marks. The present practice, however, is to use bronze tablets (Fig. 1-10) bearing the name of the organization and other information; these may be countersunk in ledge or in concrete and cemented in place.

Fig. 1-10. Triangulation Station Marker.

Where the point is not on ledge the station may be marked by a stone or concrete monument for a surface mark and a disk of earthenware placed several feet below the surface to preserve the location in case the surface mark is moved or lost.

In addition to the surface and sub-surface marks there should be several reference marks placed at some distance from the station and preferably near boundary walls or fences where the marks will probably not be disturbed. These reference marks may consist of drill holes or bronze tablets. The azimuth and distance to each mark should be recorded in the notes.

The importance of station descriptions cannot be overstated. The point is worthless for future work unless it can be found. It is customary when revisiting frequently used stations to prepare a recovery note telling whether the station has been found or whether it is apparently lost. These recovery notes should also include any changes or errors in the original description.

1-13. Description of Station. When a station is established or a signal built, a complete written description of the station and signal should be made. This description should enable any other surveyor who wishes to use the station to find it without difficulty. The description should start with the general location and then proceed to describe the definite point. Thus, the state, county, city or town should be given first, followed by the portion of the town, if it has a local name, and the local name of the hill and the name of the property owner. Then specific directions, such as distances from the station to fences, and natural landmarks should be given. Finally, the device used to mark the point, such as an iron pipe, a bronze disk or copper bolt, should

be described in detail. The description of reference marks, together with azimuths and distances to them, should also be given. If the observing at the station has been done from a tower, it is good practice to list those distant stations or prominent landmarks which may be seen with an instrument set at the usual observing height above the ground.

Station descriptions are available from a number of Government agencies such as U. S. Coast and Geodetic Survey, U. S. Geological Survey, Corps of Engineers, Army Map Service, Bureau of Land Management, Tennessee Valley Authority, National Park Service and Hydrographic Office, Dept. of the Navy.

The following station description is typical of those prepared by the U. S. Coast and Geodetic Survey.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 525
Rev. Aug. 1948

DESCRIPTION OF TRIANGULATION STATION

NAME OF STATION: MUNSON   STATE: Arizona   COUNTY: Maricopa

CHIEF OF PARTY: Steven L. Hollis Jr. YEAR: 1959   Described by: L.O.Y.

NOTE.* HEIGHT OF TELESCOPE ABOVE STATION MARK 1 METERS.† HEIGHT OF LIGHT ABOVE STATION MARK METERS.

1a Surface-station mark, 7a Underground-station mark — DISTANCES AND DIRECTIONS TO AZIMUTH MARK, REFERENCE MARKS AND PROMINENT OBJECTS WHICH CAN BE SEEN FROM THE GROUND AT THE STATION

NOTE	OBJECT	BEARING	DISTANCE feet	DISTANCE meters	DIRECTION‡
	SKUNK 1947				0° 00′ 00.″0
16a	Azimuth mark	S	(approx.	0.3 mile)	01 48 37.8
11a	Reference mark 2	S	59.85	18.242	03 01 51
11a	Reference mark 1	E	25.42	7.748	270 09 33

The station is about 17 miles north of Phoenix on the west side of Arizona State Highway 69 and on the highway right-of-way.

Detailed descriptions

To reach the station from the intersection of Camelback Road and State Highway 69, go north on State Highway 69 for 17 miles to the station on the left, west.

The station is a standard disk, stamped MUNSON 1959, set in a 12-inch square concrete monument which projects 2 inches. It is 7.6 feet north of a witness post and 3.9 feet east of the west right-of-way fence.

Reference mark 1 is a standard disk, stamped MUNSON NO 1 1959, set in a 12-inch square concrete monument which projects 1 inch and is 29.1 feet northeast of the witness post.

Reference mark 2 is a standard disk, stamped MUNSON NO 2 1959, set in a 12-inch square concrete monument which is flush with the ground. It is 52.2 feet south of the witness post and 2.7 feet east of the right-of-way fence.

The azimuth mark is a standard disk, stamped MUNSON 1959, set in a 12-inch square concrete monument which is flush with the ground. It is 60 feet west of the centerline of State Highway 69, 8.5 feet south of a gate post, 416 feet southeast of a witness post and 3 feet east of the right-of-way fence.

To reach the azimuth mark from the station, go south on State Highway 69 for 0.3 mile to the azimuth mark on the right, west.

* Refers to notes in manuals of triangulation and state publications of triangulation.
† To nearest meter only, when no trigonometric leveling is being done.
‡ Direction-angle measured clockwise, referred to initial station.

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When preparing to make use of a triangulation station, its geodetic position and azimuth to adjacent triangulation stations, and its State plane coordinates should be secured in addition to the station description.

1-14. **Signals.** The kind of signal or target used at the triangulation station will depend upon the length of line to be sighted over, the frequency with which the station is to be occupied, the material available for building the signal, the difficulty of transportation, and the nature of the ground at the station. The actual target point to be sighted should be sharp, distinctive, and as free from phase as possible considering the distance involved and telescope characteristics; magnification, type of cross hairs, etc. It is frequently desirable to "dress" other parts of the signal in some larger and more conspicuous fashion to facilitate ready identification from a distance, unless observations are made at night using signal lamps. The signal should be easily dismounted and reassembled for occupation of the station by the instrument, or its construction should be such that the station can be occupied without disturbing the target and without interference with making observations.

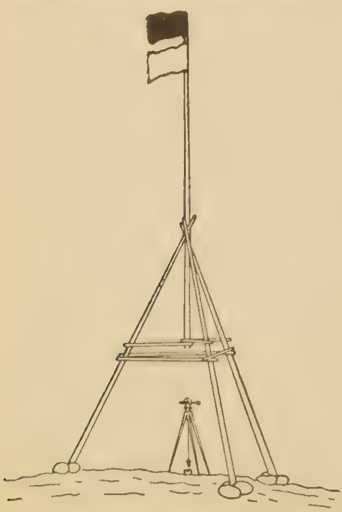

FIG. 1-11. TEMPORARY TRIPOD.

1-15. **Tripod Signals.** If the triangulation station is on a bare summit and the point is to be occupied frequently, as in the case of a first or a second order station, the tripod signal shown in Fig. 1-11 may be used. It consists of a mast 16 to 24 feet long, made of 4 by 4-inch joist, supported by three legs of about the same size as the mast, the parts being secured by six braces. Three of these braces secure the foot of the mast to the tripod legs; the other three stiffen the tripod. The mast and the tripod legs are fastened together by means of a bolt.

The lower end of the mast is usually 7 or 8 feet above the ground so that the instrument used for measuring the angles can be set beneath the signal. The center of the lower end of the mast is carefully placed over the point used to mark the station, and the mast is made vertical by means of either a plumb-line or a transit. The mast may be painted with black and white stripes about 2 feet wide to aid in identifying the signal and in making pointings. Such a tripod signal may be used for distances up to about 10 or 15 miles.

1-16. Guyed Mast Signal. When the hill is wooded or the signal is to be set in a low place surrounded by trees, a high signal may be erected by splicing two or three poles and bracing this mast in position by several sets of wire guys. Sights are taken on a definite point near the top of the mast, marked by black and white cloth bands. This portion of the mast is held vertically over the station mark by a set of guys. The rest of the pole need not be over the station. If the foot of the mast is a little to one side of the station mark the instrument can be set up without disturbing the signal. When lines are short and visibility is clear, a single short pole may be erected and held vertically over the mark by three or more guys. Such a signal can be lowered and re-erected by a one-man observer.

1-17. Heliotropes. On lines which are much longer than ten or fifteen miles tripod signals are not practicable. In such cases the position of the point to be sighted may be shown by means of sunlight reflected by an instrument called a *heliotrope*, which consists of a plane mirror and some device for pointing the mirror so that the light may be seen at the distant station. Heliotropes are used not only to show the position of the station but also as a means of sending messages between the observer and the heliotroper. For these messages a simple code of flashes is improvised, as there will in general be but few messages required, such as increasing and diminishing the light, name of next station to occupy, signal that the work is completed, etc.

Portable radio-telephone sets are sometimes used for communication.

Heliotropes may be provided with a telescope, mirrors, and facilities for mounting and pointing. If a commercial heliotrope

is not at hand, one may be easily improvised with small ordinary mirrors as shown in Fig. 1-12. As the sun moves, its reflected image will move off the station. Therefore care must be taken to make sure that the mirrors are adjusted from time to time to maintain the correct pointing on the observing station.

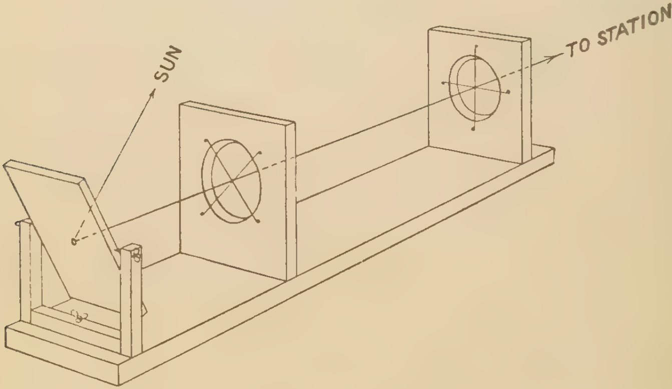

FIG. 1-12. IMPROVISED HELIOTROPE.

1-18. Improvised Heliotrope. A serviceable heliotrope may be made by constructing a wooden apparatus similar to that shown in Fig. 1-12, having two holes bored in the upright pieces. The rear hole should be slightly larger than the front one. Threads are stretched at right angles across the holes to mark their centers. For reflecting the sunlight any ordinary mirror so mounted that it can be moved in altitude and in azimuth will serve the purpose. Such a heliotrope has proved satisfactory on lines 25 miles or more in length.

1-19. Signals for Observing at Night. A large amount of the triangulation work carried on by the Coast and Geodetic Survey is done at night, experience having shown that this is more accurate and more economical than day observations. Battery-supplied signal lamps (Fig. 1-13) are commonly used for this purpose. They are provided with powerful reflectors and with means of pointing to the observing station. Some of these are automatic, an eight-day clock turning the lights on at a given

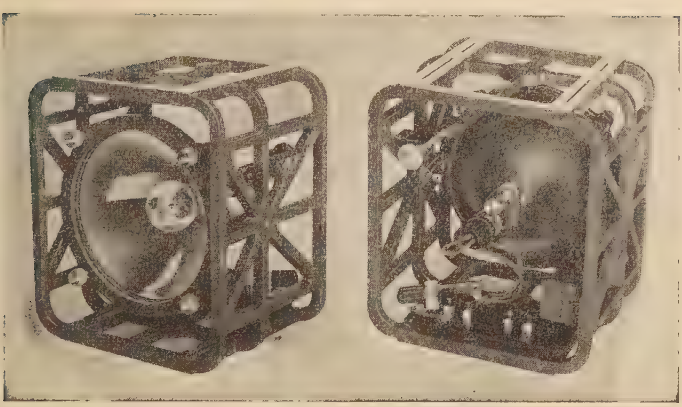

FIG. 1-13. SIGNAL LAMPS FOR TRIANGULATION AT NIGHT.

time and turning them off after elapse of a set time interval.

Other types of target are illustrated in Fig. 1-14. The right hand view shows one that fits into the same head as the theodolite and is interchangeable with it. This type is well adapted to precise traversing when sighting is done on tripod-mounted targets.

The target shown in the left view of Fig. 1-14 is equipped with a battery and light for use at night. A level bubble is used to

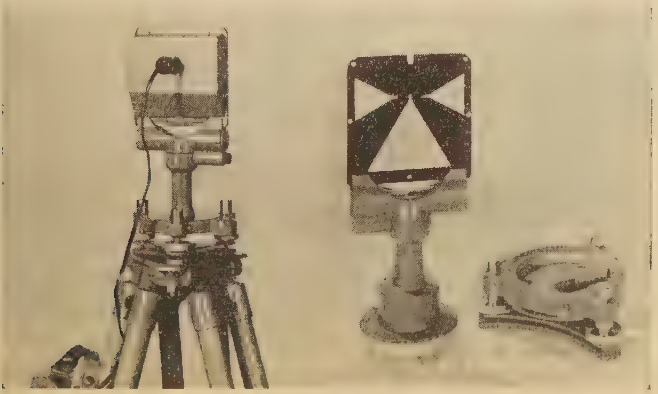

FIG. 1-14. TRIPOD TARGETS.
(Courtesy, Wild Heerbrugg Instruments, Inc.)

plumb the target, and an optical device is provided to center it over the point. This type of target can be used for triangulation as well as traversing where lines are of limited length.

1-20. Observing Towers. In flat country towers are necessary because of the earth's curvature, and to see over woods or hills.

Such towers usually consist of an inner tripod to support the instrument and an outer stand for the observer (Fig. 1-15). The

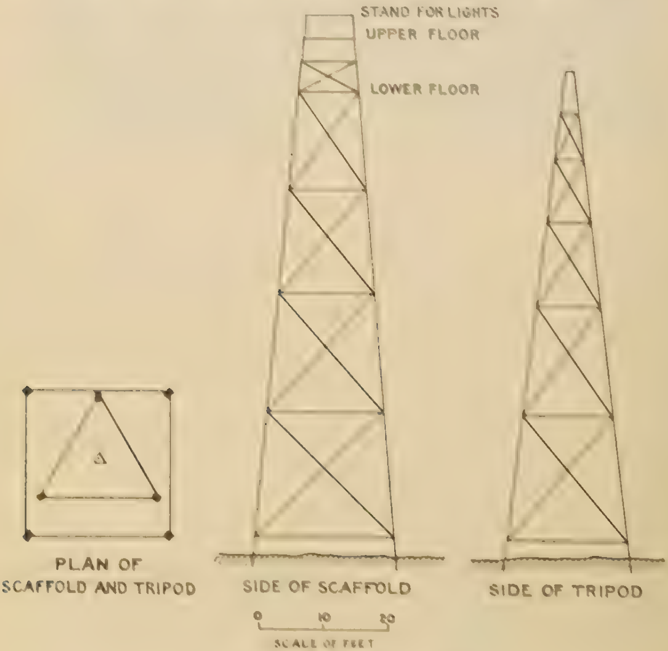

Fig. 1-15. Observing Tower.

two are built entirely disconnected so that the observer will not disturb the instrument as he moves about on the observing stand. If the tower is high the outer structure is secured by wire guys.

Since 1927 the Coast Survey has used steel towers, of the type designed by Mr. J. S. Bilby. They are constructed of angle irons similar to those used in high windmills. Parts are replaceable and the tower may be taken down and erected as many times as desired. See Fig. 1-16.

Fig. 1-16. Bilby Steel Tower for Triangulation. Top Unit on Larger Scale at Right.

A theodolite in a tower is usually centered over the ground point by means of a vertical collimator. In one type the collimator is centered over the station mark by means of a plumb bob or small vertical telescope, and the upward sight taken through a horizontal telescope using a prism to direct the sight vertically. The center of the base of the theodolite is placed at the intersection of the collimator telescope cross hairs. In another type (Fig. 1-17), both the station mark below and the center of the instrument above can be viewed in one telescope at the same time by a prism system, and brought into vertical alignment.

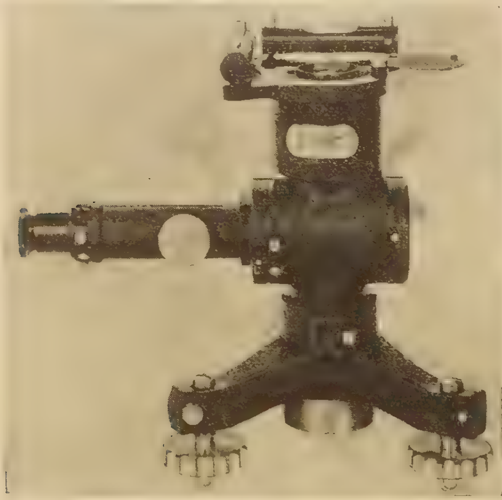

FIG. 1-17. VERTICAL COLLIMATOR.
(Courtesy, C. L. Berger & Sons.)

1-21. Earth's Curvature. Obstruction to Line of Sight. The offset from the tangent to the curve, due to curvature of the earth and refraction combined, is about 0.57 feet for a point a mile away, and varies as the square of the distance. (Offsets are given in Table I, p. 490.)

This formula may be derived as follows: If a sight is taken to a distant point, that point will appear lower than it really is on account of the earth's curvature, and it will appear slightly

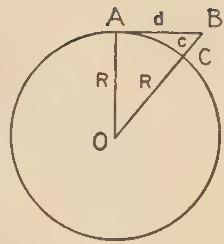

FIG. 1-18.

higher than it really is on account of the effect of atmospheric refraction. The combined correction (curvature and refraction) must be applied in order to obtain the correct result.

The curvature effect may be seen from Fig. 1-18. Point B appears to be at the same height as A because it is on the horizon of A. Actually B is above sea-level by the distance BC; consequently BC is the correction to the height for the effect of curvature for a distance AC or AB. If $OA = OC = R$, $BC = c$ and $AB = d$ then

$$R^2 + d^2 = (R + c)^2 = R^2 + 2Rc + c^2$$

and

$$c = \frac{d^2}{2R + c} = \frac{d^2}{2R} \text{ (approx.)} = \frac{d^2}{D} \text{ (approx.)} \qquad (1\text{-}1)$$

That is, the effect of curvature on the apparent elevation is equal to the square of the distance divided by the earth's diameter. To allow for refraction we find that we must diminish this result by about one-seventh part, the total correction for curvature and refraction being

$$(c\&r) = \frac{(\text{distance})^2}{\text{diam}}(1 - 2m) = \frac{d^2}{D} \times 0.86 \qquad (1\text{-}2)$$

in which d and D are in feet, and m is a coefficient equal to about 0.07, but varying slightly with atmospheric conditions. Using this value for m and substituting the earth's diameter (41,850,000 ft. approx.) we obtain corr. for curv. and refr. (in feet) = $0.574 \times M^2$ where M is distance in miles. Or, reversing the equation

$$M = 1.32\sqrt{\text{ht. (feet)}} \qquad (1\text{-}3)$$

In perfectly level country if towers 57 feet high are erected at each end of a line 20 miles long, the line of sight between the tops of the two towers will just clear the intervening level surface. In order to avoid the great atmospheric refraction near the surface,

as well as obstructions to the line of sight, it would be necessary in practice to make the towers several feet higher than this.

If a sight is desired from station A to a higher station B and an obstruction C is in line and interferes with the sight, the question of the height of towers to be erected on A and B to overcome this obstacle may be investigated as follows.

Let h = height of line at C, in feet.
h_1 = height of A, in feet.
h_2 = height of B, in feet.
d_1 = distance AC, in miles.
d_2 = distance CB, in miles.

Then
$$h = h_1 + (h_2 - h_1)\frac{d_1}{d_1 + d_2} - 0.574\, d_1 d_2$$

The difference between this value of h and the height of hill C is the distance by which the sight line AB passes under the summit of C. From this it is easy to estimate the height of towers necessary to erect on A and B to clear C. It is advisable to allow liberally for the clearance of the line above the ground. Grazing sights are subject to large errors caused by irregular refraction of the air.

Example. Hill C is on a line between hills A and B. Hill C is 9 miles from A and hill B is 6 miles from C. The elevations of the hills are: A 450 ft., B 500 ft., and C 450 ft.

Determine the following:

(a) Whether B is visible from A.
(b) Height of towers (to instrument) at A and B so that line of sight will clear ground by 10 feet at C.
(c) Height of tower on A so that line of sight will clear the ground at C by 10 feet if no tower on B.
(d) Height of tower on B to meet conditions in (c) if no tower on A.

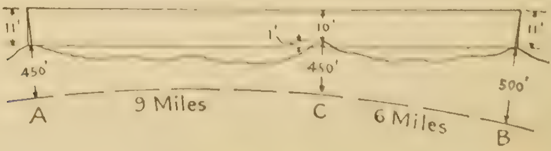

FIG. 1-19.

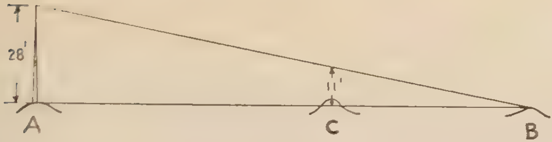

FIG. 1-20.

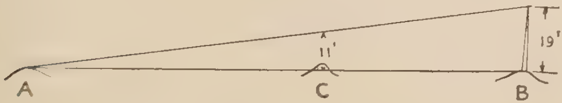

FIG. 1-21.

(a) $h_1 = 450$ feet, $h_2 = 500$ feet, $d_1 = 9$ miles, $d_2 = 6$ miles.

$$h = 450 + (500 - 450)\frac{9}{9+6} - 0.574 \times 9 \times 6$$

$$= 450 + 30 - 31 = 449 \text{ feet.}$$

Therefore, the line of sight AB fails to clear C by 1 foot.

(b) Towers on A and B, each 11 feet high, will insure 10-foot clearance at C. (Fig. 1-19.)

(c) With no tower on B, height of tower at A must be

$$\tfrac{15}{6} \times 11 = 27.5 \text{ feet (Fig. 1-20)}$$

(d) With no tower on A, height of tower at B must be

$$\tfrac{15}{9} \times 11 = 18.3 \text{ feet (Fig. 1-21)}$$

1-22. Instruments for Measuring Base-Lines. Various forms of bar apparatus have been used for base-line work of the highest accuracy, and until about 1885 these furnished the only means of making such measurements. Experiments with steel tapes showed that if the proper precautions were taken to determine the actual length of tape and the variations due to temperature these would give all the accuracy necessary for the most accurate base-lines. From about 1890 to 1907 the steel tape apparatus was used on all the principal base-lines. The difficulty of determining the temperature of the steel made it necessary, however, to make the measurements at night. In 1906 tests were made on the use of *invar* tapes for base measurement. These tapes are made of an alloy of nickel and steel having a low coefficient of expansion. The experiments showed that measurements with

these tapes were sufficiently accurate for base-line work and furthermore that the work could be done in bright sunlight without too great an error due to temperature.

1-23. **The Steel Tape Apparatus.** The steel tape apparatus, formerly used on the principal base lines and still used on work of lesser importance, consists of a 50-meter or a 100-meter steel tape supported on stakes or tripods every 25 meters. The tension is given by means of a spring balance secured to a vertical bar held firmly by an assistant as shown in Fig. 1-22. The temperature is read on thermometers fastened to the tape. The intermediate supports are lined up and brought to grade by sighting along the stakes with a field glass.

1-24. **The Invar Tape Apparatus.** The invar tape apparatus as used by the U. S. Coast and Geodetic Survey consists of a 50-meter tape about $6^{mm} \times 0.5^{mm}$ in cross section. The coefficients of expansion of these tapes are from about $\frac{1}{25}$th to $\frac{1}{30}$th that of steel.* They closely resemble steel in appearance but bend more easily and must be wound on large reels, not less than 16 inches in diameter, in order to avoid permanent bends. The tapes are 53 meters in length, the graduations being usually ruled directly on the invar.

The tension used is 15 kilograms, applied with a spring balance as shown in Fig. 1-22. This balance reads directly to 25 grams and to about 10 grams by estimation. The tape temperature is obtained by means of two thermometers fastened to the tape.

These tapes are standardized at the Bureau of Standards, Washington, on a 50-meter comparator. The length of the comparator (between microscopes) is established by measuring the distance with a 5-meter steel bar immersed in melting ice. The length of the 5-meter bar is obtained by comparing it with the standard meter bar (known as M 21) on a 5-meter comparator in the same room.

1-25. **Measuring the Base with a Tape.** Before a base can be measured, it is necessary first to clear the line of all obstructions such as trees, tall grass, hummocks of earth, etc. Then measur-

* Invar tapes have been made with coefficients nearly zero, or even negative, but it is found that these are not always reliable and should be used with caution.

FIG. 1-22. U. S. COAST AND GEODETIC SURVEY BASE-LINE MEASUREMENT.

ing stakes about $4'' \times 4''$ are set carefully in line 50 meters apart. The elevation of the top of each stake is then determined by careful leveling. In making the final measurements, the zero mark of the tape is placed vertically above the initial mark of the base by means of a theodolite or a special plumb bob. It may also be done by means of a vertical collimator (Fig. 1-17). The ends of base-lines are marked by bronze markers set in concrete or stone. The middle point of the tape is supported by means of a light stake lined in by eye for both line and grade. In light winds it is sometimes necessary to use two additional supports at the quarter points of the tape. On top of the measuring stake is nailed a piece of copper with one edge on line on which reference marks can be scratched. The tension (15 kilograms) is given by means of a spring balance at the forward end of the tape. When the tension has been applied the zero mark is adjusted to position by means of a bar which holds the tape in position. The end of the tape may be slid up or down on the bar, and may be moved right or left, forward or backward, by moving the upper end of the bar. When the zero is in correct position, the spring balance reads correct tension, and alignment and grade are correct, a scratch is made on the copper on post No. 1 at the 50-meter graduation. The two thermometers are then read, and all the measurements are recorded. The tape is then carried forward and the process repeated. If any stake is so placed that the 50-meter mark does not fall on the copper strip, the rear end can be set ahead or back a few centimeters and the length of this set-up or set-back measured and recorded. By the above process a speed of about two kilometers (of completed base-line) per hour can be attained.

1-26. Use of Tripods in Taping. For second or third order base-line or precise traverse measurements, standardized 100- or 200-ft. steel tapes may be used, and tripods, known as taping or chaining bucks, may be employed at intermediate points on line. For this purpose a special taping head has been developed which screws on to the standard Johnson-type plane-table tripod. The head (Fig. 1-23) consists of a circular plate of dural metal, 5 inches in diameter and $\frac{3}{4}$ inches in thickness. In the center a thread is

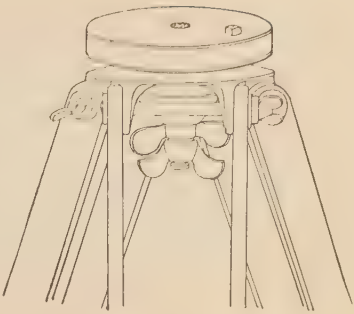

FIG. 1-23. TAPING BUCK.

tapped to fit the screw in the top of the Johnson tripod. At a distance of 2 inches from the center of the head, a round-head cap screw is set in the plate. In the center of the top of this screw a fine hole is drilled. This hole serves as a reference mark for taping. The tripods are set approximately on line and approximately one tape-length apart. Tapes with an extra foot graduated in advance of zero are preferred so that the measurements may be either a little over or a little under one tape-length. As slope measurements are to be made, the circular head is first tilted to conform to the approximate slope of the taping and is held in this position by means of the top clamp of the Johnson motion. The hole in the top of the eccentric cap screw is then accurately placed in line by turning the circular head. The lower clamp is then tightened. The measurements are taken between reference holes in the heads of adjacent tripods which have been set up along the line.

Tripods consisting of triangular metal frames are also used for taping lines where more precision is desired than can be obtained by ordinary slope-taping or by pumbing.

1-27. Base-Line Precision. The base-line is usually divided into sections about one kilometer in length. Each section is measured twice, once forward and once backward. If the two results differ by less than $10^{mm}\sqrt{K}$ (K being the length in kilometers) the mean value is adopted as correct. If the discrepancy is greater then more measurements are made. The final estimated error of the base should not be greater than one part in a million. In practice a higher precision is usually reached on the first trial. See also Arts. 1-7 and 10-10.

1-28. Corrections to Base-Line Measurements. Correction for Slope. In order to reduce the inclined distance to the corresponding horizontal distance a correction must be subtracted.

(Fig. 1-24.) Since the elevations of the stakes are known the difference in height h is known. We may therefore make this reduction by the well-known formula

$$\text{Horizontal Distance} = L - \frac{h^2}{2L} \qquad (1\text{-}4)$$

where L is the length of the section being corrected. (See footnote, Vol. I, p. 18.) If the grade exceeds about 3 per cent, the correction should be made by means of the more accurate expression

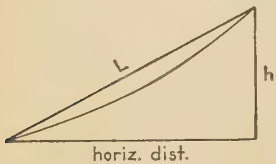

FIG. 1-24. SLOPE CORRECTION.

$$\left(\frac{h^2}{2L} + \frac{h^4}{8L^3}\right)$$

Sometimes the slope is determined in degrees by means of a clinometer. In this event the correction may be computed from the formula $-2L \sin \frac{\alpha}{2}$, α being the angle of slope. Tables for making this correction will be found in Coast and Geodetic Survey Special Publication No. 247.

1-29. Temperature Correction. The correction for temperature is made by adding to the measured length the quantity $L \times k \times (t_1^\circ - t_0^\circ)$, where L is the measured length; k, the coefficient of expansion for 1°; t_0°, the temperature at which the tape is standardized; and t_1°, the observed temperature. The coefficient must, of course, be that corresponding to the scale of the thermometer used in taking the temperature (Centigrade or Fahrenheit).

1-30. Standardization of Invar Tape. A certificate of standardization for an invar tape is shown on the following page; it includes the length of the tape when supported at three points and at five points. The length when supported horizontally throughout is not observed but is calculated from the above two observations. The thermal expansion in millimeters per 50 meters for each degree centigrade is given as part of the certificate. Coefficients of expansion for invar tapes vary widely, although they are never more than a small fraction of the coefficient for a steel tape.

NBS 584
(Rev. 5/13/48)

UNITED STATES DEPARTMENT OF COMMERCE
WASHINGTON

National Bureau of Standards
Certificate

FOR

50-METER IRON-NICKEL ALLOY TAPE

(Low Expansion Coefficient)

NBS No. 9115

Maker: Keuffel & Esser Co.
Maker's No. 6460

SUBMITTED BY

United States Coast & Geodetic Survey
Washington 25, D. C.

This tape has been compared with the standards of the United States under a horizontal tension of 15 kilograms. The interval (0 to 50 meters) has the following lengths at 25 °C under the conditions given below:

Supported at the 0-, 25-, and 50-meter points: 50.00025 meters.

Supported at the 0-, 12.5-, 37.5-, and 50-meter points, with the 12.5- and 37.5-meter points 6 inches above the plane of the 0- and 50-meter supports: 49.99990 meters.

The weight per meter of this tape, previously determined, is 26.0grams.

Thermometers weighing 45 grams were attached at points 1 meter inside the terminal marks.

These comparisons were made on the section of the lines near the end on the edge of the tape marked with a small "x" or "v" or dots near the graduation.

The values for the lengths are not in error by more than 1 part in 500,000 ; the probable error does not exceed 1 part in 1,500.000.

The values for the lengths were obtained from measurements made at 24.2 °C, and in reducing to 25 °C, the thermal expansion of +0.023 millimeter per 50 meters per degree centigrade was used.

For the Director *by*

B. L. Page

Chief, Length Section,
Metrology Division.

Test No. 2.4/G-27805
Test completed: November 7, 1960

U. S. GOVERNMENT PRINTING OFFICE 16—55418-1

If much base-line measurement is to be done it is customary to have the tape standardized before the beginning and at the close of the season.

1-31. Reduction to Sea Level. In order that the lines of the triangulation shall refer to the surface of the earth at sea level their lengths will all require a slight reduction. Since all other lengths are derived from that of the base by calculation, all the lines will be reduced to sea level without further correction if the base itself is first reduced. The approximate formula ordinarily used for this reduction is $-B\frac{h}{R}$, which may be readily deduced from the fact that in a sector of a given angle, arcs are proportional to the radii. (Fig. 1-25.) B is the measured length, h the altitude above the sea level obtained by leveling, and R the radius of a section of the earth at the point in question. [An average value for log R (in feet) = 7.32068; log R (in meters) = 6.80470.]

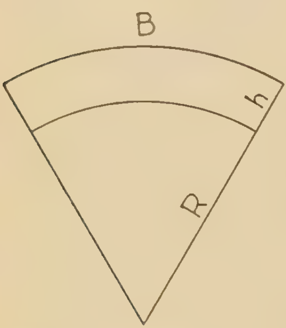

FIG. 1-25. REDUCTION TO SEA LEVEL.

If the length or the mean elevation of the base justifies the use of a more accurate formula, the correction is calculated by means of the following expression.

$$\text{Corr.} = -B\left(\frac{h}{R} - \frac{h^2}{R^2}\right) \quad (1\text{-}5)$$

Approximate reduction factors are given in Art. 1-65.

1-32. Correction for Sag. The tape hangs between supports in a curve known as the **catenary.** For the purpose of computing the small difference in length between the curve and the chord this curve may be regarded as a **parabola** (ACB, Fig. 1-26) having a vertical axis. The difference in length between the curve and the chord between consecutive supports is given by the expression

$$\frac{w^2l^3}{24t^2} \quad (1\text{-}6)$$

in which l is the length of a span, w is the weight of a unit length of tape, and t is the tension; w and t must be in the same units.

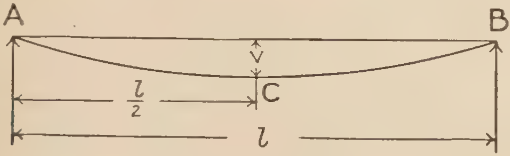

FIG. 1-26. CORRECTION FOR SAG.

If there are n spans then the shortening of the total distance is

$$\frac{nl}{24}\left(\frac{wl}{t}\right)^2$$

and since the whole tape length $L = nl$, we have

$$\frac{L}{24}\left(\frac{wl}{t}\right)^2 \tag{1-7}$$

This expression may be derived as follows. Considering the forces acting on a section of the tape halfway between supports, and taking moments about one of the supports we have

$$\frac{wl}{2}\cdot\frac{l}{4} = tv$$

v being the vertical sag of the middle point of the section of tape below the line of supports. Therefore,

$$v = \frac{wl^2}{8t} \tag{1-8}$$

The parabola ABC (vertex at C) has the equation

$$x^2 = \frac{l^2}{4v}\cdot y$$

If we express the length of the parabola as a converging series, neglecting terms of 3d and higher powers we obtain

$$P = l\left(1 + \frac{8}{3}\cdot\frac{v^2}{l^2} + \cdots\right) \tag{1-9}$$

Substituting in (1-9) the value of v from (1-8) and computing $P - l$, the shortening due to sag, we find

$$P - l = \frac{l}{24}\left(\frac{wl}{t}\right)^2$$

or, since $nl = L$, the correction for sag for the whole tape, C_s, is given by

$$C_s = \frac{L}{24}\left(\frac{wl}{t}\right)^2 \tag{1-10}$$

If there is no intermediate support, then $L = l$, and if we place $W = wl$, the weight of the whole tape, then

$$C_s = \frac{LW^2}{24t^2} \tag{1-11}$$

1-33. Tension. The modulus of elasticity of the tape, E, is equal to the stress per unit of area, $t \div S$, divided by the elongation per unit of length, $a \div L$, that is

$$E = \frac{Lt}{Sa}$$

The increase, a, in the length of the tape, above its unstretched length L, when a tension t is applied, may therefore be computed by the equation

$$C_p = \frac{Lt}{SE} \tag{1-12}$$

in which S is the cross-sectional area of the tape, and E is the modulus of elasticity in the same units as L and w, and C_p is the correction for pull. If we write this equation for the two tensions t_1 and t_2, and take the difference, then

$$C_p = \frac{L(t_2 - t_1)}{SE} \tag{1-13}$$

C_p being the increase in length. The modulus of elasticity of steel tapes may ordinarily be taken as about 28,000,000 pounds per

square inch. For invar tapes it is about 22,000,000. It is difficult to measure S directly, so it is preferable to compute the quantity SE by the equation

$$SE = \frac{L(t_2 - t_1)}{\Delta L}$$

based on an actual test. ΔL, the observed elongation, should be measured carefully. A difference in tension of at least 15 pounds should be used in this determination.

If small variations in tension (Δt) occur during fieldwork, their effect may be calculated by the equation

$$\Delta L = \frac{L \cdot \Delta t}{S \cdot E} + \frac{L}{12}\left(\frac{wl}{t}\right)^2 \cdot \frac{\Delta t}{t} \qquad (1\text{-}14)$$

obtained by differentiating (1-10) and (1-12). Results obtained from direct tests are usually considered more reliable than those found by this formula, and are to be preferred.

1-34. Normal Tension. If we equate (1-10) and (1-13), calling the actual tension t_n (normal tension) and t_1 the tension at which the tape was standardized, we have

$$\frac{L}{24}\left(\frac{wl}{t_n}\right)^2 = \frac{L(t_n - t_1)}{SE} \qquad (1\text{-}15)$$

The solution of this equation for t_n will give that tension for which the sag and tension corrections exactly balance each other. If the tape is supported at the ends only, then $wl = W$. Cancelling L we have

$$t_n^2(t_n - t_1) = \frac{W^2SE}{24}$$

or

$$t_n\sqrt{t_n - t_1} = W\sqrt{\frac{SE}{24}} \qquad (1\text{-}16)$$

This equation may be solved for t_n by any method of solving a cubic equation, or by "trial and error." If solved by means of the slide rule, the value of the constant is first set by the runner

on the bottom (D) scale, and then by trial a position of the slide is found such that t_n is at the C index on the bottom (D) scale when $(t_n - t_1)$ is at runner on the upper (B) scale of slide.

Equations (1-10), (1-11) and (1-13) will give results that usually are reliable to less than 0.001 foot, and therefore are sufficiently accurate for ordinary work. They are often not considered as accurate enough for first or second order work. This will be the case if the assumptions underlying the formula and the conditions of the fieldwork are not in exact agreement. The surest way to determine the length of the tape is to have it tested at the Bureau of Standards with the same tension and the same manner of support that are to be used in the fieldwork. The normal tension for heavy tapes is often so great (sometimes as much as 50 pounds) as to render the method impractical.

1-35. Electronic Distance Measurement. Several types of electronic devices for measuring distances have been developed. Early models, designed for military and navigational uses, produced results which were too approximate for surveying purposes. Improvements and refinements have since produced instruments which permit a degree of accuracy suitable for geodetic and plane surveying, especially for measuring long distances over rough terrain. These instruments operate by transmitting beams of electromagnetic waves, visible light or micro-waves, between instruments located at the ends of the line being measured. The beams are modulated at precisely controlled frequencies so that by comparing the transmitted and received beams, at one or both ends of the line, the distance can be accurately determined. Accurate distance determination depends upon accurate knowledge of the velocity of the particular electromagnetic radiation being used, involving corrections for temperature, atmospheric pressure, and relative humidity. These instruments generally require that the ends of the line be intervisible and occupiable. The distance actually determined is the slope length of the line.

Two electronic distance measuring instruments in common use are the Geodimeter and the Tellurometer; other instruments, such as the Cubic Electrotape, are similar to these in principle. The units at each end of the line may or may not be identical, depending on the particular system. The electronics of these

instruments and the details of their operation are too complex to be described fully in this text. A detailed description of the instruments and their use will be found in Tellurometer Manual and Geodimeter Manual, U. S. Coast and Geodetic Survey, Publications 62-1 and 62-2, respectively.*

To provide the reader with some understanding of electronic distance measurement concept, the technique of the Geodimeter system will be described in a simplified form.

1-36. The Geodimeter. The Geodimeter system consists of an electronic-optical instrumentation unit at one end of the line

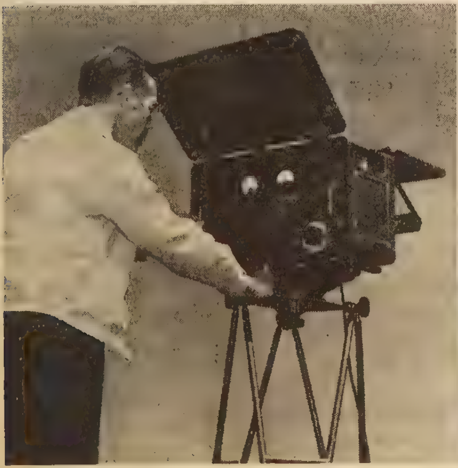

FIG. 1-27a. GEODIMETER UNIT MODEL 3.

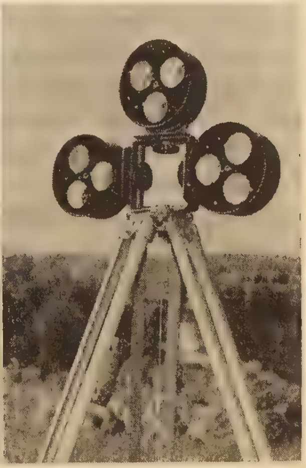

FIG. 1-27b. RETRO-DIRECTIVE PRISMS.

(Courtesy, The Geodimeter Company)

(Fig. 1-27a) and an optical instrumentation unit at the other end of the line (Fig. 1-27b). The electronic-optical unit, hereafter referred to as the Geodimeter, is set up and oriented over one of the end points of the line from which it transmits a highly collimated, electrically modulated light beam to the optical unit set over the other end of the line. The optical unit consists of

* Also "Electronic Surveying and Mapping" by Simo Laurila, The Ohio State University, 1960.

reflecting or retro-directive elements which return the light beam to the Geodimeter. Retro-directive prisms, such as shown in Fig. 1-27b, have the property of returning incident light to its source. For short distances flat mirrors may be effectively used, however, careful angular orientation is required. The light beam returned to the Geodimeter is then compared with the modulation of the beam being transmitted. From this comparison, for two or three separately used modulating frequencies, the distance may be accurately determined.

A light beam is formed in the Geodimeter from an incandescent light source, and passed through an electronic shutter consisting of a pair of crossed polaroids and a Kerr cell. The operation of the electronic shutter is controlled by the application of an electric field across the Kerr cell. Hence, the intensity of the transmitted beam may be controlled or modulated in any desired fashion. The modulated light beam transmitted by the Geodimeter may be visualized as a number of tapelengths, placed end to end, and moving through space at the velocity of light. These tapelengths are emitted at a rate equal to the frequency of the Kerr cell modulation. Hence, the length of these tapelengths is equal to the velocity of the light divided by the modulating frequency, i.e., the modulation wavelength λ. The beam of tapelengths is

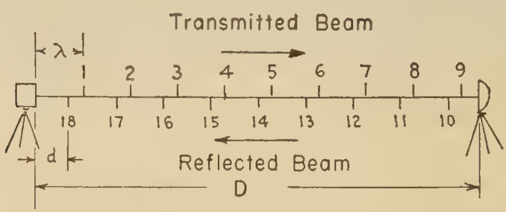

Fig. 1-28.

returned to the Geodimeter by the reflector located at the other end of the line being measured.

A schematic representation of these tapelengths is shown in Fig. 1-28. Consider an instant in time when the Geodimeter has just emitted a full tapelength, as indicated in Fig. 1-28. In this illustration the transmitted and reflected beams contain 18 full

tapelengths over the pathlength of $2D - d$, where D is actual distance being measured and d is the remaining fraction of a tapelength. Note that the sum of the parts of a tapelength in the transmitted and reflected beams at the reflector is equal to a full tapelength. Hence, $D = \frac{1}{2}(n\lambda + d)$, where n is the total number of full tapelengths of length λ in both transmitted and reflected beams. The circuitry which compares the transmitted and reflected beams at the Geodimeter *determines only d*. The Geodimeter has no way to determine the number of full tapelengths. Therefore, the number of full tapelengths must be determined independently of the direct measurements made by the Geodimeter.

This can be done if the distance $2D$ is known in advance to within $\pm\frac{1}{2}\lambda$, or equivalently, if the length D of line being measured can be obtained from a map by other means to within $\pm\frac{1}{4}\lambda$. At a currently used modulating frequency of 10 megacycles the tapelength is approximately 30 meters in length. This would require that the distance to be known to ± 7.5 meters, and would limit the use of the instrument, since lines are not usually known this close in advance.

1-37. Determining Number of Full Wavelengths. A practical solution to the problem of obtaining the number of full tapelengths is achieved by providing two or three, separately used, modulating frequencies. These will be referred to as the primary frequency and the secondary frequency. The length of the secondary frequency tapelength will be denoted by λ', and the length of the fractional part of the remaining tapelength when the secondary frequency is being used will be denoted by d'. If the secondary frequency is such that $100\lambda = 101\lambda'$, then it is necessary to know D to within $\pm\frac{1}{4}(100\lambda) = \pm 25\lambda$ in advance. For the 10 megacycle primary modulating frequency, the line being measured would have to be known to within 750 meters. The proof of this is as follows.

Fig. 1-29a shows how the primary beam tapelengths compare with the secondary beam tapelengths in space. It is quite clear that the points 100λ, 200λ, 300λ, 400λ, etc. are the same points as $101\lambda'$, $202\lambda'$, $303\lambda'$, $404\lambda'$, etc. Now suppose that Geodimeter measurements have been made using both the primary and secondary frequencies obtaining values of d and d' and that $n = 145$,

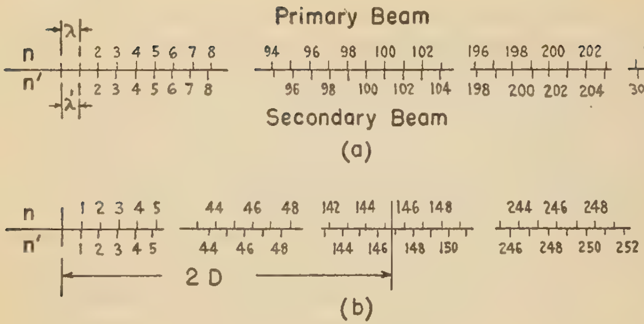

FIG. 1-29.

hence $2D = 145 + d$. Visualize in the schematic diagram of Fig. 1-29b that the transmitted and reflected beams, for both frequencies, are shown as a continuous straight line. The Geodimeter transmitting elements are located at the left end of the $2D$ line, the receiving elements are located at the right end of $2D$ line, and the reflector is located at the midpoint. The distance $2D$, indicated on the diagram in Fig. 1-29b, is equal to $146\lambda' + d'$ as well as $145\lambda + d$. However, now notice that $45\lambda + d$ is equal to $45\lambda' + d'$, and that $245\lambda + d$, is likewise equal to $247\lambda' + d'$, and so on at $345\lambda + d$, $445\lambda + d$, etc., since these are corresponding points in the identical 100λ long sections of the two beams. It is also apparent that $d' - d$ is constant between $n = 145$ and $n' = 147$ and that this is the only region between $n = 100$ and $n = 200$ having this value of $d' - d$; but, in the corresponding regions of the other 100λ long sections of the beam the value of $d' - d$ is the same as between $n = 145$ and $n' = 147$. Therefore, for a pair of values of d and d' there exists a unique region, defined by adjacent full tapelength points of each frequency, in every 100λ long section of the light path, which may contain the end point of the line. Therefore, if the light path length is known to within $\pm\frac{1}{2}(100\lambda) = \pm 50\lambda$ in advance, then the actual distance may be determined. For instruments using 10 megacycle modulation the length of the line being measured must therefore be known to within 750 meters ($\frac{1}{4} \times 100\lambda$) in advance.

There are two distinct and different values of $d' - d$ within each primary tapelength, one of which is positive and the other negative, resulting in a total of 200 distinct possible values of $d' - d$. These values are the same in every set of 100 primary tapelengths. For each one of these values the number of full tapelengths from the last integral number of 100 primary tapelengths can be determined. This is illustrated in Fig. 1-30 which is an enlargement of the portion of Fig. 1-29b between primary tapelengths 144 and 146. The number of primary tapelengths in excess of an even 100 is denoted by n_e, and the number of secondary tapelengths in excess of whole multiples of 101 is denoted by n_e'.

Two situations are demonstrated in Fig. 1-30. The first (Fig. 1-30a) is similar to that in Fig. 1-29b where $d < d'$ and

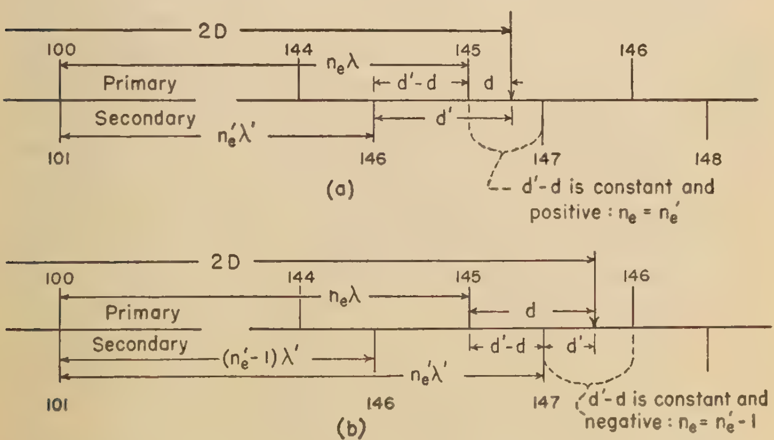

FIG. 1-30.

$n_e = n_e'$. Also, $n_e\lambda + d = n_e'\lambda' + d'$ or $n_e\lambda + d = n_e\lambda' + d'$; hence $n_e = (d - d')/\lambda' - \lambda$.

Substituting $\lambda/1.01$ for λ', the expression for n_e becomes $(d - d')/(\lambda/1.01 - \lambda)$ which simplifies to

$$n_e = \frac{101(d' - d)}{\lambda}$$

In the second case (Fig. 1-30b) $d > d'$ and $n_e = n_e' - 1$. Then $n_e' = n_e + 1$ and $n_e\lambda + d = (n_e + 1)\lambda' + d'$. Substituting $\lambda/1.01$ for λ' and simplifying the expression reduces to

$$n_e = 101 \frac{d' - d}{\lambda} + 100$$

Note that in the first case (Fig. 1-30a) $d' - d$ is positive and constant between primary tapelength 145 and secondary tapelength 147, and that in the second case (Fig. 1-30b), $d' - d$ is negative and constant between secondary tapelength 147 and primary tapelength 146.

When $d' - d$ is negative the second expression must be used, since n_e must be a positive whole number.

To clarify the above procedure consider the following example. From U.S.G.S. maps, or other sources, it is found that the actual slope length of a line to be measured is about $5\frac{3}{4}$ miles which is about 9,250 meters. Using a primary modulating frequency of 10 megacycles per second (tapelengths of 30 meters), there are about 308 tapelengths in the line. Therefore, the actual length of the line must be between 283 and 333 tapelengths (308 ± 25), and the light path must contain between 566 and 666 full tapelengths. The Geodimeter measurements obtained are $d = 21.56$ meters and $d' = 20.11$ meters, hence $d' - d = 20.11 - 21.56 = -1.45$ meters. Now since $d' - d$ is negative the expression $101(d' - d)/\lambda + 100$ must be used for n_e. Therefore, $n_e = 101(-1.45/\lambda) + 100 = -4.9 + 100 = 95.1$. Since n_e must be an integer it is clear that $n_e = 95$ and that n is 595. The actual distance may now be computed from $D = \frac{1}{2}(n\lambda + d)$, which becomes $\frac{1}{2}(595 \times 30 + 21.56) = 8{,}935.78$ meters. In practice, slope corrections must be made and the exact tapelength length determined for the prevailing meterological conditions.

Several models of the Geodimeter have two secondary frequencies. If the second secondary frequency is closer to the primary frequency than the first one, the tolerance to which the distance need be known in advance is increased. For example, if the second secondary frequency is such that $\lambda/\lambda' = 1.005$, then the distance needs only to be known within 1500 meters

rather than the 750 meters required for $\lambda/\lambda' = 1.01$. However, it should be noted that the first secondary frequency must still be used to guarantee high accuracy.

1-38. Use of the Geodimeter. There are currently four models of the Geodimeter. The first two models are essentially the same with the second having some improved circuitry and a third frequency. The Geodimeter Models 2, 3 and 4 each have their own area of application because of their individual range and accuracy characteristics. The Model 2 Geodimeter is suitable for the most precise geodetic measurements having a range of up to 30 miles with an average error of 0.4 inches ± one millionth of the distance being measured. The complete instrument weighs about 225 pounds and requires about an hour to set up and make a single set of measurements. The Model 3 Geodimeter has a range of about 20 miles and an average error of 2 inches ± two millionths of the distance being measured. This instrument weighs about 80 pounds and required about a half an hour to set up and make a complete set of measurements. The Model 4 Geodimeter is suitable for engineering surveys and traverse measurements, having a range of up to 5 miles, with an average error of 0.5 of an inch ± five millionths of the distance being measured. This model weighs about 75 pounds and requires about 20 minutes to set up and make a complete set of measurements. All of the instruments are powered by a gasoline engine driven generator supplying 110 or 220 volts at 50 or 60 cycles per second. Models 3 and 4 also operate from a battery with a converter.

The Geodimeter should be used at night, however, using special procedures, satisfastory results have been obtained during day time. Satisfactory results have also been obtained with the line partially obscured by brush and light rain; wind is no problem. Reflected or stray light from other sources, especially car headlights, very close to the line, may cause considerable difficulty. Radio telephone units are commonly employed for communication between the Geodimeter operator and reflex tender.

1-39. The Tellurometer. The Tellurometer distance measuring system utilizes micro-wave radiation as the measuring

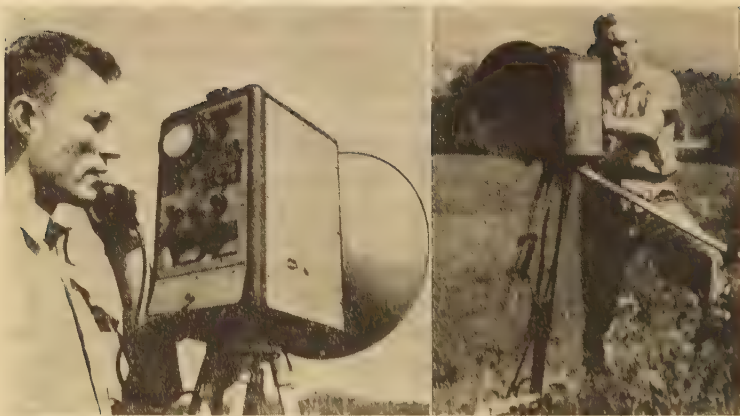

FIG. 1-31. TELLUROMETER MODEL MRA 1 — MASTER AND REMOTE STATIONS.
(Courtesy, Tellurometer, Inc.)

medium. Figure 1-31 shows two units of the Model MRA-1 Tellurometer. The master unit is located at one end of the line and the remote unit is located at the other end. Each unit transmits a modulated beam which is independent of the beam being received from the other unit. This beam, as described for the Geodimeter, may be visualized as constructing a series of tapelengths placed end to end traveling through space at the velocity of the micro waves. The beam being received at each end of the line is compared with that being transmitted. The result of this comparison at the remote station is relayed to the master station. The two comparisons are then combined to provide the measurement data. A circle is generated on the oscilloscope face of the master unit. The circumference of the circle may be considered to represent a full tapelength in space. The fractional part of a full tapelength in space is displayed by a break or gap in the circle generated on the oscilloscope. The face of the oscilloscope is graduated into 100 parts, therefore, the decimal part of a full tapelength may be read from the oscilloscope. As with the Geodimeter several modulating frequencies are used which construct different tapelength lengths in space.

The Tellurometer is capable of measuring distances from 500 feet to 40 miles. Under ideal conditions, distances of 90 miles

have been measured. The uncertainty of the meterological conditions between the instruments may be a source of considerable error when measuring over the longer distances. Measurements may be successfully made in haze, mist and light rain. Measurements may also be made through light brush and trees but there should be no intervening terrain. Some difficulty may be encountered with lines over large flat surfaces, such as bodies of water and highway surfaces, which reflect the micro-wave radiation creating pathlengths greater than that between the instruments. The use of towers frequently reduces the effect of reflected waves. The modulating frequencies provided in the Tellurometer are such that the distance being measured must be known in advance to within only ±5 miles. A complete set of measurements may be made in about one half an hour including setting up and aligning the instruments. The probable error of Model MRA-1 (Fig. 1-31) is 2 inches ± three millionths of the distance being measured. Each unit of the MRA-1 weighs about 50 pounds including carrying case and power pack.

A smaller Tellurometer (Model MRA-2) is shown in Fig. 1-32. Each unit of this model weighs about 30 pounds and does not

FIG. 1-32. TELLUROMETER MODEL MRA-2.
(Courtesy, Tellurometer, Inc.)

have a carrying case or separate power pack. The MRA-2 units are identical and may be used as either master or remote units. The Model MRA-1 units are not identical. The probable error of the Model MRA-2 is 2 inches ± three millionths of the distance being measured. Each unit of the MRA-2 weighs about 30 pounds and is smaller than the MRA-1.

Both models are powered by an automobile type storage battery. Voice communication is provided between the master and remote units, except while measurements are being made.

The Tellurometer and the Geodimeter are both very complex electronic instruments, and, as in any measuring instrument, there always exists potential sources of systematic error (Art. 10-3). Special devices are provided and procedures specified to reduce or eliminate the effects of these error sources. The procedure is lengthy, but not necessarily difficult. Meterological measurements for correction purposes are always made at the beginning and end of the measurement sequences at both ends of the line.

1-40. Instruments for Measuring Horizontal Angles. There are two types of instruments used in triangulation work for measuring horizontal angles, the *repeating theodolite* and the *direction instrument.* The former is constructed on the same principles as the ordinary transit and read by means of verniers. The direction instrument is used for reading the directions of all the stations, the circle remaining unchanged in position during one whole set of observations. Micrometer microscopes or optical micrometers are used instead of verniers for taking the circle readings. The repeater might be used for reading directions, but the direction instrument could not be used for repeating angles. The diameter of the circle varies from 8 to 12 inches. In repeating instruments intended for third order work, a 6-inch or a 7-inch circle may be used. These instruments usually have three leveling screws instead of four as in the surveyor's transit.

1-41. Repeating Instrument.* With the repeating instrument the angle is read on two opposite verniers either to 5 or to

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* The repeating theodolite is rarely used for measuring angles in surveys of higher than third order. Normally three sets of direct and reversed readings are required to obtain desired precision, not higher than second order.

10 seconds, the accuracy of the measurements being increased by repeating the angles. Such an instrument is shown in Fig. 1-33. In all its essential parts it is like the ordinary engineer's transit,

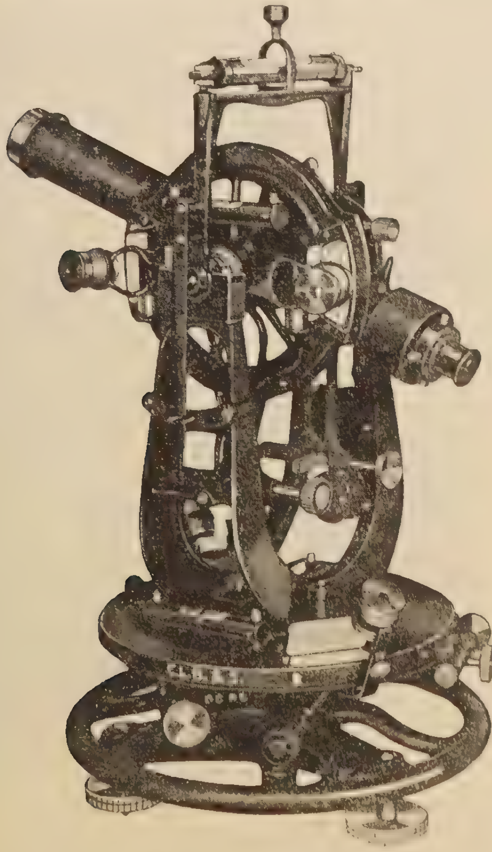

FIG. 1-33. REPEATING INSTRUMENT.
(Courtesy, C. L. Berger & Sons.)

excepting that like all other triangulation instruments it has an extra heavy tripod with a broad head. The leveling screws rest in radial slots in the tripod head and the instrument is secured to the tripod by means of a central clamp and spring. A striding

level is usually provided for leveling the horizontal axis in sighting at high altitudes or in astronomical work.

1-42. Direction Instrument. The direction instrument (Fig. 1-34) has a circle graduated usually to 5-minute divisions. This

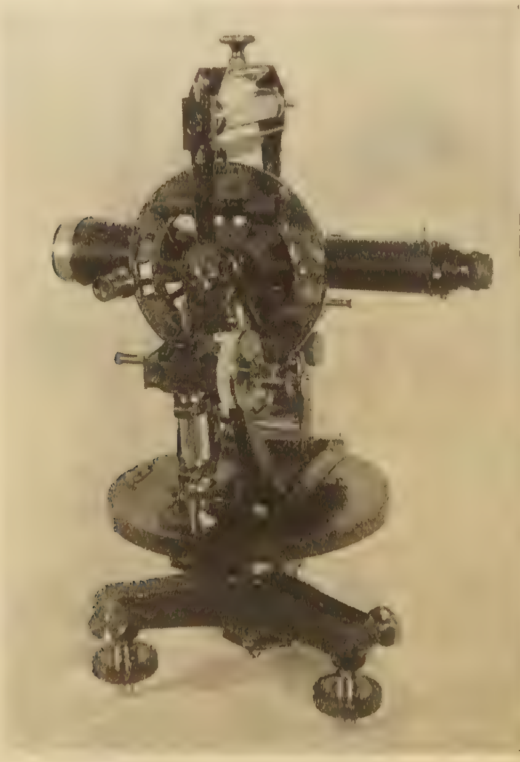

FIG. 1-34. PARKHURST THEODOLITE.

(U. S Coast & Geodetic Survey.)

circle can be turned and clamped in any desired position, independently of the upper portion of the instrument. In some instruments the circle is held in place by friction alone. On the frame supporting the telescope is a set of micrometer microscopes (usually two or three), placed at equal distances apart, by means of which the fractional parts of the 5-minute spaces are read. On

large instruments there is usually a small index microscope intended for reading the degrees and 5′ divisions only, the subdivisions being read by the micrometers.

1-43. The Micrometer Microscope. In the field of each microscope two or more of the graduations of the circle can be seen (Fig. 1-35). In the focal plane of the eyepiece of each microscope is a micrometer screw which moves a frame carrying a set of cross-hairs. These either are in the form of an X or else consist

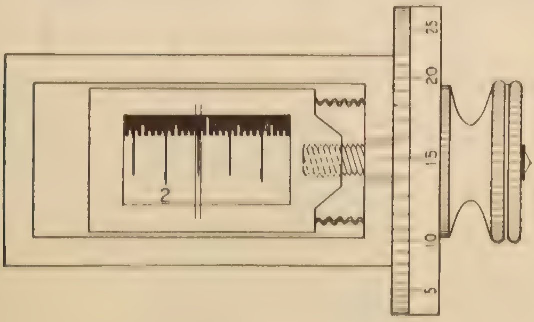

FIG. 1-35. MICROMETER MICROSCOPE.

of two parallel hairs set far enough apart so that a line of the graduation will not quite fill the space between them. The pitch of the screw and the focal length of the objective of the microscope are so related that some whole number of turns equals one space on the graduated circle. The head of the micrometer is graduated so that angles can be read to seconds or to fractions of a second. In the field of view is a notched scale for counting the whole turns of the screw. In the center of the scale is a deeper notch which represents the zero point of the micrometer readings. For any pointing of the telescope the direction is read by simply turning the micrometer until the hairs are symmetrical with respect to the preceding line of the graduation, reading the scale of the micrometer screw head (called in this position the *forward reading*), and adding this reading to the direct reading of the degrees and minutes on the circle. For the purpose of checking the reading, and also to determine the error of the microscope, a

reading is also taken on the next following graduation (called the *back reading*).

In Fig. 1-36 the zero (deep) notch of the scale has passed the 2° 30′ line and over 2′ more. There are two notches between the 2° 30′ graduation and the zero notch, indicating that two whole minutes have been passed. The drum reads 20″.0. The

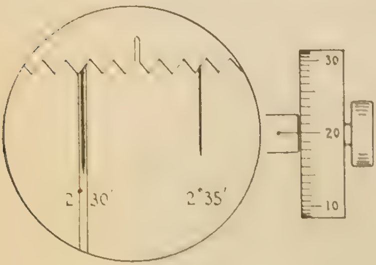

FIG. 1-36. FORWARD READING (2′ 20″.0).

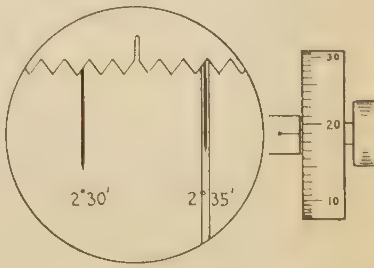

FIG. 1-37. BACK READING (2′ 19″.0).

whole reading is therefore

$$2° \; 30' + 2' \; 20''.0 = 2° \; 32' \; 20''.0$$

For the purpose of checking the reading and also determining the error of run, a reading is taken also on the next following graduation, called the back reading. (Fig. 1-37.) In this case the drum reads 10″.0, showing that the drum has made 1 division more than 5 whole turns from the 30′ to the 35′ mark.

In some theodolites the microscopes are provided with two sets of parallel hairs placed about 4 minutes apart. This makes it unnecessary to turn the screw five whole turns when passing from the forward to the back reading. The left-hand pair is used with the forward reading and the right-hand pair is used with the back reading. This results in a saving of time.

Direction instruments have been used extensively for high-order triangulation by the U. S. Coast and Geodetic Survey and the U. S. Geological Survey. For lower orders, the repeating instrument will give sufficient accuracy. Since its operation is familiar to surveys it is to be preferred for ordinary work. One

objection to its use is the systematic error due to the action of the clamps and tangent screws, which is not easy to eliminate. This error is usually not serious unless the instrument is in poor condition or very refined measurements are desired.

Micrometer microscope direction theodolites and the older repeating instruments are being largely replaced by instruments employing optical micrometer systems which can be read to a fraction of a second.

1-44. Theodolites with Optical Systems. Several types of theodolites are manufactured in the United States and other

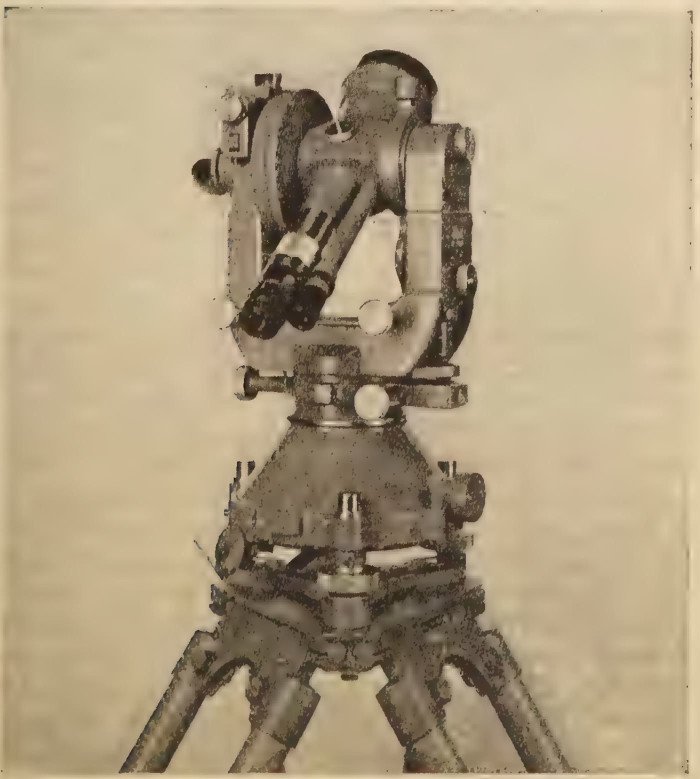

FIG. 1-38. WILD THEODOLITE

(Courtesy, Henry Wild Surveying Instrument Supply Co. Ltd.)

countries having optical systems which provide more precise and convenient angle readings than conventional types. They have small circles, are very compact and easy to carry. Several models are available, both of the repeating and directional type. The Wild T-3, Fig. 1-38, is typical of a direction type satisfactory for first and second order triangulation.

In this instrument the horizontal graduations are ruled on a glass circle 5.5 inches in diameter. Through the use of a coincidence micrometer, horizontal angles may be read directly to 0″.2. Both the horizontal and vertical circles are read through a mi-

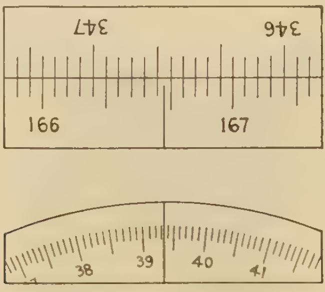

FIG. 1-39.

croscope mounted so that its eyepiece is alongside of the eyepiece of the telescope. By means of an optical train, two opposite portions of the graduated circle are viewed in juxtaposition (side by side). As first viewed through the microscope, after setting on the target, the adjacent circle readings do not coincide. They are made to coincide by turning a graduated micrometer drum. The degrees and minutes are read at an index appearing on the circle graduations, and the seconds are obtained by reading the micrometer drum. In bringing the circle graduations into coincidence, the drum measures the fractional angle to be added to the circle reading. As the circle divisions are 4 minutes apart, the maximum movement of the drum required to bring them together (from one coincidence to another) is 2 minutes, or 120 seconds. The drum, however, is graduated into 60 seconds, so that it is necessary either to double the reading to get the total reading, or else to make two settings of the drum and add these two readings. In Fig. 1-39 (upper view) the circle graduations have

been brought into coincidence. The reading of the index is $166° 38'$ and the drum reading (lower view) is $39''.3$. A second setting of the drum gives $39''.4$. The total reading is, therefore, $166° 38' + 39''.3 + 39''.4 = 166° 39' 18''.7$. This reading is an average of readings on opposite sides of the circle and is therefore free from eccentricity.

The vertical circle is graduated on a glass circle 3.8 inches in diameter. The procedure in reading vertical angles is the same as for horizontal angles except that diametrically opposite graduations are numbered in such a way that a vertical angle reading is given by the difference between the readings obtained with the telescope direct and with the telescope reversed. These positions are often referred to as circle left and circle right, since the vertical circle is on the left side for one position of the telescope and on the right side for the other.

The telescope has a large aperture object-glass giving both ample illumination and a wide field. Illumination for the optical mechanism is obtained by use of adjustable reflectors in daytime use and by an electrical illumination system at night. Another distinctive feature is a vertical collimator, called an optical plummet, whereby the instrument may be set over a station without using a plumb-bob.

This instrument is much speedier in operation than the conventional direction instrument because of the ease with which it may be read. The lightness in weight is another important advantage. However, because of the complexity of its design, emergency repairs while in the field are virtually impossible.

Other types of theodolites with optical micrometer reading facilities vary primarily in the details of obtaining the horizontal and vertical angles. Normally the circles are read through a fixed eyepiece located close to the telescope. When the telescope is reversed a second eyepiece, coaxial with, and opposed to, the first, is used to read the circles. A lever mounted between the two reading eyepieces switches the circle image from one to the other.

Both horizontal and vertical circles are visible simultaneously in the reading eyepiece. Readings are obtained by the double circle principle. In the reading eyepiece a portion of one set of

graduations appears adjacent to the diametrically opposite portion of the other set; when the instrument is rotated these portions appear to be moving in opposite direstions. An optical micrometer is used to obtain fractional readings of the circle graduation interval. The setting of the micrometer is made by "symmetry" instead of by "coincidence." This is possible because one set of graduations consists of double lines while the other consists of single lines. The optical micrometer is set when one of the single lines is placed exactly in the center of a double graduation. The reading automatically gives the mean of the two diametrically opposed circle points. The vertical circle is in the top part of the view, the horizontal in the middle, and the micrometer in the lower part: the three are seen together in one window.

This type of reading is displayed in Fig. 1-40, which illustrates the reading of a vertical circle on a Kern DKM3 theodolite. The upper scale has been centered on the parallel circle graduations by a symmetry, and is read to the nearest 10′ with reference to

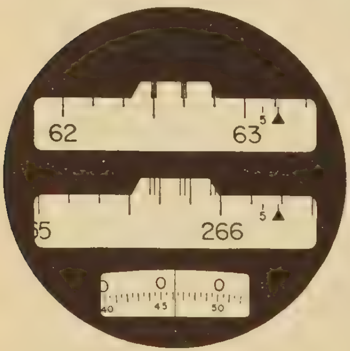

Fig. 1-40. Vertical Circle Reading Kern DKM3.

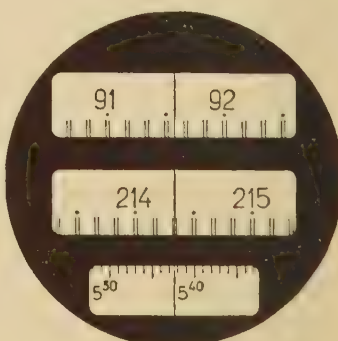

Fig. 1-41. Horizontal Angle Reading Kern DKM2.

the triangular index mark, giving 63° 10′. (The short graduation "5" near the index mark is exactly 5° from that mark.) The micrometer scale at the bottom reads 0′ 46″.2, giving a total angle of 63° 10′ 46″.2 (zenith angle measured from the vertical).

In another type of instrument, Fig. 1-41, only one set of graduations is numbered. In the circle image two lines, one from each set of graduations, are brought together by an optical system so that they appear as double lines, which actually represent diagrammetrically opposite points on the circle, as before. These double lines are moved in the image plane by an optical micrometer until the fixed index splits the nearest double lines. Degrees and tens of minutes are read on the main scale, and the subdivision of tens of minutes and the seconds appear on the micrometer drum.

Fig. 1-41 illustrates this type of display in the Kern DKM2 theodolite. The horizontal angle is read on the middle scale where the fixed vertical index line has been centered on a pair of parallel circle divisions, and reads $214° 20'$. The micrometer scale reads $5' 37''.6$, giving a total reading of $214° 25' 37''.6$.

1-45. Preparation for Occupying the Station. The observer should be provided with a list of directions of the stations which he is to sight, referred to some conspicuous object as the 0° direction; for example, a nearby church spire or a water tower may be used as the initial direction. The observer should have descriptions of the stations (Art. 1-13) he is to occupy and the signals he is to sight. He should also be provided with maps, if any exist, to aid him in identifying the stations. He should have with him a heliotrope or a signal lamp and an assistant to operate it, or two-way portable radio, so that he can, upon arriving at the station, promptly establish communication with all the signal men at the distant stations. After establishing the directions from the instrument station to the distant stations he can show light to each of them and thus enable the signal men at the distant stations to ascertain exactly the direction in which they should point their lights. If its direction is accurately known a signal can be seen through the telescope at times when it would otherwise be impossible to find it. Observations may often be made on a heliotrope when the hill itself is almost entirely obscured by haze. All of the signal men should be provided with maps of the region showing the scheme of triangulation, with a program of the observations, and a code of signals. When the signal man arrives at his station he first finds the directions of

the stations toward which he is to show lights and then places his heliotropes or signal lamps in position. Sometimes the lights are placed on line a few feet from the station point (by sighting in with a plumb line), sometimes exactly over the center mark. The directions of the instrument stations become known as soon as the observers show their lights. It is important that lights should be so placed and so operated that an observer cannot sight by mistake on a light showing on another line.

1-46. Measuring Angles with Repeating Instrument. A common method of measuring an angle with a repeating instrument is to measure, say, six times with the telescope direct, then six times with the telescope reversed; this is called a "set."

In measuring the angles always turn the instrument in a **clockwise direction** (whether the upper or the lower motion is being used). The angle itself is first measured by six repetitions with the telescope direct; then its *explement* (360 degrees — the angle) is measured by six repetitions with the telescope reversed. Both verniers are read on the first and the sixth readings. With proper handling this will result, except for usual observational errors, in readings in the reversed and direct positions that agree quite closely.

It is evident that if the error due to the action of the clamps and tangent screws is nearly constant, the mean of the angle as found directly and through its explement will be nearly free from such error.

The tangent screws should always be brought up to the desired position by means of a right-hand turn in order to insure proper tension of the opposing spring. But if the cylinder and spring have been properly cleaned and oiled this is found to make little or no difference.

The Coast Survey instructions to observers recommend 5 or 6 complete sets on first order work, both on the angle and on the explement; for second order work, 2 to 3 sets; for third order, one set when an 8-inch horizontal circle is used, or 2 to 4 sets with a 7-inch circle. The horizon should be closed in all cases, i.e., the series should cover a complete circle.

In order to eliminate the errors of graduation of the circle the readings are to be distributed over the circle by increasing the

initial reading each time by an amount equal to $360° \div mn$, where m is the number of sets, and n is the number of verniers. If, however, the theodolite has 3 verniers or microscopes, the formula $360° \div 2\,mn$ may be used, because when the telescope is reversed the microscopes fall in three new places on the circle, making six in all for a complete direct and reversed reading. Errors in the vernier are eliminated by setting the index of the vernier ahead at the beginning of each set by $\frac{1}{m}$th part of the smallest division of the circle. For example, if 6 sets are to be taken with a two-vernier instrument having the circle graduated to 10-minute spaces the first setting would be 0° 00′ 00″, the second 30° 01′ 40″, the third 60° 03′ 20″, etc.

1-47. Measuring Angles with Direction Instrument. Some line (such as a line to a prominent signal) is chosen as a reference line and the directions of the various signals read in order, beginning with this one. After this series is completed the telescope is reversed, the pivots remaining in the same bearings, and the signals sighted in the same order. This completes the readings for this "position" of the circle. The Coast Survey instructions require observations in 16 positions for first order work. For second order work 6 to 12 positions are required; for third order work, 2 to 4 positions.

In order to distribute the readings over the circle the initial setting is increased by an amount indicated by the above formulas for each new position. For 16 positions (3 micrometers) this would be $3\frac{3}{4}°$. Actually the advance is made more rapidly around the circle by advancing the initial reading 15° each time. The micrometer readings are grouped in fours and placed symmetrically about the middle point of a space on the circle. This results in the readings 00′ 40″, 01′ 50″, 03′ 10″, and 04′ 20″. By this arrangement the plus and minus corrections for run balance each other and no correction is necessary provided no pointings are lost in the set. This results in the following initial settings: 0° 00′ 40″, 15° 01′ 50″, 30° 03′ 10″, 45° 04′ 20″, 64° 00′ 40″, etc. The last setting is 237° 04′ 20″. This distributes the readings around the circle at intervals of nearly

$4°$, and eliminates errors of graduation. These settings are approximate and it is necessary to make exact pointings and readings after the setting has been made nearly right.

If a 2-micrometer instrument is used, the settings (16 positions) would be $0° 00' 40''$, $11° 01' 50''$, $22° 03' 10''$, $33° 04' 20''$, $45° 00' 40''$, etc. For 8 positions, 3 micrometers, we would have $0° 00' 40''$, $15° 01' 50''$, $30° 03' 10''$, $45° 04' 20''$, $52° 00' 40''$. For a 2-micrometer instrument, $0° 00' 40''$, $22° 01' 50''$, $45° 03' 10''$, $67° 04' 20''$, $90° 00' 40''$, etc.

1-48. Precautions in Measuring Angles. Great care should be used in securing a steady support for the instrument. If it is necessary to set the tripod on soft ground, heavy pegs should be driven into the ground and holes bored in the tops to receive the points of the tripod legs. The clamp screws controlling the leveling screws should not be loose but should be set up firmly after the instrument has been leveled. The instrument should be protected from wind and sun by a tent or by an umbrella. The observer should be careful not to disturb the instrument or to allow the heat from his body to affect it.

Care should be taken not to observe on a signal whose position is made doubtful by difficult seeing. Where there are two or more signals in the same general direction care should be taken to observe the correct one. If a signal is illuminated on one side only, so that the error of pointing will be appreciable, observations should be delayed or else the error computed and allowed for. This is known as the correction for "phase."

1-49. Time for Observing. The best "seeing" for ordinary work (sighting on poles, etc.) is obtained on cloudy days, or on bright days late in the afternoon. Observations on heliotropes made during the middle of the day, however, when the signals appear quite unsteady, have been found to give about as accurate results as those made under apparently better conditions. The safest guide is not the appearance of the signals, but the "probable errors" of the angles or the directions, as shown by the measurements themselves. Observations made very early in the morning are usually unsatisfactory on account of the irregular refraction caused by rising vapors. The practice of the U. S. Coast and Geodetic Survey is to do a large part of the first and

second order triangulation at night. This results in greater accuracy owing to the fact that horizontal refraction is not so great as during the day.

1-50. Forms of Records. The tables on pp. 65 and 66 are forms of records which may be used with the direction instrument and the repeating instrument respectively. In the form on p. 66 the stations are set down in order. The approximate time and the position of the telescope (Direct or Reversed) are given. The degrees and minutes are read on microscope *A* only. The seconds of the two microscopes both forward and backward are entered as read. The mean of the four values is put in the next column. Next is the mean for the two positions of the

TRIANGULATION NOTES — REPEATING INSTRUMENT

Station: Camp | Instrument: B. & B. #14016
Observer: A.T.G. | Date: Aug. 30, 1957; 3:15 P.M.

Objects Observed	Tel.	Rep.	Angle	Vernier A	Vernier B	Mean A & B	Mean Angle D & R	Mean	Adj.	Adj. Secs.
			° ′	″	″	″	° ′ ″	″	″	″
High Head —	D	0	0 00	20	10	15.0				
Spruce		1	141 47	00						
	D	6	* 130 40	40	40	40.0	141 46 44.2			
	R	6	0 00	30	20	25.0	141 46 42.5	43.4	+0.3	43.7
Spruce —	R	0	0 00	30	20	25.0				
Dowling		1	83 00	00						
	R	6	137 57	50	60	55.0	82 59 35.0			
	D	6	0 00	30	20	25.0	82 59 35.0	35.0	+0.3	35.3
Dowling —	D	0	0 00	30	20	25.0				
Gardner		1	7 29	40						
	D	6	44 58	10	10	10.0	7 29 37.5			
	R	6	0 00	10	10	10.0	7 29 40.0	38.8	+0.3	39.1
Gardner —	R	0	0 00	10	10	10.0				
High Head		1	127 44	10						
	R	6	46 24	30	30	30.0	127 44 03.3			
	D	6	0 00	30	30	30.0	127 44 00.0	01.6	+0.3	01.9
							359 59	58.8	+1.2	60.0

* Since the angle is over 120 degrees, the A vernier has passed 360 degrees twice in the six repetitions. In computing the mean we divide the 720 degrees by 6 mentally, giving 120 degrees. Then divide the 130° 40′ 25″.0 (130° 40′ 40″.0 − 0° 00′ 15″.0) by 6, then add to 120 degrees, giving 141° 46′ 44″.2. Observe that when six repetitions are used, the remainder, after dividing the degrees by 6, gives the first figure of the minutes, i.e., 130° ÷ 6 = 21 degrees in the mean, plus 4 degrees to be carried to the minutes column, giving 40 minutes. Similarly, in dividing the minutes by 6, the remainder in the tens place is the tens place in the seconds.

Triangulation Notes — Direction Instrument

Station: High Head
Observer: A. J. B.

Instrument: Hildebrand No. 72174
Date: August 10, 1957.

Position	Objects Observed	Time	Tel. D. or R.	Mic.		Back	Forward	Mean	Mean D. and R.	Direction	Remarks
					° ′	″	″	″	″	″	
4	Spruce	1:50	D	A	67 04	07	08				
		P.M.		B		06	06	06.8			
			R	A	247 04	08	10				
				B		09	10	09.2	08.0	00.0	
									+.1		
									08.1		
	Dowling		D	A	111 10	54	53				
				B		52	52	52.8			
			R	A	291 10	54	56				
				B		58	58	56.5	54.6	46.5	
	Gardner		D	A	115 46	08	08				
				B		06	04	06.5			
			R	A	295 46	01	04				
				B		01	04	02.5	04.5	$\overline{56.4}$*	
	Chase		D	A	166 48	03	01				
				B		01	02	01.8			
			R	A	346 47	60	59				
				B		59	58	59.0	60.4	52.3	
	Spruce		D	A	67 04	07	08				
				B		08	08	07.8			
			R	A	247 04	08	08				
				B		09	09	08.5	08.2	00.0	
									−.1		
									08.1		

* The single bar (vinculum) over this entry indicates that the seconds so marked refer to a value of minutes one less than recorded. Hence, 115° 46′ $\overline{56''.4}$ should read 115° 45′ 56″.4. Subtracting 67° 04′ 00″.0 from each of the observed directions, using the seconds given in the direction column, gives the directions to other stations referred to Spruce as the initial direction, as follows:

Station	Direction
Spruce	0° 00′ 00″.0
Dowling	44 06 46 .5
Gardner	48 41 56 .4
Chase	99 43 52 .3

telescope. In this instance the first station is read again (to close the horizon) as is customary on second order work. The discrepancy of $0''.2$ is divided between the two to give the correct reading for Station Spruce. The seconds of the directions of the stations are found by subtracting the seconds of the initial station from the seconds of each station in succession.

In the form on p. 65 the angles are measured with a $10''$ repeating instrument. Six repetitions of the angle were made with the telescope direct and six with the telescope inverted. Only the initial setting of the vernier on the horizontal plate, the value of the first angle, and the sixth reading with the telescope direct and reversed are recorded. Both verniers are read on the initial setting and sixth readings. The first reading is recorded simply as a rough check of the sixth reading. No intermediate values of the angle such as the second, third, etc., are recorded, as they are not used in computing the mean angle. It will be noted that in the illustration the initial setting is zero. Where high precision is desired the initial settings should be determined as described in Art. 1-46. The form shown is suitable for use with any number of repetitions, the maximum number is usually six repetitions, direct and reversed, for one angle.

1-51. Reduction to Center. It sometimes happens that the instrument cannot be placed exactly over the station point, but from a point near by, called the *eccentric station*, the angles can be observed. This may occur when such a point as a church spire or a standpipe is used as a triangulation point. If the angles are taken with exactly the same care that they would be at the center, and if, in addition, the direction and distance of the center mark from the instrument (at the eccentric station) be observed, then the angles at the center can be accurately calculated, provided the approximate distances to the observed signals are known. Even though accurate values of the triangle sides cannot be obtained it is always possible to obtain approximate values, and these are sufficiently accurate for this reduction.

In making these corrections it is convenient to correct the *directions* of the separate lines rather than to correct the angles themselves directly; that is, the *direction* of each signal is to be corrected for the effect of eccentricity and the correct angles

between signals may be found afterward from these corrected directions. In Fig. 1-42 let C be the station point, E the eccentric station where the angles are observed, and S one of the observed signals. First compute the direction of each signal referred to the center C as 0°. This is usually done by turning an angle from the signal C to the next signal to the right. This is the direction of this first signal; the others are found by adding in succession the angles between consecutive signals. Then for each signal form a triangle CES. In this triangle

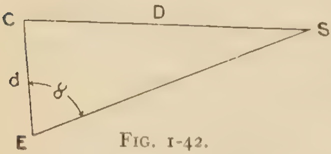

Fig. 1-42.

$$\frac{\sin S}{\sin E} = \frac{CE}{CS}$$

or, approximately,

$$S'' = \frac{d \sin \alpha}{D \sin 1''} \tag{1-17}$$

in which the angle S is expressed in seconds; * this angle is sometimes called the angle of " swing " and represents the change in direction from the observed to the true direction; it is to be added algebraically to the direction ES. The angle α should be taken as the *angle to the right* from C (from 0° to 360°); sin α should be given the proper algebraic sign; D is the distance between stations; and d is the eccentric distance, measured with a tape. After S'' has been calculated and added (algebraically) to each direction the true angles at the center may be found by taking the differences between these corrected directions. The correction to any measured angle is of course the difference between the corrections found for the two observed directions.

---

* To reduce S to seconds we should divide by the arc of $1''$ (= 0.000004848 . . .) but since this is identical with $\sin 1''$ for 10 places the latter is used in practice as a matter of convenience. In estimating small angles in triangulation it is convenient to remember that three tenths of an inch subtends an angle of very nearly one second at a distance of one mile.

EXAMPLE OF REDUCTION TO CENTER.

HARPERS

(Eccentric Sta. No. 1)

$d = 1.342^m$ log = 0.12775
colog sin 1″ = 5.31443
log const. = 5.44218

Measured angles: — Center to Smith's Cupola = 42° 14′ 20″; Smith's Cupola to Cotton's = 62° 33′ 10″.1; Methodist Church to Cotton's = 58° 45′ 31″.0; Cotton's to White Flag = 56° 22′ 36″.1; White Flag to Baldwin = 43° 59′ 57″.4.

The directions from the center are computed and the computation of corrections tabulated as follows:

Station	Smith's Cupola	Methodist Church	Cotton's	White Flag	Baldwin
Direction	42° 14′ 20″	46° 01′ 59″.1	104° 47′ 30″.1	161° 10′ 06″.2	205° 10′ 03″.6
Log sin	9.8275	9.8572	9.9853	9.5090	9.6286
Colog D	6.1052	6.1025	6.0640	6.2672	6.0909
Log Const.	5.4422	5.4422	5.4422	5.4422	5.4422
Log S	1.3749	1.4019	1.4915	1.2184	1.1617
S''	+ 23″.7	+ 25″.2	+ 31″.0	+ 16″.5	− 14″.5
Direction	42° 14′ 43″.7	46° 02′ 24″.3	104° 48′ 01″.1	161° 10′ 22″.7	205° 09′ 49″.1

The reduced angles are therefore: — Smith's Cupola to Cotton's = 62° 33′ 17″.4, Methodist Church to Cotton's = 58° 45′ 36″.8, Cotton's to White Flag = 56° 22′ 21″.6, White Flag to Baldwin = 43° 59′ 26″.4.

1-52. Calculation of Results. After the angles have been measured in the field an abstract of the results should be prepared. If the angles have been measured with a repeating instrument and the horizon closed in each set similar to the observations shown in the example on p. 65, then averages of the various sets may be easily found. If a direction instrument has been used according to the program outlined in Art. 1-47, p. 63, then the abstract will consist of the averages of the directions observed, using the various positions of the circle.

For the preliminary computation of the distances, before the general or figure adjustment is made, the triangles should be tested for error of closure. In every triangle the sum of the angles should equal 180° plus the spherical excess of the triangle. The amount that the actual sum exceeds or falls short of this quantity is the "closing error." The amount of the spherical excess depends upon the area of the triangle and also upon the

latitudes of the vertices: it is approximately equal to one second for every 75.5 square miles. It may be more accurately computed from the formula

$$\epsilon'' = \frac{\text{Area of triangle}}{R^2 \sin 1''} = \frac{bc \sin A}{2R^2 \sin 1''} \qquad (1\text{-}18)$$

b and c being sides of the triangle in linear units, R the mean radius of the earth, and ϵ'' the spherical excess in seconds of angle. [Log R (in feet) may be taken as 7.32008; log R (in meters) = 6.80470.] In order to give the sphere the same (average) curvature as the spheroid at the place in question, we may substitute for the radius of the sphere R the value $\sqrt{R_m N}$, in which R_m is the radius of curvature of the meridian section and N is the radius of curvature of a section at right angles to the meridian. This gives for the spherical excess

$$\epsilon'' = \frac{1}{2R_m N \sin 1''} \cdot bc \sin A = mbc \sin A \qquad (1\text{-}19)$$

The log of m, a constant for any given latitude, is given in Table II, p. 401. In determining m from the table the latitude given for any one of the vertices of the triangle may be used.

1-53. Computation of the Triangle Sides. As an illustration of the computation of the triangle sides take the triangle shown in Fig. 1-43 for which the observed angles are

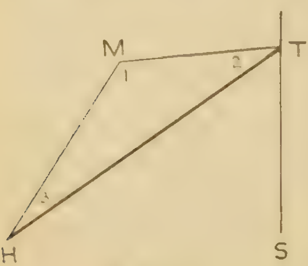

FIG. 1-43.

McFaul (M) =	130° 47′ 20″.7
Trescott (T) =	26 04 15 .8
Howard (H) =	23 08 10 .1
Sum =	180 00 01 .6

In this triangle TH is the known side and MT and MH are to be computed. The spherical excess of this triangle is 0″.4 (Art. 1-52). Hence the error of closure of the triangle is 1″.2. Subtracting

0″.4 from each angle we obtain for the adjusted spherical angles

$$
\begin{aligned}
M &= 130^\circ\ 47'\ 26''.3\\
T &= 26\ \ 04\ \ 15\ .4\\
H &= 23\ \ 08\ \ 18\ .7\\
\hline
\text{Sum} &= 180\ \ 00\ \ 00\ .4
\end{aligned}
$$

Find the plane angles by subtracting 0″.2 from the first angle and 0″.1 from the others, obtaining

$$
\begin{aligned}
M &= 130^\circ\ 47'\ 26''.1\\
T &= 26\ \ 04\ \ 15\ .3\\
H &= 23\ \ 08\ \ 18\ .6\\
\hline
\text{Sum} &= 180\ \ 00\ \ 00\ .0
\end{aligned}
$$

Having given the log TH = 4.430 2718 we find the sides TH and MH by the formula $\frac{a}{b} = \frac{\sin A}{\sin B}$ as shown below.

No.	Stations	Obs'd Angle	Corr.	Sph. Ang.	Sph. Exc.	Plane Angle and Dists.	Logarithms
2–3	T to H					26932.20 m.	4.430 2718
1	M	130° 47′ 26″.7	−0″.4	26″.3	−0″.2	130° 47′ 26″.1	0.120 8450
2	T	26 04 15 .8	−0 .4	15 .4	−0 .1	26 04 15 .3	9.642 9424
3	H	23 08 19 .1	−0 .4	18 .7	−0 .1	23 08 18 .6	9.594 3429
		180 00 01 .6	−1 .2	00 .4	−0 .4	00 .0	
1–3	M to H					15633.6 m.	4.194 0592
1–2	M to T					13978.5 m.	4.145 4597

1-54. Adjusting Triangulation. In important systems of triangulation where high accuracy is necessary, the angles should be adjusted by the method of least squares. An exact adjustment requires a lengthy solution which is expensive by manual methods. Electronic computer programs have been developed by Government agencies which make precise adjustments feasible. For some purposes, however, approximate methods give acceptable results with less computation.

Instead of being adjusted as a whole, sometimes triangulation systems are divided into smaller units, each unit being adjusted

individually. It is always convenient to have a system that is adjusted, since this makes all the results consistent and we are not troubled with having several different values for the length of the same line. (See Appendix B.)

1-55. Computation of Geodetic Positions.* After all of the triangle sides are known and the azimuth of some line is determined, the geodetic latitude and longitude of each triangulation station is computed. The calculation of the latitude and longitude is made by means of **differences** rather than by the direct solution. The triangle formed by joining the two stations with the pole, which would be required in the direct solution, is so large that 10-place logarithms are needed to give the required accuracy in the latitude, so the difference in latitude is calculated.

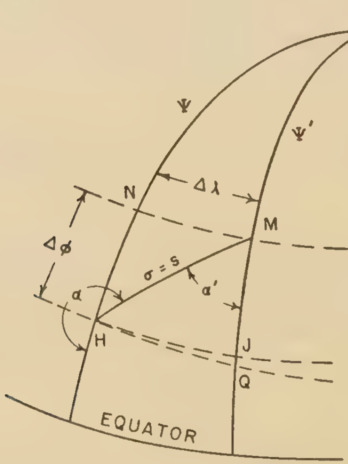

Fig. 1-44.

In Fig. 1-44 let H be the position of a point whose latitude and longitude are known; let M be the unknown point whose position is to be found; α, the azimuth of M from H; σ, the arc distance,

* For the method of deriving the formulas and computing the constants for computing the geodetic positions, see U. S. Coast and Geodetic Survey Special Publication No. 8.

expressed in circular measure, HM. In the triangle PHM the side PM is evidently a function of σ hence an expression may be obtained for it by means of Maclaurin's Theorem. Letting $PH = \Psi$ and $PM = \Psi'$

then
$$\Psi' = \Psi + \frac{d\Psi}{d\sigma}\sigma + \frac{d^2\Psi}{d\sigma^2}\cdot\frac{\sigma^2}{2} + \frac{d^3\Psi}{d\sigma^3}\cdot\frac{\sigma^3}{\sigma} \tag{1-20}$$

In the spherical triangle PHM, $PM = \Psi'$, and $HM = \sigma$, then by trigonometry

$$\cos\Psi' = \cos\Psi\cos\sigma + \sin\Psi\sin\sigma\cos H \tag{1-21}$$

By successive differentiation of (1-21) values may be obtained of the differential coefficients in terms of Ψ and H. If these are substituted in (1-20) an expression for the difference in latitude of the points H and M is obtained. To put this series in convenient form for computation, σ is first changed from an arc to the corresponding distance in meters (s) on the surface of a sphere tangent to the spheroid at H. Its radius is the normal terminating in the minor axis of the spheroid. A change is then made from the radius of this sphere to the actual radius of curvature of the meridian at H. A new term (the D term) is inserted to compensate for the fact that we use ϕ, the only latitude known, for computing these radii of curvature, whereas we should use $\frac{1}{2}(\phi + \phi')$. The final form of the series is

$$-\Delta\phi = Bs\cos\alpha + Cs^2\sin^2\alpha + (\delta\phi)^2 D - Ehs^2\sin^2\alpha \tag{1-22}$$

in which
$s =$ the distance HM in meters
$\alpha =$ the azimuth of M from H
$\phi =$ the latitude of H
$\Delta\phi =$ the difference in latitude
$\delta\phi =$ the sum of the first two terms
and $h = Bs\cos\alpha$

B, C, D, and E are constants for any given latitude; their logarithms are given in Table III, p. 492. All these terms are in seconds of arc.

The first term in the series is the plane distance $s \cos \alpha$; the second term is the offset from the tangent great circle to the parallel of latitude. The third term is a still closer approximation.

The difference in longitude of M and H may be obtained directly from the triangle PHM by the equation

$$\sin P = \frac{\sin \sigma}{\cos \phi'} \sin \alpha$$

which becomes

$$\Delta\lambda = A's \sin \alpha \sec \phi' \tag{1-23}$$

in which $\Delta\lambda$ is the difference in longitude in seconds of arc, and A' is another tabulated quantity which must be taken out for ϕ', the latitude of M, because this equation involves ϕ, as found by equation (1-22).*

The difference in azimuth due to convergence of the meridians is found as follows. From Napier's analogies

$$\tan \tfrac{1}{2}(H + M) = \cot \tfrac{1}{2}\Delta\lambda \frac{\cos \frac{1}{2}(\Psi' - \Psi)}{\cos \frac{1}{2}(\Psi' + \Psi)}$$

Since $\frac{1}{2}(H + M) = \frac{1}{2}(180° - \Delta\alpha)$ this may be put in the form

$$-\Delta\alpha = \Delta\lambda \sin \tfrac{1}{2}(\phi + \phi') \sec \tfrac{1}{2}(\Delta\phi) \tag{1-24}$$

in which $-\Delta\alpha$ is the correction to the azimuth $180° + \alpha$ to reduce it to α'.

The properties of the unknown point M are determined by the following relations:

$$\phi' = \phi + \Delta\phi$$

$$\lambda' = \lambda + \Delta\lambda$$

$$\alpha' = 180° + \alpha + \Delta\alpha$$

The primes in each case refer to the unknown point M.

---

* Symbols or letters that carry a prime (such as A') denote that the latitude of the new station ϕ' is to be used as the argument in finding values from tables.

It is evident that when the azimuth of the unknown direction (HM in this case) is between 90° and 270° the difference in latitude $\Delta\phi$ must be added to the latitude of the known point (H); but since the algebraic sign of $\cos\alpha$ is negative for such angles the sign of $\Delta\phi$ in the formula must be negative.

From Fig. 1-45 it is evident that if M is east of H, i.e., if α is between 180° and 360°, the correction $\Delta\alpha$ must be added to

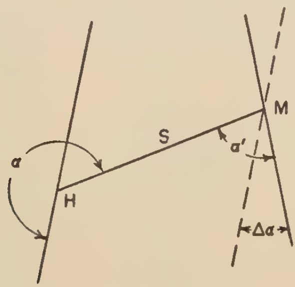

FIG. 1-45.

$(\alpha - 180)$ to refer the back-azimuth to the meridian through M (in the northern hemisphere).

For triangle sides not exceeding 10 statute miles (or $\log s =$ 4.23 . . .) the last term in the expression for $\Delta\phi$ may be omitted, and also the factor $\sec \frac{1}{2}\Delta\phi$ in the expression for $\Delta\alpha$. The quantity h^2 may be substituted for $(\delta\phi)^2$ when $\log s$ is not greater than 4.93 . . .

When the difference in longitude is very large it is necessary to apply a small correction to allow for the difference between the arc and the sine, an approximation which was made in deducing the formula for $\Delta\lambda$. This difference is given in Table IV, p. 497. To use this table take out the " difference " opposite $\log s$ and also the " difference " opposite $\log \Delta\lambda$, giving each the sign shown at the top of the column. The algebraic sum of the two is to be added to $\log \Delta\lambda$.

POSITION COMPUTATION, THIRD-ORDER TRIANGULATION

					°	′	″
α	2	T	to 3	H	57	50	48.0
2d∠		H	&	M	+ 26	04	15.4
α	2	T	to 1	M	83	55	03.4
Δα					−	7	24.8
					180	*00*	*00.0*
α′	1	M	to 2	T	263	47	38.6
		First Angle of Triangle			130	47	26.3

	°	′	″				°	′	″
φ	44	45	31.470	2	T	λ	67	06	30.870
Δφ		−	48.468			Δλ	+	10	31.826
φ′	44	44	43.002	1	M	λ′	67	17	02.696

	Logarithms	Values in seconds	
s	4.1454597		
Cos α	9.0251355		
B	8.5104863		
h	1.6810815	1st term	+ 47.9824″
s²	8.29092		
Sin²α	9.99510		
C	1.40037		
	9.68639	2d term	+ 0.4857
h²	3.3622		
D	2.3926		
	5.7548	3d term	+ 0.0001
		−Δφ	+ 48.4682

	° ′ ″	
½(φ+φ′)	44 45 07.2	
	Logarithms	**Values in seconds**
s	4.1454597	
Sin α	9.9975483	
A′	8.5089970	
Sec φ′	0.1485928	
Δλ	2.8005978	+ 631.826″
Sin ½(φ+φ′)	9.8475970	
−Δα	2.6481948	+ 444.8

					°	′	″
α	3	H	to 2	T	237	38	40.5
3d∠		M	&	T	− 23	08	18.7
α	3	H	to 1	M	214	30	21.8
Δα					+	4	43.1
					180	*00*	*00.0*
α′	1	M	to 3	H	34	35	04.9

	°	′	″				°	′	″
φ	44	37	45.837	3	H	λ	67	23	45.268
Δφ	+	6	57.166			Δλ	−	6	42.572
φ′	44	44	43.003¹	1	M	λ′	67	17	02.696

	Logarithms	Values in seconds	
s	4.1940592		
Cos α	9.9159622n		
B	8.5104964		
h	2.6205178n	1st term	− 417.3667″
s²	8.38812		
Sin²α	9.50639		
C	1.39842		
	9.29293	2d term	+ 0.1963
h²	5.2410		
D	2.3926		
	7.6336	3d term	+ 0.0043
		−Δφ	− 417.1661

	° ′ ″	
½(φ+φ′)	44 41 14.4	
	Logarithms	**Values in seconds**
s	4.1940592	
Sin α	9.7531948n	
A′	8.5089970	
Sec φ′	0.1485928	
Δλ	2.6048438n	− 402.572″
Sin ½(φ+φ′)	9.8471020	
−Δα	2.4519458	− 283.1

Example of Computation of Geodetic Position

From the table on page 71, the adjusted spherical angles are

$M = 130° 47' 26''.3, \quad T = 26° 04' 15''.4, \quad H = 23° 08' 18''.7$

The computation of the position of M from the base-line TH may be put in the form shown on p. 76.

In these forms, adapted for third order work, the computation of the E term has been omitted. If the line is sufficiently long so that this term becomes appreciable it may be computed as a separate term.

It will be observed that in taking out the value of A' in computing $\Delta\lambda$ this log is taken out for ϕ', **not** for ϕ. In computing a large number of positions over a small area such as a state or a county, it will be convenient to prepare a table of log $A' \sec \phi'$ for intervals of, say, 10 seconds of latitude and for values of the latitudes covering the limits of the survey.

1-56. Machine Computation of Geodetic Position. The demonstrations of calculations of geodetic position presented in this Chapter are in terms of logarithms. This is the basis upon which they were originally derived. The form is such that a student may compute a position using the formulas and tables in this book. As a practical matter any office concerned with a number of calculations would use calculating machines and follow a somewhat different procedure based on natural trigonometric functions and actual distances.

The U. S. Coast and Geodetic Survey has issued Special Publication No. 241, "Natural Tables for the Computation of Geodetic Position." This contains formulas and procedures for machine computation of geodetic positions in terms of latitude and longitude, and of plane coordinates in meters. The instructions and tables in this publication are too voluminous to reproduce in this text. It may be obtained from U. S. Government Printing Office, Washington 25, D. C.

1-57. Three-Point Problem. It sometimes becomes necessary to establish a new station and to locate it accurately without occupying any of the other stations. If three signals can be seen from the proposed station and two angles measured between

them, the position of the point may be accurately determined by means of the Three-Point Problem. One solution of this problem is as follows. In Fig. 1-46, H, M, and T represent the three signals sighted, and P is the instrument station, the point whose

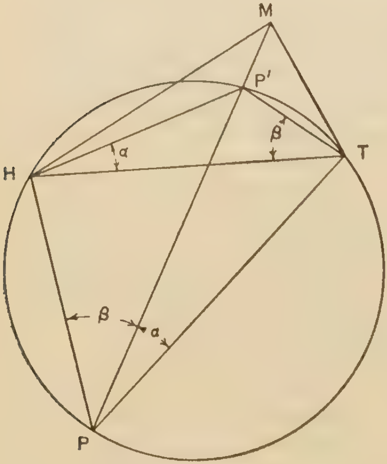

FIG. 1-46. TRIGONOMETRIC SOLUTION OF THREE-POINT PROBLEM.

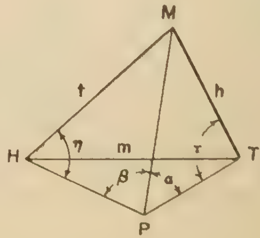

FIG. 1-47.

position is sought. A circle is passed through H, T and P, and the lines PM, HP' and $P'T$ are drawn.

Every side and angle of the triangle HMT may be computed from triangulation data.

In Fig. 1-46, let angle $P'HT = \alpha$, $P'TH = \beta$. Since these angles intercept equal arcs, angle $MPT = \alpha$, and $HPM = \beta$.

In triangle $HP'T$ one side HT and two angles (α and β) are known, and HP' may be calculated.

In triangle $HP'T$, two sides HP' and HM are known, and the included angle $NHP' = MHT - \alpha$. Therefore angle $HMP' = HMP$ may be computed.

Finally in triangle HMP, one side HM and two angles HMP and α are known, from which HP and MP may be found; and from triangle MPT, TP may be found.

A second solution is presented which is shorter in calculation time, and better arranged for logarithmic computation.

In the quadrilateral $HMTP$, Fig. 1-47, angles

$$\eta + \tau + M + \alpha + \beta = 360°$$

and
$$\tfrac{1}{2}(\eta + \tau) = 180° - \tfrac{1}{2}(M + \alpha + \beta) = S$$

Angles η and τ are required.

Referring to Fig. 1-47, from triangles HMP and TPM,

$$\frac{\sin \eta}{\sin \beta} = \frac{MP}{t} \quad \text{and} \quad \frac{\sin \tau}{\sin \alpha} = \frac{MP}{h}$$

From which

$$\frac{t \sin \alpha}{h \sin \beta} = \frac{\sin \tau}{\sin \eta}$$

In order to derive a convenient expression for computation, $\dfrac{\sin \tau}{\sin \eta}$ is made equal to $\tan z$, in which z has no physical meaning. The expression is then transposed to $\dfrac{\tan S}{\tan (z + 45°)}$, as follows:

$$\tan (z + 45) = \frac{\tan z + \tan 45}{1 - \tan z \tan 45} = \frac{\tan z + 1}{1 - \tan z}$$

$$\frac{\dfrac{\sin \tau}{\sin \eta} + 1}{1 - \dfrac{\sin \tau}{\sin \eta}} = \frac{\sin \tau + \sin \eta}{\sin \eta - \sin \tau}$$

But
$$\sin \eta + \sin \tau = 2 \sin \tfrac{1}{2}(\eta + \tau) \cos \tfrac{1}{2}(\eta - \tau)$$

and
$$\sin \eta - \sin \tau = 2 \sin \tfrac{1}{2}(\eta - \tau) \cos \tfrac{1}{2}(\eta + \tau)$$

Therefore

$$\tan (z + 45) = \frac{\sin \tfrac{1}{2}(\eta + \tau) \cos \tfrac{1}{2}(\eta - \tau)}{\sin \tfrac{1}{2}(\eta - \tau) \cos \tfrac{1}{2}(\eta + \tau)}$$

$$= \frac{\tan \tfrac{1}{2}(\eta + \tau)}{\tan \tfrac{1}{2}(\eta - \tau)} = \frac{\tan S}{\tan \tfrac{1}{2}(\eta - \tau)}$$

and
$$\tan \tfrac{1}{2}(\eta - \tau) = \frac{\tan S}{\tan (z + 45)}$$

Three geometric cases can occur with respect to the location of P relative to triangle MHT, as shown in Figs. 1-47, 1-48 and 1-49. The following example illustrates the condition in Figs. 1-46 and 1-47.

Example

Triangulation data: $HMT = 86^\circ\ 29'\ 42''.3$ $MTH = 74^\circ\ 14'\ 06''.3$ $THM = 19^\circ\ 16'\ 11''.4$ $t = 12859.41$ ft. $h = 4409.70$ ft. $m = 13337.13$ ft.

Measured angles: $\alpha = 21^\circ\ 38'\ 06''.8$ $\beta = 84^\circ\ 12'\ 57''.9$

$$\begin{aligned}
\alpha &= 21^\circ\ 38'\ 06''.8 \\
\beta &= 84\ \ 12\ \ 57\ .9 \\
HMT &= 86\ \ 29\ \ 42\ .3 \\
\hline
\text{Sum} &= 192^\circ\ 20'\ 47''.0 \\
S = 180^\circ - \tfrac{1}{2}\,\text{Sum} &= 83^\circ\ 49'\ 36''.5
\end{aligned}$$

$$\begin{aligned}
\log t &= 4.109221 \\
\log \sin \alpha &= 9.566668 \\
\operatorname{colog} h &= 6.355591 \\
\operatorname{colog} \sin \beta &= 0.002217 \\
\hline
\log \tan z &= 0.033697 \\
z &= 47^\circ\ 13'\ 14''.0 \\
z + 45 &= 92^\circ\ 13'\ 14''.0
\end{aligned}$$

$$\begin{aligned}
\log \tan S &= 0.965928 \\
\log \tan (z + 45) &= 1.411444 \text{ neg.} \\
\hline
\log \tan \tfrac{1}{2}(\eta - \tau) &= 9.554484 \text{ neg.}
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}(\eta - \tau) &= -19^\circ\ 43'\ 21''.1 \\
S = \tfrac{1}{2}(\eta + \tau) &= 83^\circ\ 49'\ 36''.5 \\
\hline
\eta = MHP & \qquad 64^\circ\ 06'\ 15''.4 \\
\tau = MTP & \qquad 103^\circ\ 32'\ 57''.6
\end{aligned}$$

Angles in triangles PHM, PTM and PHT are

HPM	$84^\circ\ 12'\ 57''.9$	MPT	$21^\circ\ 38'\ 06''.8$	PTH	$29^\circ\ 18'\ 51''.3$ $(\tau - MTH)$
MHP	64 06 15 .4	PTM	103 32 57 .6	THP	44 50 04 .0 $(\eta - THM)$
PMH	31 40 46 .7	TMP	54 48 55 .6	HPT	105 51 04 .7 $(\alpha + \beta)$
	$180^\circ\ 00'\ 00''.0$		$180^\circ\ 00'\ 00''.0$		$180^\circ\ 00'\ 00''.0$

From these angles and the known sides, any other desired distances may be calculated.

If the point P is situated as in Fig. 1-48, the solution is carried out in the same way. If P is situated as in Fig. 1-49 then $S = \frac{1}{2}(M - \alpha - \beta)$.

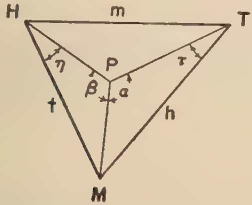

FIG. 1-48.

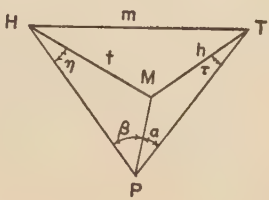

FIG. 1-49.

When combining $\frac{1}{2}(\eta + \tau)$ and $\frac{1}{2}(\eta - \tau)$ it should be remembered that if $\tan \frac{1}{2}(\eta - \tau)$ is positive, then $\eta = S + \frac{1}{2}(\eta - \tau)$ and $\tau = S - \frac{1}{2}(\eta - \tau)$. But if $\tan \frac{1}{2}(\eta - \tau)$ is negative, then $\eta = S - \frac{1}{2}(\eta - \tau)$ and $\tau = S + \frac{1}{2}(\eta - \tau)$.

The information obtained from the above solutions can be used to obtain the coordinates of the instrument station from those of the known points. Solutions of the three point problem have also been worked directly from coordinates of known points.*

1-58. Astronomical Determination of Position. In order that a triangulation net may be accurately located upon the earth's surface it is necessary that the geographical positions of a few triangulation points should be accurately determined by astronomical observations for their latitudes and longitudes. The methods of making these observations are fully discussed in works on geodesy and practical astronomy and in U. S. Coast and Geodetic Survey Special Publication No. 237. Longitude and latitude observations are also treated in Chapter 2, Arts. 2-34 to 2-41.

The latitude is usually determined by observations with the zenith telescope. By means of this instrument the latitude is

* "A New Solution of the Three Point Problem," by Jean Smith, Publication No. 1, Maryland State Roads Commission, in cooperation with the Pennsylvania Water and Power Co.

found from the measured difference in the zenith distances of two stars, one of which is north of the zenith and the other south. The stars to be observed are so selected that this difference is small enough to be measured with an eyepiece micrometer. Any change in the inclination of the telescope is measured by means of a sensitive spirit level. By this method a latitude can readily be determined in one night with a probable error of about $0''.10$ of arc or less, which is equivalent to about 10 feet or less, on the earth's surface.

The longitude is determined by observing, with a portable astronomical transit instrument, the error of a chronometer on local sidereal time and then comparing the chronometer time by telegraph with that of another (known) field station where similar observations have been made, or, by radio, directly with the U. S. Naval observatory clock at Washington. The difference in time thus found is the difference of longitude expressed in time units. The accuracy with which longitude can be determined is nearly as great as that for latitude observations.

From the observed latitudes and longitudes the triangulation net is located on the surface of the spheroid. Since the earth's surface is not a true spheroid, the latitude and longitude of any station as computed from the position of another station will be found to disagree with the latitude and longitude as directly observed. This discrepancy is principally due, not to errors of observation, but to irregularities in the density of the surface and to the effects of topography. The difference between the observed and the computed positions is known as the **station error**; it often amounts to 8 or 10 seconds of arc and averages about 2 or 3 seconds (equivalent to 200 or 300 feet on the earth's surface). For this reason the latitude and longitude of a single station will not be sufficient to locate accurately the triangulation system. A large number of points must be located, so that the station errors are nearly eliminated from the mean result and the true positions on the spheroid closely approximated.

The triangulation of the United States is computed on the surface known as the "Clarke Spheroid of 1866" and all geodetic positions are derived from that of a single station, namely,

Meades Ranch (Kansas). The adopted position of this station is

Latitude, 39° 13′ 26″.686; Longitude, 98° 32′ 30″.506

Azimuth to *Waldo*, 75° 28′ 14″.52

This adopted position of a single station on a specified spheroid is known as the **geodetic datum.** Since Canada and Mexico have agreed to employ this same datum for their triangulation it is called (since 1913) the *North American Datum.*

At any triangulation station where an azimuth and a longitude have been determined astronomically, the effect of local error of the plumb-line becomes known. This furnishes a means of correcting the accumulated errors of azimuth in the triangulation. Such stations are known as "Laplace" stations. Since 1909 it has been the practice to adopt the Laplace azimuth at all points where it is observed and to adjust to it the azimuths carried through the triangulation.

In 1927 a general readjustment was started of the triangulation systems in the United States and since that time positions based on this new adjustment are said to be referred to the North American Datum of 1927.

All triangulation which is connected with this initial point may be calculated on the N. A. Datum, and no astronomical observations are needed for determining the positions of new stations. They are needed, however, for determining the "station error" at these stations. If the survey in question is not connected with the main triangulation, such as surveys of islands or outlying territory, then astonomical positions must be determined in order to locate and orient the survey.

1-59. Azimuths. In addition to observations for latitude and longitude of stations it is also necessary to observe the azimuths of several of the triangle sides in order that the triangulation may be correctly oriented. Methods which will apply to nearly all cases are given in Chapter 2.

1-60. The Inverse Problem. When the latitude and longitude of two points are given it is possible, by means of these same formulas, to compute the length of the line, s, and the azimuths

α and α'. Solving (1-22) and (1-23) for $s \cos \alpha$ and $s \sin \alpha$, we have

$$s \cos \alpha = y = -\frac{1}{B}[\Delta\phi + Cx^2 + D(\Delta\phi)^2 + E(\Delta\phi)x^2] \quad (1\text{-}25)$$

$$s \sin \alpha = x = \frac{\Delta\lambda \cos \phi'}{A'} \quad (1\text{-}26)$$

$$\tan \alpha = \frac{x}{y};\ s = y \sec \alpha = x \csc \alpha \quad (1\text{-}27)$$

The same printed forms used for the computation of position may be used conveniently for this inverse computation.

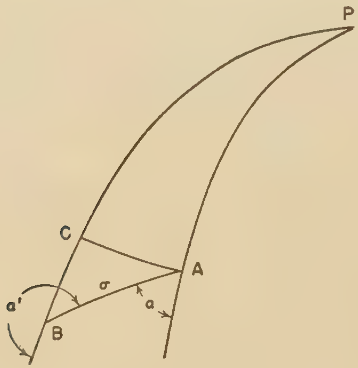

FIG. 1-50.

A second solution of this problem is as follows. In Fig. 1-50, we have, by Delambre's Equations

$$\cos \tfrac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}c} \cdot \sin \frac{C}{2}$$

$$\sin \tfrac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}c} \cdot \cos \frac{C}{2}$$

But $A = 180° - \phi$, $B = \alpha - \Delta\alpha$, from which we may find $\frac{1}{2}(A - B) = 90° - \alpha - \frac{\Delta\alpha}{2}$. Also, arc AB subtends an angle σ at the center. Then

$$\sin\frac{\sigma}{2}\cdot\sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\cos\tfrac{1}{2}(\phi+\phi')\sin\frac{\Delta\lambda}{2}$$

$$\sin\frac{\sigma}{2}\cdot\cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\sin\tfrac{1}{2}(\phi-\phi')\cos\frac{\Delta\lambda}{2}$$

Expressing σ as a distance in meters (s), and putting small angles for their sines,

$$s\sin\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{\Delta\lambda\cos\phi_m}{A_m}$$

$$s\cos\left(\alpha+\frac{\Delta\alpha}{2}\right)=\frac{-\Delta\phi\cdot\cos\dfrac{\Delta\lambda}{2}}{B_m}$$

in which A_m and B_m are the factors for the middle latitude, $\frac{1}{2}(\phi+\phi')$. For the most precise results corrections should be made to compensate for placing the sines equal to their angles. (See Coast Survey Spec. Publ. No. 145.)

1-61. Application of Triangulation to Small Surveys. The methods which have been described in this chapter are chiefly those which apply to extensive triangulation. In conducting smaller surveys the same general methods will apply, but many of the refinements may profitably be omitted. In such work a repeating instrument is always to be preferred, as it is simpler to use and is always amply accurate for the purpose. An engineer's transit with a 6-inch or a 7-inch circle, and with verniers reading to 30″, to 20″, or to 10″, is suitable for the triangulation here considered. Instead of carrying out the more elaborate program for measuring angles it will often be sufficient to take but one or two sets of angles, six repetitions being taken with the telescope direct and six with the telescope reversed.

While many of the refinements may be omitted, the work should not be done carelessly, and all of the general precautions in regard to protection and manipulation of the instrument should be carefully observed. The system should be planned so as to provide checks on the accuracy of the results. On account of the relatively short lines in such a survey special attention should

be given to centering the instrument and the signals over the stations. When using pole signals it is best to have a definite point on the mast (such as a white band) which is plumbed exactly over the station mark.

In measuring the base-line either an invar or a steel tape should be used. The tension should be given with a tension-handle or a spring balance. The temperature correction should be determined with an accuracy consistent with the requirements of the survey in question. There should be at least two measurements of the entire base so as to afford a check, since an error in the base will affect the entire triangulation.

The triangulation net may be oriented by observing the azimuth of one or more lines by the methods described in Chapter 2.

1-62. Plane Rectangular Coordinate Systems. When conducting surveys for such purposes as mapping and engineering projects, the use of plane rectangular coordinate systems with as wide-spread extent as possible offers numerous field and computational advantages. The ideal plane coordinate system would possess the following properties:

1. The X- and Y-coordinates of a point in the plane system are obtained from the corresponding latitude and longitude, and vice versa.

2. Distances between points obtained from their coordinates (grid distances) are equal to, or closely relatable to, the corresponding distances on the ground.

3. The azimuths obtained from coordinates (grid azimuths) are either equal to the corresponding true values or the interrelation between grid and true can be established.

4. The forward azimuth of a line on the grid is exactly 180° different from the back-azimuth.

Through the establishment in the United States of state-wide rectangular plane coordinate systems, as described in the following articles, the objectives cited above have been solved in such a manner as to present coordinate systems suitable for large-scale projects. The first and fourth conditions are completely satisfied.

However, since distances on a plane reference surface cannot everywhere equal the corresponding distances on the ground,

scale factors are established giving the appropriate relationships. These factors if, ignored, cause errors of little importance except in the most precise surveys; in which case, corrections may be applied. If the coordinate system is made rectangular, grid north will differ from true north, except along a selected meridian. The difference, however, is subject to a progressive and systematic change depending upon the departure from the meridian where true and grid norths are made the same. Through employing the property of *conformality* (Art. 9-2) in the selected systems, the equality of angles measured on the ground to the corresponding angles on the projection is maintained.

The plane rectangular coordinate grid systems used in the several states are indicated below.

LAMBERT SYSTEM

Arkansas
California
Colorado
Connecticut
Iowa
Kansas
Kentucky
Louisiana
Maryland
Massachusetts
Minnesota
Montana
Nebraska
North Carolina
North Dakota
Ohio
Oklahoma
Oregon
Pennsylvania
South Carolina
South Dakota
Tennessee
Texas
Utah
Virginia
Washington
West Virginia
Wisconsin
Long Island, N. Y.
Nantucket and Martha's Vineyard, Mass.

TRANSVERSE MERCATOR SYSTEM

Alabama
Arizona
Delaware
Georgia
Idaho
Illinois
Indiana
Maine
Michigan
Missouri
Nevada
New Hampshire
New Jersey
New Mexico
New York
Rhode Island
Vermont
Wyoming

BOTH SYSTEMS

Florida

1-63. The Transverse Mercator Projection.* Several states having a predominating north and south extent have adopted a system of plane coordinates based on the transverse Mercator projection. This is the ordinary Mercator projection (Art. 9-6) turned through an angle of 90° so that a selected central meridian takes the place of the equator in the ordinary Mercator projection. Whereas the family of straight lines perpendicular to each other are labeled as parallels of latitude and meridians of longitude in the ordinary Mercator, when rotated 90° they merely form the basic framework upon which a transverse Mercator projection can be made. Tabular data for computing positions on the transverse Mercator projection are given in U. S. Coast and Geodetic Survey Special Publication No. 67. The result is a system in which latitudes and longitudes plot as curved lines; the parallels curving upward toward the poles and the meridians converging toward the poles. True scale is maintained along two small circles whose planes are parallel to the central meridian plane and at some miles distant from it. The selection of these small circle planes aims to have the scales at the east and west margins of the area just as much too large as it is too small along the central meridian.

In the ordinary Mercator projection a cylinder is assumed to be wrapped around the earth tangent to the equator of the earth and the meridians and parallels are projected on the surface of the cylinder in accordance with certain mathematical relations so that the ratio of the length of a minute of latitude to that of a minute of longitude of any point of the projection is the same as the ratio existing at the corresponding point on the sphere. The cylinder is then developed (rolled flat), the meridians appearing as parallel equally spaced lines while the distances between the parallels of latitude increase toward the poles. In the transverse Mercator the cylinder is placed tangent to the central meridian of the area to be developed with the same mathematical relations referred to the central meridian instead of the equator.

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* See "The State Coordinate Systems — A Manual for Surveyors," Spec. Publication No. 235, U. S. Coast and Geodetic Survey. Also other Special Publications Nos. 68, 193 and 195.

It is a well-known fact that the scale varies all over a Mercator projection. The further we get from the equator on the projection the greater will be the distance on a scale that represents a mile; i.e., if $\frac{1}{8}$ inch represents a mile in latitude 0°, in latitude 60° (on a sphere) a mile would correspond to $\frac{1}{4}$ inch on the scale. In the vicinity of the equator the scale variation for a given distance north and south is less than the variation in scale for the same distance north or south in a higher latitude, say 45°.

As the central meridian in the transverse Mercator corresponds to the equator in the ordinary Mercator the scale along the meridian will be correct and the error will be greater the further we go east or west from the central meridian. It is for this reason that the transverse Mercator is particularly applicable to areas whose predominating dimension is north and south.

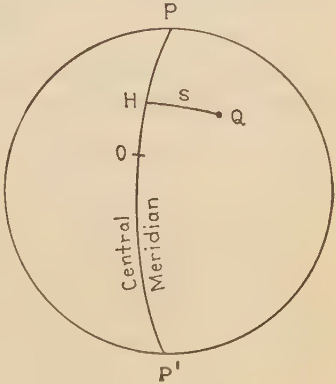

FIG. 1-51.

To develop the formulas for use in a transverse Mercator projection let Fig. 1-51 represent the earth, POP' the chosen central meridian of the transverse Mercator which is taken as the Y axis and O the chosen origin whose latitude and longitude are known. The point Q, whose latitude and longitude are known, is on the great circle HQ which is perpendicular to the meridian POP', at H. We first compute the geodetic length of the perpendicular $HQ = s$ and the latitude of H.

To compute the geodetic distance $HQ = s$ we may use formula 1-23.

$$\Delta\lambda = s \sin \alpha \sec \phi' \cdot A'$$

or $$s = \frac{\Delta\lambda}{\sin \alpha \sec \phi' \cdot A'} = \frac{\Delta\lambda \cos \phi'}{A'}, \text{ since } \alpha = 90° \text{ or } 270°;$$

the log s and log λ having been corrected for the assumption in

deriving the formula that the sine is equal to the arc. (See Art. 1-55, 4th paragraph on p. 75.)

To compute the latitude of H from Q, we use formula (1-22), page 73

$$-\Delta\phi = s \cos \alpha \cdot B + s^2 \sin^2 \alpha \cdot C + (\delta\phi)^2 D - hs^2 \sin^2 \alpha \cdot E$$

Since $\alpha = 90°$ or $270°$ and the D term is inappreciable this formula reduces to

$$-\Delta\phi = s^2 \cdot C$$

The factor C should be taken from Table III, p. 492, for the latitude of H, which is only known approximately; if the latitude of Q is used it will give an approximate difference of latitude. With the resulting approximate latitude of H a new value of C may be obtained and the latitude of H evaluated.

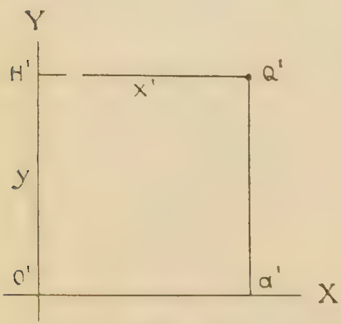

FIG. 1-52.

With the above two values, s and latitude of H known, we next proceed to determine the x and y coordinates on the projection of the point Q.

Developing the projection we obtain Fig. 1-52 in which $a'Q' = O'H' = y$, and $H'Q' = x'$. The general expression for the X coordinate on a Mercator projection is

$$x = s + \frac{s^3}{6R^2}$$

where s is the distance from the central meridian to a point and R is the radius of the earth.

To adapt this formula to the spheroid and the area in question the relation

$$x = s + \frac{s^3}{6\rho^2}$$

is used where ρ is the mean radius of curvature for the mean lati-

tude of the system and is equal to the square root of the product of the radius of curvature of the normal section (prime vertical) and the radius of curvature of the meridian.

The scale of the projection varies. In order to minimize this scale variation over the area to be mapped the scale of the projection is reduced; i.e., instead of having a tangent cylinder the cylinder is made to intersect the sphere at equal distances from the central meridian. The intersections are two small circles parallel to the central meridian (Fig. 1-53).

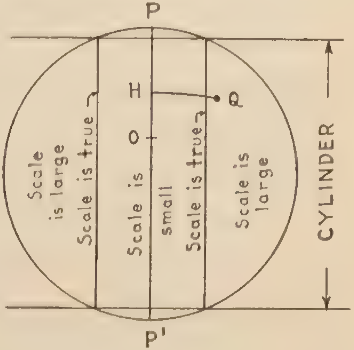

FIG. 1-53.

The ratio of the reduction in scale to the original is called the scale reduction factor. The scale is held true only along the intersections. Between the intersections the scale of the projection is too small while outside it is too large.

If S is the scale reduction factor the formula for the X coordinate (x') on the projection is

$$x' = s \cdot S + \frac{s^3 S^3}{6\rho^2 S^2}$$

Let $s \cdot S = s'$, the distance on the projection; then the formula becomes

$$x' = s' + \frac{s'^3}{6\rho^2 S^2}$$

in which $\frac{1}{6\rho^2 S^2}$ is a constant for any given projection system.

As the above values are in meters and the projection tables are usually given in feet, the factors in the above formula are multiplied by $\frac{3937}{1200}$.

Hence the formula becomes

$$x'_{\text{ft.}} = \frac{3937}{1200}s' + \frac{3937}{1200}\frac{s'^3}{6\rho^2 S^2}$$

The y coordinate is the distance on the projection along the central meridian from the origin O' to the foot of the perpendicular at H' (Fig. 1 52). Since the geodetic latitudes of O and H are known the geodetic distance between the points may be computed by the arc distance between the two points reduced to distance units multiplied by scale factor S. Hence $y = S(O'H')$ meters.

The U. S. Coast and Geodetic Survey has developed tables for states using the transverse Mercator projection which give the values of y in feet for minutes of latitudes of H. The y coordinates of O on the assumed projection may be readily interpolated by use of these tables.

This system is *conformal;* i.e., it has the characteristic that the scale is the same in all directions at any given point. Small areas are therefore represented in their true shape.

EXAMPLE: State of New Jersey.

Compute the transverse Mercator coordinates of Station Newark (Lat = ϕ = 40° 44′ 12″.771 N Long = λ = 74° 10′ 13″.651 W) with reference to an origin whose Lat = ϕ_0 = 38° 50′ N and Long = λ_0 (Central Meridian) = 74° 40′ W. Log S (log of scale reduction factor) = −108.6 (in 7th place of decimals) and the log $\frac{1}{6\rho^2 S^2}$ = 4.5810213 − 20.

$$s = \frac{\Delta\lambda \cos\phi'}{A'}$$

λ =	74° 40′ 00″.000
λ =	74° 10′ 13″.651
$\Delta\lambda$ =	+0° 29′ 46″.349
$\Delta\lambda$ =	1786″.349

log 1786″.349	3.25196631
Corr. arc − sin	−543
	3.25196088
colog A'	1.40000028
log cos 40° 44′ 12″.771	9.87050556
log s_1	4.62236672
Corr. arc − sin	+312
log s	4.62236984

$-\Delta\phi = s^2 \cdot C$ (C is taken for latitude of Sta. Newark and then corrected)

	1st Approx.	2nd Approx.
log s^2	= 9.244740	9.244740
log C	= 1.339570	1.339586
log $\Delta\phi$	= 0.584310	0.584326
$\Delta\phi$	= 3″.840	$\Delta\phi$ = 3″.840

Lat. (Newark)	$= 40° 44' 12''.771$		$40° 44' 12''.771$
	3.840		3.840
Approx. lat. H	$= 40° 44' 16''.611$		$40° 44' 16''.611$
Computing s' in feet		Computing $\frac{s'^3}{6\rho^2 S^2}$ in feet	
$\log s$	4.62236984	$\log (s'_{ft.})^3$	15.4150294
$\log S$	−1086	$\log \frac{1}{6\rho^2 S^2}$	4.5810213 − 20
$\log s'$	4.62235898		
$\log \frac{3937}{1200}$	0.51598417	$\log \frac{s'^3}{6\rho^2 S^2}$	19.9960507
$\log s'$ ft.	5.13834315		0.991 ft.
	137512.809 ft.		

$x' = 137512.809 + 0.991 = 137513.80$ ft.

Assumed x of central meridian = 500,000 ft. Therefore the x coördinate of Newark = **637513.80 ft.**

From the projection tables for the State of New Jersey, using the latitude at the foot of the perpendicular ($40° 44' 16''.611$, a y value of **693768.99** ft. is obtained).

1-64. Lambert Conformal Projection. If the area surveyed extends chiefly east and west these systems would not be satisfactory. For such an area the Lambert projection on the natural scale is well suited, for the error of scale does not change as we go east or west. The scale error may be fixed at any desired limit, according to the north-and-south limit chosen for the area, and then the survey may be extended east and west as far as desired.

The Lambert projection is a conical projection based on two standard parallels of latitude. On these two parallels the scale is exact; points lying on these parallels will therefore be represented at their correct distance apart. In between these parallels the scale is too small. Outside these standard parallels the scale is too large. On any parallel, however, the scale factor may be definitely determined, and proper allowance made for the error, so that correct distances may be obtained in any part of the area covered.

The parallels on the Lambert projection are concentric circles and the meridians are straight lines radial from the center of these circles (see Fig. 1-54). The radius r of any circle may be computed by the formula *

$$r = K \tan^l \frac{z}{2}$$

in which z is the complement of the geocentric latitude, and l is a

* See U. S. Coast & Geodetic Survey Spec. Publ. Nos. 52, 194 and 246.

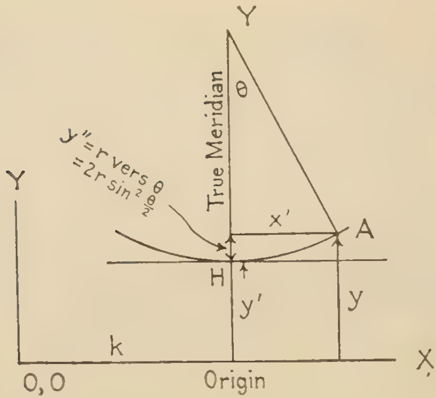

FIG. 1-54.

constant for this projection given by

$$l = \frac{\log \cos \phi_1 - \log \cos \phi_2 - \log A_1 + \log A_2}{\log \tan \frac{z_1}{2} - \log \tan \frac{z_2}{2}}$$

ϕ_1 and ϕ_2 are the latitudes of the standard parallels, A_1 and A_2 the longitude factors (Table III) and z_1 amd z_2 the complements of the geocentric latitudes for these parallels; K is a constant given by

$$K = \frac{\cos \phi_1}{A_1 \sin 1'' \, l \tan^l \frac{z_1}{2}} = \frac{\cos \phi_2}{A_2 \sin 1'' \, l \tan^l \frac{z_2}{2}}$$

From the above equations values of r may be found for every minute of latitude (or any other convenient interval) for the entire range of latitude. The origin may be selected at a point on the central meridian at some distance south of the area to be surveyed, thus avoiding negative values of the y coordinate.

The center of the arcs of the parallel is fixed by the position of the origin and the value of r for the latitude of the origin. If the coordinates of any point such as A, Fig. 1-54 are desired we first find the angle θ between the meridian of A and the central

meridian of the projection using the equation

$$\theta = l \Delta\lambda$$

l being a constant for any particular projection, and $\Delta\lambda$ the difference in longitude between A and the origin. y' is the ordinate of the parallel of A at the central meridian (point H) and is found from the difference between the two values of r.

Then $$y'' = r \operatorname{vers} \theta = 2r \sin^2 \frac{\theta}{2}$$

and $$y = y' + y''$$

Suppose k to be the assumed distance of the (0, 0) point west of the origin, then

$$x = xo + r \sin \theta$$

If the point A is west of the origin then θ is negative.

A Lambert conformal system devised by the U. S. Coast and Geodetic Survey for the State of Massachusetts has a maximum error of about 1 in 27,000. The standard parallels are 41° 43′ and 42° 41′. On the parallel 42° 12′ the scale is 1 part in 27,600 too small. Along parallels 41° 30′ and 42° 54′ it is 1 part in 26,200 too large. The origin is in latitude 41° 00′ N, 71° 30′ W. A list of scale factors has been computed for this range of latitude, enabling the engineer to obtain the correct distance from the distance found from his coordinates.

For the State of North Carolina the system in use has its standard parallels in latitude 34° 20′ and 36° 10′, the origin being at 33° 45′ N, 79° W. This point is assigned the coordinates x = 2,000,000 feet, y = 0 feet to avoid negative values. The maximum scale error is 1 in 6000. A list of scale factors has been computed for this projection, enabling the engineer to correct any distance.

Similar systems have been developed for all states which extend principally in an east and west direction. For states which extend

north and south, systems based on the transverse Mercator projection have been established.

EXAMPLE: State of Massachusetts.

Compute the Lambert conformal conic projection coordinates of Station White (Lat. $= \phi =$ $42° 21' 14''.105$ N, Long. $= \lambda = 71° 06' 34''.191$ W) with reference to an origin whose Lat. $= \phi_0$, ($y = 0$ ft.) $= 41° 00'$ N and Long. $= \lambda_0$, Central Meridian, ($x = 600{,}000$ ft.) $= 71° 30'$ W.

From tables prepared by the U. S. Coast and Geodetic Survey, using $\phi = 42° 21' 14''.105$, it will be found that $r = 23{,}056{,}103.61$ ft. and $y' = 403{,}373.71$ ft. For $\lambda = 71° 06' 34''.191$, we will find that $\theta = +0° 15' 44''.3222$.

$$\begin{aligned} x' &= r \sin \theta \\ &= (23{,}056{,}103.61)(+0.0045781872) = +105{,}555.16. \end{aligned}$$

Then

$$\begin{aligned} x &= x_0 + x' \\ &= 600{,}000.00 + 105{,}555.16 = \mathbf{705{,}555.16} \text{ ft.} \end{aligned}$$

$$\begin{aligned} y'' &= 2r \sin^2 \frac{\theta}{2} \\ &= 2(23{,}056{,}103.61)(0.00228\ 90996)^2 \\ &= 241.63 \text{ ft.} \end{aligned}$$

Then

$$\begin{aligned} y &= y' + y'' \\ &= 403{,}373.71 + 241.63 = \mathbf{493{,}615.34} \text{ ft.} \end{aligned}$$

1-65. Effect of Elevation of Land. It should be observed that all triangulation distances have been reduced to sea-level automatically with the reduction of the measured base-line (Art. 1-31). Consequently all x, y coordinates derived from the triangulation correspond to sea-level distances, and the distances computed from the coordinates are sea-level distances. If the actual surface of the ground is high above sea-level there will be a discrepancy between all measured distances and the computed distances. In order to make the system as useful and convenient as possible to local engineers it is customary to increase the sea-level distances to those corresponding to the mean elevation of the surface to be surveyed, so that no further attention need be given to this matter. In the city of Denver, Colorado, for example, with a mean elevation of nearly a mile, all horizontal distances would be nearly one 4000th part greater than the corresponding sea-level distances.

The following table gives factors for reducing distances at different elevations to sea-level. Inversely they may be used to adjust sea-level values to other elevations.

APPROXIMATE SEA-LEVEL REDUCTION TABLE

Elevation Feet	Sea-level Factor	Elevation Feet	Sea-level Factor
Sea-level	1.0000000	3,000	0.9998565
500	.9999761	3,500	.9998326
1,000	.9999522	4,000	.9998087
1,500	.9999283	4,500	.9997848
2,000	.9999043	5,000	.9997609
2,500	.9998804	5,500	.9997370

1-66. Triangulation for a Bridge. In triangulating across a river for locating a bridge it is important that the distance along the center line of the structure be determined accurately and very important that it be checked. The stations should be so located that one of the lines across the river is either the center line of the bridge or else sufficiently near to it to serve as a reference line from which the center line may be located.

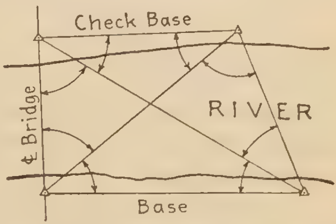

FIG. 1-55.

A base may be measured along one bank of the river directly between two of the instrument stations (Fig. 1-55). All of the eight angles should be measured if possible and the figure should be adjusted by the method given in Appendix B. The distance between the two stations on the opposite bank may then be accurately measured as a check base and the result compared with the computed length of this line; this will give a good idea of the precision of the computed distance across the river. In deciding upon a final value for the distance across the river the lengths of both bases may be taken into account. This may be done by using for the length of the check base a value intermediate between its measured and computed lengths, and then changing all dimensions in a constant ratio so as to be consistent with this adopted base.

For close results the two distances should be measured with a standardized invar tape, used as described in Art. 125, and the angles taken with a ten second repeating instrument, using 12 repetitions, 6 in each position of the telescope.

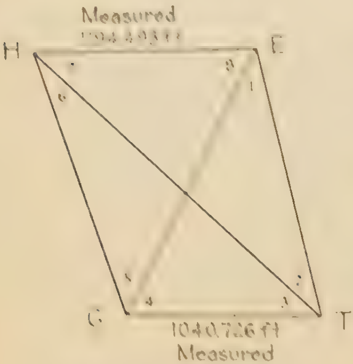

Fig. 150

With this equipment there should be no difficulty in measuring a 3000-ft. base within about 0.01 or 0.02 ft. and the angles within about one second. The distances may be carried through the triangles with an error of perhaps 0.03 or 0.04 ft. The distance across should therefore be within 0.04 or 0.05 ft. After adjusting the quadrilateral the above error should be somewhat reduced. The success with which this work can be carried out will depend considerably upon the kind of signals used and the care with which they are centered.

Example. In Fig. 150, the two base lines *H F* and *G T* have been measured with an invar tape; all angles were measured by means of a 10 second repeating instrument. The observed angles were:

1.	30° 34′ 11″.0	5.	45° 37′ 09″.3
2.	34 01 35 .8	6.	27 58 45 .2
3.	42 03 40 .0	7.	43 36 49 .2
4.	64 20 23 .1	8.	62 47 16 .3

Final angles after adjustment by first method of Appendix B.

1.	30° 34′ 18″.8	5.	45° 37′ 13″.3
2.	34 01 32 .9	6.	27 58 38 .4
3.	42 03 51 .0	7.	43 36 56 .0
4.	64 20 17 .3	8.	62 47 12 .3

Starting from *H F* with measured base 1104.403 ft., the distance *GT* is calculated through two sets of triangles (about the

two diagonals). The results for the logarithms are both equal to 3.0173356, giving 1040.724 ft., which checks the measured length GT.

PROBLEMS

1. The first tape-length of a base is read as 100.1085^m; the observed temperature is 78° F.; the difference in elevation of the end points is 20.80 ft. The tape is standard at 68° F.; the coefficient of expansion is .0000063 for 1° F. What is the true length of this section?

2. An interval between two base line stakes was measured with a 50-meter invar tape, the certificate of length for which is shown on page 37. If the difference of elevation of the ends of the tape was 6.21 ft. and the temperature of the tape at the time of measurement was 26° C., what is the length of this section at sea-level? The mean elevation of this section of base-line is 1210 ft.

3. Reduction to Center (Fig. 1-57).

Angles measured.

Pitman to West Hill	144° 19′ 56″.7
Pitman to Island	64° 05′ 20″.0
Pitman to Flag Pole	105° 06′ 07″.6

Distances in feet.

North Base to Pitman	3235.1
North Base to West Hill	8720.5
North Base to Island	10485.0
North Base to Flag Pole	7295.6

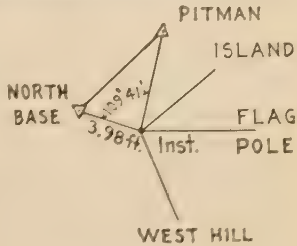

FIG. 1-57.

Reduce these angles to North Base.

4. Position of pt. L; latitude 42° 26′ 13″.276, longitude 70° 55′ 54″.088. Distance L to N, 3012.0 meters (log = 3.478 8600). Azimuth L to N, 314° 34′ 00″; back-azimuth, 134° 35′ 03″. Position of pt. N, latitude 42° 25′ 04″.764, longitude 70° 54′ 18″.232. Angle at L, 36° 15′ 07″; at N, 63° 44′ 59″; at E, 79° 59′ 57″. Compute the position of E from both lines, LE and NE, and also the back-azimuths of both lines. (E is east of LN.)

5. Position of point B { lat. 39° 13′ 26″.686 / long. 98° 32′ 30″.506 }

Position of point C { lat. 38° 51′ 50″.913 / long. 98° 29′ 15″.508 }

Azimuth B to C 353° 17′ 21″.81. Dist. 40232.35 meters. (log = 4.604 5734)

Back-azimuth 173° 19′ 24″.64

The spherical angles are A 57° 53′ 14″.39
B 62° 23′ 31″.40
C 59° 43′ 17″.93

Compute position of A for both lines and the back azimuths. (A is east of BC.)

6. A straight line is run due west from a point A in latitude 40° N, for a distance of 10 miles to point B. Compute the distance in feet that point B is due south of a true parallel of latitude through A. See equations (1-22), (1-23), and (1-24) of this chapter and Table III, p. 492.

7. Position of point B $\begin{cases} \text{lat. } 41°\ 38'\ 11''.330 \\ \text{long. } 98°\ 32'\ 45''.067 \end{cases}$

Position of point C $\begin{cases} \text{lat. } 41°\ 27'\ 58''.053 \\ \text{long. } 98°\ 33'\ 34''.309 \end{cases}$

Compute the distance BC and the forward and back-azimuths.

8. Referring to the triangle LNE in problem 4, compute the distance of a point X from stations L, N, and E, the angles being $LXN = 155°\ 29'\ 10''$, $LXE = 100°\ 38'\ 20''$ and $NXE = 103°\ 52'\ 30''$.

9. Compute the number of square miles in a triangle on the surface of the earth which has a spherical excess of one second.

10. A distance was measured with a Geodimeter and a reflector using a primary modulated frequency of 10 megacycles per second and a secondary frequency 1.01 times greater. The mean readings obtained were 15.62 meters for d and 16.13 meters for d'. The length of the light wave corrected for temperature, pressure and humidity was found to be 29.9727 meters. The approximate distance scaled from a map was 7.76 miles. The elevation of the Geodimeter instrument was 1589 feet and of the reflecting unit 2021 feet. Approximate radius of the earth's curvature is 6,371,000 meters. Length of one meter is 3.2808 feet.

(a) Find the length of the measured distance to 0.1 meter.
(b) Find the equivalent sea level distance.

11. What is the angle subtended by a 4-inch signal pole at a distance of seven miles from the instrument?

12. It is desired to sight from hill A (eleva. 600 ft.) to hill C (eleva. 650 ft.) 25 miles away. Hill B (eleva. 550 ft.), 10 miles away, obstructs the line. If towers of equal height on A and C are to be raised so that the line just clears B what will be the height of these towers? How high a tower would be required on C alone? On A alone?

13. If a traverse starts from a point B which is 12 miles from point A, in latitude 40° N, and follows the circumference of a circle of 12 miles radius about A what will be the error of closure when calculated on the plane tangent at A instead of on the sphere? Assume that there is no error in the measurements.

14. Compute the strength factors R_1 and R_2 for a regular pentagon with an interior station, the first side (base) being one side of the pentagon and the final line being a side not adjacent to the base. The pentagon has 18 new directions and 6 geometric conditions.

15. Compute the factors R_1 and R_2 for a regular hexagon. The number of conditions is 7.

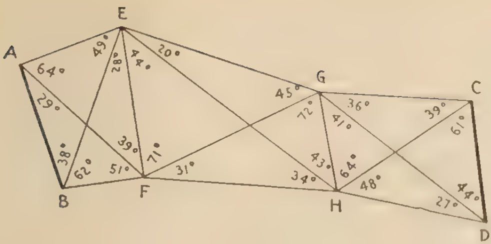

FIG. 1-58.

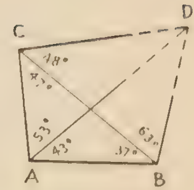

FIG. 1-59.

16. In Fig. 1-58 compute R_1 and R_2 between the bases AB and CD.

17. Compute the strength factors R_1 and R_2 for the quadrilateral shown in Fig. 1-59. Note that no directions have been observed at station D.

CHAPTER 2

ASTRONOMICAL OBSERVATIONS

2-1. Astronomical Observations. A complete treatment of the field methods for the precise determination of the latitude and longitude of a triangulation station and the azimuth of a line on a geodetic survey is beyond the scope of this book.* In this chapter the treatment of the subject will be confined chiefly to such observations as can be carried out with ordinary instruments and which apply to smaller and less precise surveys.

Before discussing these methods in detail it will be necessary to consider the fundamental principles of spherical astronomy.

2-2. Apparent Motions of Celestial Objects. The daily rotation of the earth on its axis and its annual revolution around the sun produce two apparent motions of the heavenly bodies which should be clearly understood. As a consequence of the rotation of the earth, which carries the observer continuously from west to east, all objects in the sky appear to him to revolve around the earth in the opposite direction, that is, they apparently rise in the east and set in the west. The annual revolution of the earth around the sun in a counterclockwise direction (when viewed from above the north) causes the sun to appear to the observer on the earth to move slowly eastward in the sky as judged by the position of the stars. If the sun's position were plotted every day on a star map, it would be seen to shift its apparent position eastward among the stars about 1° per day, completing its revolution in just one year. This causes the ap-

* For a treatment of these subjects the student is referred to the following works: Chauvenet, "Spherical and Practical Astronomy," J. B. Lippincott Company. Doolittle, "Practical Astronomy," John Wiley & Sons. Hayford, "Geodetic Astronomy," John Wiley & Sons. U. S. Coast and Geodetic Survey Special Publication No. 237, "Manual of Geodetic Astronomy." Hosmer and Robbins, "Practical Astronomy," John Wiley & Sons.

parent daily rate of motion of the stars to differ slightly from that of the sun. For this reason we see different constellations at different seasons of the year. This revolution of the earth around the sun also causes the planets to shift their positions among the stars, the character of the motion depending upon whether the planet is nearer to or farther from the sun than we are. The motion we see is a combination of the actual motion of the planet and an apparent motion produced by the orbital motion of the earth.

2-3. Definitions. In the problems of practical astronomy all heavenly bodies are considered as being projected onto the surface of a great sphere, whose center is at the center of the earth and whose radius is infinite. This is known as the *celestial sphere.*

The different properties of the celestial sphere depend in part upon the observer's position and in part upon the position of celestial bodies as projected on the surface of this sphere. Since problems of practical astronomy are based upon the interrelationships of these properties, a knowledge of the following definitions is required for a clear understanding of the subject.

Vertical. A *vertical line* at any place is the direction of gravity, i.e., the direction assumed by a plumb line at rest (Z to center of earth, Fig. 2-1). This line may be thought of as the extension of the plumb line of a transit.

Zenith. The *zenith* is the point where a vertical line, produced upward, pierces the surface of the celestial sphere (Z, Fig. 2-1).

Horizon. The *horizon* is the great circle on the celestial sphere cut by a plane (through the earth's center) at right angles to the vertical ($NESW$, Fig. 2-1). Since the radius of the celestial sphere is infinite, the horizon may be regarded as the great circle formed on the surface of the celestial sphere by an indefinite extension of the leveled horizontal plate of an engineer's transit.

Vertical Circle. A *vertical circle* is a great circle passing through the zenith ($ZS'B$ in Fig. 2-1 is an arc of a vertical circle). This circle is perpendicular to the horizon, and between the horizon and the zenith, points on any given vertical circle have the same direction or bearing. A vertical circle may be thought of as the

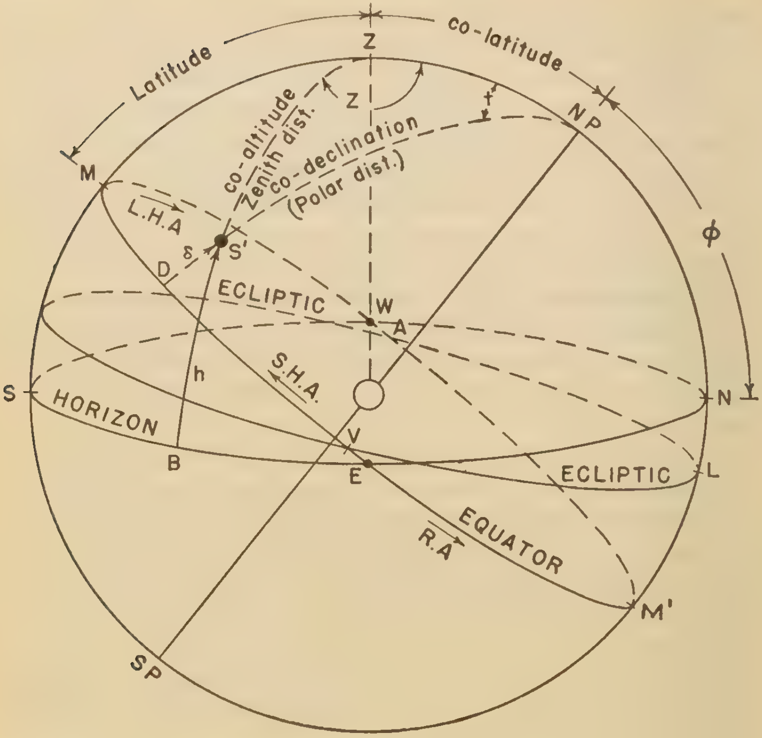

Fig. 2-1. The Celestial Sphere.

great circle formed on the celestial sphere by the line of sight of an engineer's transit when the telescope is rotated around its horizontal axis.

Parallel of Altitude. A *parallel of altitude* is a small circle on the celestial sphere, the plane of which is parallel to that of the horizon. This circle may be imagined to be formed by the line of sight of an engineer's transit when the telescope is clamped at a constant angle of elevation and the instrument is then rotated about its vertical axis.

Poles. The *celestial poles* are the points on the surface of the celestial sphere pierced by the extension of the earth's polar axis (*NP* and *SP*, Fig. 2-1)

Equator. The *equator* is that great circle on the celestial sphere formed by the extension of the plane of the earth's equator ($MWM'E$, Fig. 2-1). The points where the equator intersects the horizon are the east and west points of the horizon.

Hour Circle. An *hour circle* is a great circle passing through the poles and is therefore perpendicular to the equator ($NP - S'D$, Fig. 2-1, is an arc of an hour circle). Hour circles may be thought of as great circles on the celestial sphere formed by the indefinite extension of the planes of meridians of longitude.

Parallels of Declination. A *parallel of declination* is a small circle on the surface of the celestial sphere, the plane of which is parallel to that of the equator. Parallels of declination may be visualized as circles formed on the celestial sphere by the projections of parallels of latitude onto that sphere.

Meridian. The *meridian* of an observer is the great circle of the celestial sphere which passes through the poles and the observer's zenith. This great circle is both a vertical and an hour circle. Where the meridian intersects the horizon to the north and the south, respectively, are the north and south points of the horizon. The observer's meridian is the great circle formed by the extension of the plane of the particular meridian of longitude on which he is located.

Ecliptic. The *ecliptic* is the great circle of the celestial sphere which the sun appears to describe in its annual (eastward) motion among the stars (VLA, Fig. 2-1). The ecliptic is inclined to the equator by a nearly constant angle of about 23° 27′.

On account of the apparent motion of the sun in the ecliptic, it is carried from a point 23° 27′ North of the equator in June to a point 23° 27′ South of the equator in December, then back again to the starting point. For this reason, the sun will rise, in midsummer, at a place that is about 45° N latitude, nearly in the northeast. It will pass the meridian at an altitude of about 68°, and will set nearly in the northwest. In March and September, when the sun is on the equator, it will rise at the east point, cross the meridian at an altitude of 45° (noon) and set at the west point. In winter (December) it will rise near the southeast, will have an altitude of about 22° at noon, and set near the southwest.

Equinoxes. The points of intersection of the equator and the ecliptic are called the equinoxes. Where the sun apparently crosses the equator when going from south to north (which occurs in March) is called the *vernal equinox*. The other intersection (where the sun apparently crosses the equator in September) is called the *autumnal equinox*. In Fig. 2-1 V is the vernal and A the autumnal equinox.

2-4. Spherical Coordinates. The positions of celestial bodies and of points of reference on the celestial sphere are defined by spherical coordinates. The circles of reference in any spherical coordinate system are (1) a great circle of reference called the *primary*, and (2) a family of great circles called *secondaries* which pass through the pole of the primary and are, as a result, at right angles to the primary.

The coordinates must be referred to an *origin* somewhere on the primary circle, and the positive and negative directions of measurement from that origin must be specified. The primary coordinate is measured in the positive direction along the primary great circle from the origin to the foot of the secondary great circle passing through the body or point whose coordinates are desired. The secondary coordinate is measured along the secondary great circle from the primary to the body or point in question. The direction of measurement from the primary is determined by the sign of the coordinate, positive or negative.

2-5. The Horizon System. In the *horizon system* the primary circle is that of the horizon and the secondaries are vertical circles. The primary coordinate is *azimuth* and the secondary, *altitude*. The azimuth of a point is the arc of the horizon between the meridian and the vertical circle through the point. In Fig. 2-1 the azimuth of point S' is $SWNB$. In astronomical work azimuth is sometimes reckoned from the south point, as in surveying, sometimes from the north point, and is usually measured in a clockwise direction; the method used should always be specified. The altitude of a point is its angular distance measured along a vertical circle from the horizon to the point, and is considered positive when above the horizon. In Fig. 2-1 the altitude of point S' is BS'. The complement of the altitude is called the *zenith distance* (ZS' in Fig. 2-1). The position of an observer's

horizon system on the celestial sphere with reference to other systems is entirely dependent on his position on the earth.

2-6. The Equatorial Systems. The *equatorial systems*, of which there are several, have identical primary and secondary great circles. The primary great circle is that of the celestial equator and the secondaries are those of hour circles. Some of the equatorial systems are independent of the observer's position and one is dependent. They all have in common the same secondary coordinate called *declination*. The declination of a point is the angular distance measured along an hour circle from the equator to the point. In Fig. 2-1 the declination of point S' is DS'. Points north of the equator are considered as having plus declinations and points south of the equator as having minus declinations. The complement of the declination is called the *polar distance* ($NP - S'$ in Fig. 2-1).

2-7. Local Hour Angle System. The primary coordinate in this system is that of *local hour angle* ($L.H.A.$ or t). The local hour angle is the angular distance along the equator measured from the meridian (M in Fig. 2-1) to the foot of the hour circle (D) through the point in a **westward** direction (clockwise, when viewed from a position above the North Pole). In Fig. 2-1, the hour angle of point S' is $MWED$.

2-8. Right Ascension System. The *right ascension* system and the following two are quite independent of the observer's position, that is, their properties do not change when he changes his latitude and longitude. They are therefore suitable for listing the positions of stars, the sun, moon, and planets in an almanac or star catalogue. The *right ascension* of a point is the angular distance measured along the equator from the vernal equinox (V) **eastward** to the foot of the hour angle (D) through the point (counterclockwise, as viewed from a position above the North Pole). In Fig. 2-1 the right ascension of point S' is $VM'MD$. When hour angles are based on right ascensions the units employed are usually hours, minutes, and seconds **of time**.

2-9. Greenwich Hour Angle System. Another system of listing astronomical positions independent of the position of the observer is the *Greenwich Hour Angle* ($G.H.A.$) system. As this is an equatorial system, the primary and secondary great circles

are the equator and hour circles. However, the hour angles of bodies are based on the Greenwich meridian and the *G.H.A.* of a body at a given instant of time is the angular distance measured along the equator from the Greenwich meridian **westward** (clockwise, as viewed from above the North Pole), to the foot of the hour circle through the body. Thus the *G.H.A.* of a body is the longitude (west from Greenwich) in which the body is located. The *L.H.A.* of a body is similar to the *G.H.A.* with the modification that it is based on the local (observer's) meridian. The relation between *G.H.A.* and *L.H.A.* is

$$G.H.A. = L.H.A. + \lambda_W$$

or

$$G.H.A. - \lambda_W = L.H.A.$$

where λ_W is the observer's longitude west of Greenwich.

2-10. Sidereal Hour Angle System. The *Sidereal Hour Angle* (*S.H.A.*), applied only to stars, is the angular distance measured along the equator from the Vernal Equinox **westward** to the foot of the hour circle through the star. When the *G.H.A.* of the Vernal Equinox (constantly changing with time) is added to the *S.H.A.* of a star (very nearly constant), the result is the *G.H.A.* of the star. The same reasoning may be applied to local hour angles. The relations may be expressed in the following formulas:

$$G.H.A._{\text{♈}} + S.H.A._{\text{Star}} = G.H.A._{\text{Star}}$$

and

$$L.H.A._{\text{♈}} + S.H.A._{\text{Star}} = L.H.A._{\text{Star}}$$

The advantages of computations based on the *G.H.A.* and *S.H.A.* systems are that all values are in degrees, minutes, and seconds of arc, tabulations are given of *G.H.A.* values for each hour of Greenwich Civil Time, longitudes need not be converted into units of time, and the equation of time for the sun (Art. 2-19, p. 119) is automatically accounted for. The computations are therefore somewhat simplified. The disadvantage is that the listing of star data in the Nautical Almanac includes only 57 stars.

2-11. Coordinates of the Observer. The position of the observer may be defined in terms of spherical coordinates of *longitude* and *latitude*. On the terrestrial globe, the longitude of a

TABULATION OF SPHERICAL COORDINATE SYSTEMS

	Dependent on Observer's Position			Independent of Observer's Position (As viewed from above the North Pole)		
Name of System	Terrestrial	Horizon	Local Hour Angle	Right Ascension Counterclockwise	Greenwich Hour Angle Clockwise	Sidereal Hour Angle Clockwise
Primary Great Circle	Terrestrial Equator	Observer's Horizon	Celestial Equator	Celestial Equator	Celestial Equator	Celestial Equator
Secondary Great Circles	Meridians of Longitude	Vertical Circles	Hour Circles	Hour Circles	Hour Circles	Hour Circles
Primary Coordinate	Longitude (λ)	Azimuth (Z)	Local Hour Angle (L.H.A.)	Right Ascension (R)	Greenwich Hour Angle (G.H.A.)	Sidereal Hour Angle (S.H.A.)
Measured from (origin)	Meridian of Greenwich	South Point	Local Meridian	Vernal Equinox	Trace of Greenwich Meridian	Vernal Equinox
Positive Direction	Eastward or Westward	Clockwise	Westward	Eastward	Westward	Westward
Range	$0°$ to $180°$ E $0°$ to $180°$ W	$0°$ to $360°$	0^h to 24^h $0°$ to $360°$	0^h to 24^h	$0°$ to $360°$	$0°$ to $360°$
Secondary Coordinate	Latitude (L)	Altitude (h)	Declination (D)	Declination (D)	Declination (D)	Declination (D)
Range	$0°$ to $90°$ N $0°$ to $90°$ S	$0°$ to $+90°$ (+ above horizon)	$0°$ to $+90°$ N $0°$ to $-90°$ S	$0°$ to $+90°$ N $0°$ to $-90°$ S	$0°$ to $+90°$ N $0°$ to $-90°$ S	$0°$ to $+90°$ N $0°$ to $-90°$ S

NOTE: Conventionally right ascension is tabulated in units of time, and Hour Angle in units of arc.

point is the angular distance measured along the equator from a primary meridian (usually Greenwich) either westward or eastward (to 180°) to the foot of the meridian through the point. The latitude is the angular distance along the meridian of longitude from the equator north or south to the point.

On the celestial sphere the latitude is the declination of the observer's zenith. In Fig. 2-1, *MZ* is the latitude of the observer. The longitude would then be the *G.H.A.* of the observer's meridian.

The tabulation on p. 109 shows the relationships present in the various systems of spherical coordinates which are of concern in practical astronomy. It should be remembered that the primary coordinate is measured along the primary great circle from the origin to the foot of the secondary great circle through the point in whatever direction is positive, and that the secondary coordinate is measured to the point from the primary great circle along the secondary great circle through the point.

2-12. Astronomical Ephemerides. Data concerning the coordinates of celestial objects are given in a number of different publications which are issued annually. One common feature of almost all ephemerides is the tabulation of astronomical data according to *Ephemeris* or *Universal Time*.* (Universal Time is also called Greenwich Civil Time and Greenwich Mean Time.) **Therefore, before securing information relative to astonomical bodies, the Greenwich Civil Time (G.C.T.) must be found.**

The most common ephemerides are given below together with a partial listing of the included data which are most useful to the surveyor.

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* Time has normally been reckoned by the speed of rotation of the earth. This speed changes occasionally in an irregular and unpredictable fashion causing variations which cannot be calculated in advance. Beginning with 1960, Ephemeris Time, based on the revolution of the earth around the sun, has been adopted as the standard for precise timekeeping for physical measurements of high precision, and for accurate astronomical computations and theoretical investigations. Highly precise *predictions* must be referred to Ephemeris Time, but Universal Time is the basis of civil timekeeping and is the standard for highly precise astronomical *observations* and in the practical applications of astronomy. The difference between the two Times is very small, and (cannot be exactly determined until a later date) hence any data tabulated under Ephemeris Time can be used for all practical observational purposes as though it were listed under Universal Time.

1. **The American Ephemeris and Nautical Almanac,** issued annually by the Nautical Almanac Office, U. S. Naval Observatory, may be purchased from the U. S. Government Printing Office, and contains the following information:

Ephemeris of the sun daily for 0^h E.T., including right ascension, declination, semidiameter, equation of time.

Ephemeris of the moon for each hour E.T. including right ascension and declination; other data twice daily at 0^h and 12^h E.T., including semidiameter, horizontal parallax, and E.T. of transit.

Ephemeris of 7 planets daily for 0^h E.T., including right ascension, declination, semidiameter, horizontal parallax, and E.T. of transit.

Ephemeris of 1078 stars (mean places as of Jan. 1st, including Polaris).

Miscellaneous tables: sunrise, sunset, and twilight; moonrise, moonset; latitude and azimuth from observed altitude of Polaris; conversion of sidereal into mean time and vice versa; time to arc and arc to time.

2. **The Nautical Almanac,** issued by the Nautical Almanac Office, U. S. Naval Observatory, is also available from the U. S. Government Printing Office, and contains tables giving the following:

Greenwich Hour Angle (G.H.A.) and declination of the sun, moon, vernal equinox, and four planets (Venus, Mars, Jupiter, and Saturn) for each hour G.M.T.

Sidereal Hour Angle (S.H.A.) and declination of 57 navigational stars for every third day.

Sidereal Hour Angle (S.H.A.) and declination of 173 stars for each month.

Tables of sunrise, sunset, twilight, moonrise, and moonset for each day, plus meridian passage of sun and moon each day; correction tables giving increase in G.H.A. for each minute and second of time (0 to 1 hour) for the sun, moon, planets, and vernal equinox; tables for determining latitude and azimuth from an observed altitude of Polaris; observed altitude correction tables.

Star charts showing positions of certain bright stars and constellations.

3. **The Ephemeris** prepared by the Nautical Almanac Office, U. S. Naval Observatory for the Bureau of Land Management, Department of the Interior, and contains:

Ephemeris of the sun daily for *Greenwich Apparent Noon*, including declination, semidiameter, time of transit, and equation of time.

Polaris for the Meridian of Greenwich, Civil Date and Mean Time, upper culmination, elongation Latitude 40°, and declination, daily.

Apparent places of 28 stars, including time of meridian transit at Greenwich and declination twice each month.

Tables for finding latitude and azimuth by an altitude observation of Polaris at any hour angle.

Tables for altitude corrections and conversions of time to arc and vice versa.

4. **Apparent Places of Fundamental Stars,** published by order of the British Admiralty, may be purchased from Her Majesty's Stationery Office, London, England, and includes the following data:

Apparent places (declination and right ascension) for 10-day intervals of 1535 stars.

Apparent places of 52 northern and southern circumpolar stars for daily intervals.

Hour angle of vernal equinox 0^h G.C.T.

Tables for converting sidereal into mean solar time and vice versa.

5. **The Star Almanac for Land Surveyors,** prepared annually by H. M. Nautical Office and published by Her Majesty's Stationery Office, London, England. The tables give the declination of the sun to nearest $0.1''$ of arc and its G.H.A. to nearest $0^s.1$ at 6-hour intervals; and the declination to $01''$ and right ascension to $0^s.1$ of 650 of the brighter stars each month. Other tables for latitude and azimuth determinations from observations on Polaris are included as well as conversion and miscellaneous tables.

6. Several Instrument Makers issue small pamphlets yearly, listing select data on the sun, vernal equinox, and Polaris daily, and for 26 stars monthly or oftener.

2-13. Relation between Altitude of Pole and Latitude of Place. In Fig. 2-2 the arc PN = arc EZ, since PO is perpendicular to

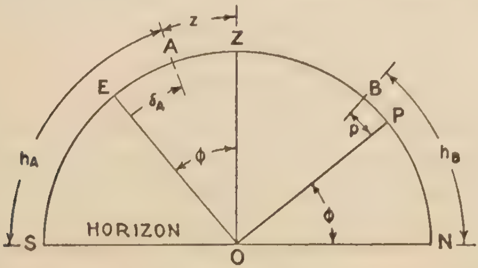

FIG. 2-2. SECTION OF HEMISPHERE ON PLANE OF MERIDIAN.

EO and NO is perpendicular to ZO. But PN is the altitude of the pole, and EZ is the latitude of the observer. Hence the **altitude of the pole equals the latitude of the observer.**

2-14. Relation between Altitude, Latitude and Declination, for an Object on the Observer's Meridian. The relation existing between the altitude of a point on the meridian, the latitude of the observer, and the declination of the point may be seen from Fig. 2-2.

Let A be the point on the meridian; then

$$EZ = \text{the latitude} = \phi$$
$$EA = \text{the declination} = \delta$$
$$SA = \text{the altitude} = h$$
$$ZA = \text{the zenith distance} = z$$

From the figure

$$ZA = EZ - EA$$

or

$$z = \phi - \delta$$

and

$$h = 90° - (\phi - \delta) \qquad (2\text{-}1)$$

This equation also holds true for a point south of the equator, provided δ is given the negative sign.

If the point is near the pole, as at B, the following relation is convenient.

$$PN = BN - BP$$

or

$$\phi = h - p \tag{2-2}$$

where p is the polar distance of B, or $90° - \delta$. If the point B were below the pole the equation would be

$$\phi = h + p \tag{2-3}$$

2-15. General Relation between Coordinate Systems. The relations existing among the coordinates of two spherical coordinate systems can usually be determined by solving the spherical triangle that is formed by joining the Pole, the Zenith, and the Celestial Object. This triangle (Fig. 2-3) may be solved by the

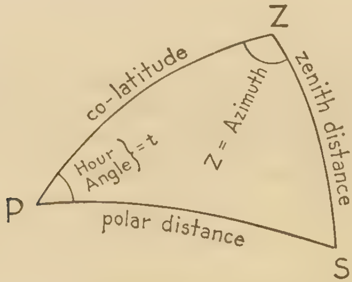

FIG. 2-3.

usual formulas of spherical trigonometry. The formulas usually given as the fundamental equations in spherical astronomy are,

1. The sine law,

$$\sin a \sin B = \sin b \sin A$$

2. The cosine law,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

and

3. The equation

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

The derivation of these may be found in any book on spherical trigonometry. As a matter of fact, all of the equations of spherical trigonometry may be derived from the cosine law alone. In practice many other formulas are used, either for convenience or for greater accuracy in computation. These special forms will be given as occasion arises.

2-16. The Astronomical Triangle. The triangle formed by joining the pole, the zenith, and the object (star, sun, etc.) by arcs of great circles, is called the astronomical triangle ($NP - Z - S'$, Fig. 2-1). The arc ZS' is the zenith distance, ZP is the complement of the latitude (= co-latitude), PS' is the polar distance. The angle at P is either the hour angle or 360° minus the hour angle, and the angle at Z is the azimuth, measured from the north point toward the east or the west.

As instances of the application of the solution of this triangle to practical problems the following may be mentioned.

1. Given the measured altitude of the sun, the sun's declination, and the observer's latitude, to calculate the sun's true bearing, and the hour angle of the sun (from which the local solar time, and therefore the longitude, is found).

2. Given the declination of a star and its hour angle, and the observer's latitude, to calculate the star's altitude and azimuth.

Formulas derived from the astronomical triangle for azimuth and hour angle are given below, in which

Z_n = azimuth from north (to east or to west)
t = hour angle (to east or to west)
ϕ = latitude
h = altitude
δ = declination
p = polar distance $(90° - \delta)$
and $s = \frac{1}{2}(\phi + h + p)$

For computing the azimuth any of the following formulas may be used.

$$\sin \frac{1}{2} Z_n = \sqrt{\frac{\sin (s - h) \sin (s - \phi)}{\cos \phi \cos h}} \qquad (2\text{-}4)$$

$$\cos \frac{1}{2} Z_n = \sqrt{\frac{\cos s \cos (s - p)}{\cos \phi \cos h}} \tag{2-5}$$

$$\tan \frac{1}{2} Z_n = \sqrt{\frac{\sin (s - \phi) \sin (s - h)}{\cos s \cos (s - p)}} \tag{2-6}$$

$$\cos Z_n = \frac{\sin \delta - \sin \phi \sin h}{\cos \phi \cos h}$$

$$\cos Z_n = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h \tag{2-7}$$

For computing the hour angle any of the following formulas may be used.

$$\sin \frac{1}{2} t = \sqrt{\frac{\cos s \sin (s - h)}{\cos \phi \sin p}} \tag{2-8}$$

$$\cos \frac{1}{2} t = \sqrt{\frac{\cos (s - p) \sin (s - \phi)}{\cos \phi \sin p}} \tag{2-9}$$

$$\tan \frac{1}{2} t = \sqrt{\frac{\cos s \sin (s - h)}{\cos (s - p) \sin (s - \phi)}} \tag{2-10}$$

$$\cos t = \frac{\sin h - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

$$= \frac{\sin h}{\cos \phi \cos \delta} - \tan \phi \tan \delta \tag{2-11}$$

2-17. Greatest Elongation. A special case of the astronomical triangle occurs when it is right angled at the star. This position is known as its "greatest elongation." This can occur only for stars whose polar distance is less than 90° − observer's Latitude

(that is, declination greater than latitude). In Fig. 2-4, point S represents a circumpolar * star at the instant of western elongation (northern hemisphere), and S' the same star when at east-

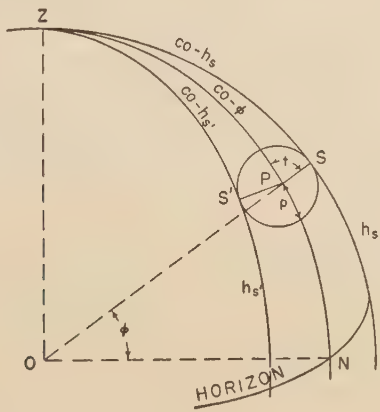

FIG. 2-4. CIRCUMPOLAR STAR AT ELONGATIONS.

ern elongation. When the star is at S the angle $PSZ = 90°$ because the great circle ZS is tangent to the star's diurnal circle at S.† In this position the azimuth of S is given by

$$\sin Z = \frac{\cos \delta}{\cos \phi} \tag{2-12}$$

the angle Z being counted from the north point N. The hour angle of the star ($ZPS = t$) at the same instant is found from the relation

$$\cos t = \frac{\tan \phi}{\tan \delta} \tag{2-13}$$

2-18. Circumpolar Star at Any Hour Angle. If the star is not at elongation but its hour angle is known, then its azimuth may

---

* A circumpolar star in any given latitude is one which never goes below the horizon; hence its polar distance must be less than the given latitude.

† Note that angle $ZPS = t$ is always somewhat less than 90° (or 6^h).

be found by the formula

$$\tan Z = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t} \tag{2-14}$$

which may be derived as follows. From the fundamental equation of spherical trigonometry we have

$$\cos h \sin Z = \cos \delta \sin t$$

and
$$\cos h \cos Z = \sin \delta \cos \phi - \cos \delta \sin \phi \cos t$$

Dividing we have

$$\tan Z = \frac{\cos \delta \sin t}{\sin \delta \cos \phi - \cos \delta \sin \phi \cos t}$$

Dividing both numerator and denominator by $\cos \delta$ we obtain equation (2-14) given above.

If the polar distance, hour angle and altitude of a circumpolar star are known, the azimuth may be found by the following

$$\sin Z = \sin p \sin t \sec h \tag{2-15}$$

2-19. Time — Definitions. The measurement of time depends directly upon the apparent motion of heavenly bodies caused by the earth's rotation on its axis. To an observer on the earth looking north the heavenly bodies appear to revolve in a *counterclockwise* direction and to cross the observer's meridian twice each day.

Transit. The instant when a body is on the meridian of an observer is called its *transit* or *culmination.* When it is on the side of the meridian containing the zenith it is called the *upper transit;* when it is on the other side it is called the *lower transit.* Except in the case of circumpolar stars the upper transit is the only one visible to the observer; hence when the transit of a star is mentioned, the upper transit is intended unless otherwise specified.

Sidereal Day. A sidereal day is the interval of time between two successive upper transits of the **vernal equinox** over the same meridian.

Sidereal Time. The sidereal time at a given meridian at any instant is the **hour angle** of the **vernal equinox.** The sidereal

time as found from observation on a star is

$$S = R + t \tag{2-16}$$

where S = the sidereal time
R = the right ascension of the star
t = the hour angle of the star

The relation expressed in (2-16) is evident from the definitions of these three angles (Art. 2-10). Referring to Fig. 2-1 we have for the position of point W,

$$MWV = VEM'W + MW$$

which becomes identical with (2-16) from the definitions.

At the instant of transit we have

$$t = 0$$

hence

$$S = R \tag{2-17}$$

i.e., **the right ascension of a star equals the sidereal time at the instant when that star is on the meridian.**

Solar Day. A solar day is the interval of time between two successive lower transits of the sun over the same meridian.

Solar Time. The solar time at any instant is the hour angle of the sun's center at that instant plus 180° or 12^h. The apparent angular motion of the sun is not uniform, hence for most purposes we use the time kept by what is known as the *fictitious sun* or *mean sun*, which is an imaginary point conceived to move at a uniform rate along the equator at such a speed as to make one revolution in the same time as the actual sun. The time kept by this mean sun is called *mean solar time*. The time kept by the real sun is called *apparent time*. This is the time shown by a sun-dial; it is also the time computed from an observed altitude of the sun. The difference between mean time and apparent time is called the *equation of time*. (Eq. T = App. T − Mean T.) It is given in the American Ephemeris and Nautical Almanac for the instant of 0^h (midnight) at Greenwich for each day in the year, and in the Nautical Almanac twice daily for 0^h and 12^h G.C.T.

Civil Time. The *civil day* begins at midnight. For astronomical purposes the civil day is divided into 24 hours, numbered from 0^h to 24^h. In this system the afternoon hours are greater than 12. For ordinary purposes the day is divided into two parts of 12 hours each; from midnight to noon is called A.M., and from noon to midnight is called P.M.; in either case the civil date changes at midnight.

Data in the Nautical Almanac are given for the instant of 0^h Greenwich Civil Time (that is, at midnight).

Local Time. When used with reference to astronomical observations, Local Time applies to the particular meridian of the observer, and is denoted by the initial "L," such as L.C.T., L.M.T. or L.S.T.

2-20. Longitude and Time. The hour angle of the mean sun counted from the lower half of the meridian at any place is the local civil time at that place. The hour angle of the sun from the lower meridian at Greenwich is the corresponding Greenwich civil time. The difference is the longitude of the place east or west of Greenwich, expressed in units of time. In Fig. 2-5, $A'B'ABC$ represents the civil time at Greenwich, and $B'ABC$ the civil time at meridian B. The difference $A'B'$, or AB, is the west longitude of meridian B.

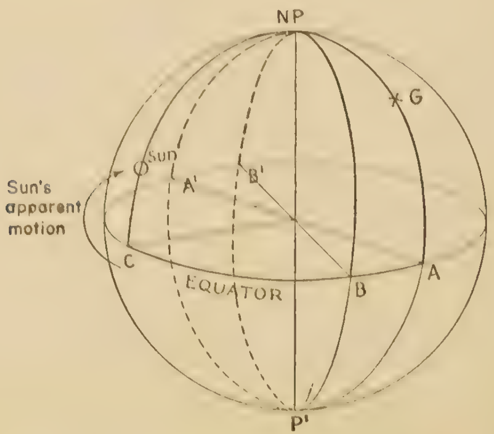

FIG. 2-5. RELATION BETWEEN LONGITUDE AND TIME.

Again, if C is the vernal equinox, the local sidereal time at B is BC, and the local sidereal time at A is AC. Notice that these are reckoned from the upper branch of the observer's meridian. The difference AB is the west longitude of B from A. If A were any other meridian than that of Greenwich the difference of the local times whether sidereal, civil or apparent would be the difference of longitude AB expressed in hours, minutes and seconds.

EXAMPLE 1. The local apparent time at a ship is $15^h\ 30^m$ at the same instant that it is $17^h\ 50^m$ apparent time at Greenwich. What is the ship's longitude? $17^h\ 50^m - 15^h\ 30^m = 2^h\ 20^m$ west longitude $= 35°$ West.

EXAMPLE 2. A star having a right ascension of 4^h passes a meridian at the instant it is $7^h\ 30^m$ sidereal time at Greenwich. What is the longitude of this meridian? $7^h\ 30^m - 4^h = 3^h\ 30^m = 52°\ 30'$ W.

Since $24^h = 360°$, then $15° = 1^h$, $15' = 1^m$, and $15'' = 1^s$. Any angle expressed in degrees, minutes, and seconds may be converted into hours, minutes, and seconds by dividing by 15. The following relations will also be found useful when making this conversion; $1° = 4^m$ and $1' = 4^s$.

EXAMPLE. Boston, Mass., is in longitude $71°\ 04'$ W and Peoria, Ill., is in $89°\ 35'$ W. What is the difference in local time?

$$\begin{array}{rl}
\text{Peoria} = & 89°\ 35' \\
\text{Boston} = & 71\ \ \ 04 \\
\hline
\text{difference} = & 18°\ 31' \\
1^h = & 15° \\
\hline
 & 3°\ 31' \\
12^m = & 3° \\
\hline
 & 31' \\
2^m = & 30' \\
\hline
 & 1' \\
4^s = & 1' \\
\hline
 & 0
\end{array}$$

Difference in time $= 1^h\ 14^m\ 04^s$ (earlier at Peoria).

2-21. Standard Time. In a country extending over many degrees of longitude, and especially where people travel extensively, local time is not convenient, since it would be necessary for a traveler to set his watch frequently and by varying amounts. In the United States, for example, the following system is in use. The country is divided into four time belts known as the Eastern, Central, Mountain, and Pacific time belts. In all parts of the Eastern belt clocks are regulated to the local mean time of the 75° meridian west of Greenwich; in the Central belt the local time of the 90° meridian is used; in the Mountain belt the 105° meridian is used; and in the Pacific belt the 120° meridian is used. Each standard meridian is 15° west of the preceding and the time is just 1 hour earlier than that of the preceeding standard meridian. In the Eastern belt the clocks are 5 hours earlier than the Greenwich clock. In the Central belt they are 6 hours earlier, in the Mountain belt 7 hours, and in the Pacific belt 8 hours earlier than Greenwich.

The time of the 60° meridian is used in the eastern provinces of Canada and the time is called *Atlantic Time.*

The time designated as *Daylight Saving Time* in any belt is the standard time of the belt lying 1 hour east of it.

It is convenient in solving astronomical problems to designate the standard time belts as time *Zones.* These zones are positive when the time belt is west of Greenwich and negative when east. Eastern Standard Time is designated as Zone $+5^h$, Central Time as Zone $+6^h$, Mountain Time as Zone $+7^h$, and Pacific Time as Zone $+8^h$. Daylight Saving Time in any of the above zones would be that of the regular standard time of the zone next to the eastward. For example, Eastern Daylight Saving Time could just as well be referred to as 60th meridian time, or Zone $+4^h$. The advantage of using time zones is that the Greenwich Civil Time may be obtained by adding the zone designation to the standard time in the zone where the time was taken.

2-22. Solar and Sidereal Intervals. Since, as viewed from a position above the North Pole, the earth's motion about the sun and its rotation about its axis are both counterclockwise, the sun has an apparent eastward motion among the stars. For this reason the sun is retarded in its apparent daily (westward) motion,

making the solar day about 4 minutes longer than the sidereal day. It will be seen that the earth makes a little more than one rotation on its axis in a solar day. Since this daily retardation is just enough to bring the sun back to its starting point at the end of one year, there will be just one more sidereal day in the year than there are solar days. The length of the tropical * year is 365.2422 mean solar days, hence

$$366.2422 \text{ sidereal days} = 365.2422 \text{ solar days}$$

or
$$1 \text{ sidereal day} = 0.99726957 \text{ solar days}$$

and
$$1 \text{ solar day} = 1.00273791 \text{ sidereal days}$$

Since the sidereal day is the shorter and is divided into the same number of hours, minutes, and seconds, these units of time are all shorter than the corresponding solar units, so that in any given interval of time there will be more sidereal units than solar units. If a chronometer were regulated to sidereal time and a watch were regulated to mean solar time, the chronometer would run faster than the watch, the gain being about 10^s per hour, or, more nearly, $3^m\ 56^s$ per day. If the two timepieces agreed at a certain date, then they would again agree just one year later, the chronometer having gained exactly one day.

If I_m is a mean solar interval and I_s is the corresponding sidereal interval,

then
$$I_s = I_m + .00273791 \times I_m$$

and
$$I_m = I_s - .00273043 \times I_s$$

These operations are readily performed by the aid of Tables V and VI in the back of this volume.† In using these tables it will

* The tropical year is the interval of time in which the sun apparently makes one revolution about the earth measured from the equinox to the equinox again.

† Tables for the conversion of sidereal into mean solar time and vice versa are also to be found in both the American Ephemeris and Nautical Almanac and in the Nautical Almanac.

be necessary to take out the corrections for the hours, minutes, and seconds separately, and add them together.

EXAMPLES. To reduce $9^h\ 23^m\ 51^s.0$ interval of sidereal time to the equivalent interval of mean solar time we enter Table V and in the column opposite 9^h we find $1^m\ 28^s.466$; in the column opposite 23^m we find $3^s.768$, and in the column opposite 51^s, we find $0^s.139$. The sum of these three, $1^m\ 32^s.373$, is the correction to be subtracted from the given interval, giving $9^h\ 22^m\ 18^s.6$ of mean solar time.

To reduce $7^h\ 10^m$ to sidereal time we find, similarly, from Table VI $1^m\ 8^s.995$ opposite the 7^h and $1^s.643$ opposite the 10^m, the sum of which, $1^m\ 10^s.6$, added to $7^h\ 10^m$, gives $7^h\ 11^m\ 10^s.6$ of sidereal time.

2-23. Relation between Sidereal and Mean Solar Time. The sidereal time at any instant is the hour angle of the vernal equinox, and civil time at the same instant is the hour angle of mean midnight, hence the relation between sidereal time and civil time is given by the equation

$$S = (\text{Hour Angle of } V\ @\ 0^h) + t_s \qquad (2\text{-}18)$$

where t_s is the hour angle of midnight, or the civil time. The Hour Angle of the Vernal Equinox is tabulated daily for 0^h U.T. in the American Ephemeris and Nautical Almanac (in units of time) * and for each hour in the Nautical Almanac (in units of arc). The hour angle of V is 0^h about September 22, and increases at a uniform rate of nearly 4^m (or $1°$) per day. Hence

$$S = (\text{H.A.}_v\ @\ 0^h) + t_s + C \qquad (2\text{-}19)$$

in which C is the increase in H.A.$_v$ for the number of hours in the interval t_s. This may be taken from Table VI.

EXAMPLES. To find the sidereal time corresponding to $9^h\ 22^m\ 18^s.60$ Eastern Standard Time (Zone $+5^h$) on Jan. 7,

* The American Ephemeris and Nautical Almanac tabulates daily the hour angle of the Vernal Equinox at 0^h U.T. (H.A.$_v$ @ 0^h) which supersedes the former listing of the Right Ascension of the Mean Sun $+ 12^h$, ($R_s + 12^h$).

1960. The first step is to find the Greenwich Civil Time (Fig. 2-6).

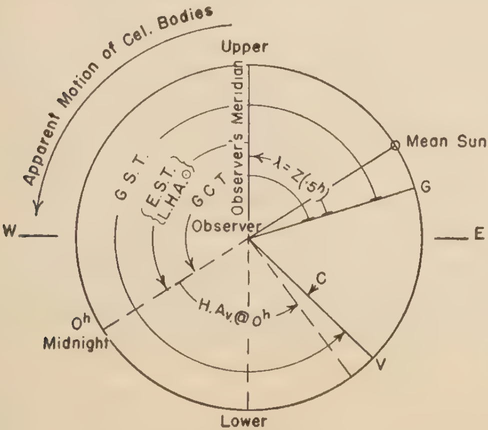

FIG. 2-6. RELATION BETWEEN SIDEREAL AND CIVIL TIME.
(As viewed facing north)

Eastern Standard Time =	$9^h\ 22^m\ 18^s.60$	Jan. 7, 1960
Time Zone Difference =	$+5$	
G.C.T. =	$14^h\ 22^m\ 18^s.60$	Jan. 7, 1960

From the American Ephemeris and Nautical Almanac, the value of H.A.$_v$ @ 0^h U.T., Jan. 7, 1960, is $7^h\ 02^m\ 17^s.66$. From Table VI we find for $14^h\ 22^m\ 18^s.60$, $C' = +2^m\ 21^s.66$.

G.C.T. =	$14^h\ 22^m\ 18^s.60$
H.A.$_v$ =	$7\ \ 02\ \ 17\ .66$
From Table VI, C =	$+\ \ 2\ \ 21\ .66$
Greenwich Sidereal Time =	$21^h\ 26^m\ 57^s.92$

NOTE. — If the sidereal time exceeds 24^h, the result should be decreased by 24^h.

If it is desired to find the civil time from the sidereal time, we may write

$$\text{Sidereal time interval from midnight} = S - (\text{H.A.}_v\ @\ 0^h)$$

or

$$\text{Civil time interval from midnight} = S - (\text{H.A.}_v\ @\ 0^h) - C' \qquad (2\text{-}20)$$

C' is the correction from Table V to reduce $S - (\text{H.A.}_v)$ to a mean solar interval. C' represents the increase in (H.A._v) during $S - (\text{H.A.}_v)$ *sidereal hours.*

To find the civil time when the Greenwich Sidereal Time is $21^h\ 26^m\ 57^s.92$ on Jan. 7, 1960.

Greenwich Sidereal Time (S) =	$21^h\ 26^m\ 57^s.92$
H.A._v at 0^h G.C.T. =	7 02 17 .66
Sid. time int. since midnight =	14 24 40 .26
From Table V, C' =	− 2 21 .66
Greenwich Civil Time =	$14^h\ 22^m\ 18^s.60$

NOTE. — If the quantity H.A._v @ 0^h is greater than the sidereal time, the latter must be increased by 24^h before subtracting.

When the above calculation is performed using the Nautical Almanac, the Greenwich Sidereal Time is first converted into angular measure giving the equivalent Greenwich sidereal hour angle which is also the hour angle of the vernal equinox. The Tables in this almanac are then used to find directly the corresponding Greenwich Civil Time. This is done in two steps; first the G.H.A._v for the nearest full hour of G.C.T. is found under the date of computation (14^h on Jan. 7, 1960, in the example below), then the minutes and seconds of G.C.T. corresponding to the remaining G.H.A._v are found from correction tables in the back of this Almanac, as illustrated below:

Greenwich Sidereal Time (S) =	$21^h\ 26^m\ 57^s.92$
G. Sid. H.A. = G.H.A._v =	$321°\ 44'.5$
G.H.A._v for 14^h G.C.T., Jan. 7, 1960 =	316 08 .9
Cor. in G.H.A._v for $22^m\ 18^s.7$ G.C.T. =	$5°\ 35'.6$ *
G.C.T. =	$14^h\ 22^m\ 18^s.7$

NOTE. — The results obtained with the Nautical Almanac may differ slightly from those obtained with the American Ephemeris because the angles in the former are given only to the nearest minute.

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* The increase in the hour angle of the vernal equinox and of stars is at the rate of $15°\ 02'.5$ per mean time hour; $15°\ 00'.0$ for sun; and varying rates for the moon and planets. The minutes and seconds of time corresponding to $5°\ 35'.6$ were obtained from tabulation of Increments and Corrections given in the yellow section of the Nautical Almanac.

The procedure for finding Greenwich Sidereal Time from Greenwich Civil Time by the Greenwich Hour Angle method is as follows:

$$
\begin{array}{rl}
\text{G.H.A.}_{\gamma} \text{ for } 14^h \text{ G.C.T., Jan. 7, 1960} = & 316^\circ\ 08'.9 \\
\text{Correction in G.H.A.}_{\gamma} \text{ for } 22^m\ 18^s.60 = & \underline{5\ \ 35\ .6} \\
 & 321^\circ\ 44'.5 \\
\text{Changing to units of time, G. Sid. T.} = & 21^h\ 26^m\ 57^s.9
\end{array}
$$

In any conversion of time, it will be found most satisfactory to convert the local standard time to Greenwich time of the same units (Arts. 2-20,21) before making the conversion. When dealing with civil times a strict account must be kept of the date as this may change between the local (or standard) meridian and Greenwich.

EXAMPLE. On May 4, 1960, the local time at San Francisco (longitude 122° 13'.4 W. = $8^h\ 08^m\ 53^s.6$) is $20^h\ 57^m\ 46^s.9$ and the Greenwich Civil Time is required (Fig. 2-7).

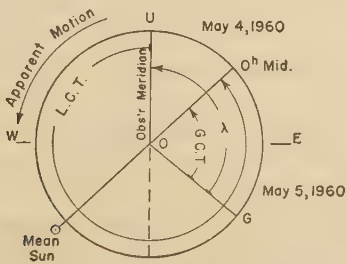


FIG. 2-7.

$$
\begin{array}{rll}
\text{Local Civil Time} = & 20^h\ 57^m\ 46^s.9 & \text{May 4, 1960} \\
\text{Longitude} = & \underline{8\ \ 08\ \ 53\ .6} & \\
\text{Greenwich Civil Time} = & 29^h\ 06^m\ 40^s.5 & \text{May 4, 1960} \\
= & 5\ \ 06\ \ 40\ .5 & \text{May 5, 1960}
\end{array}
$$

2-24. Correction of Observed Altitudes — Refraction. Rays of light from celestial bodies are refracted downward upon entering the earth's atmosphere, thus causing these bodies to appear

higher above the horizon than they actually are. In Fig. 2-8 the star S appears to the observer (O) to be at S'. The measured altitude HOS' is therefore larger than the true altitude HOS by the angle of refraction, $SOS' = r$. It may be shown that the refraction correction varies approximately as the tangent of the zenith distance, ZOS', except near the horizon. The refraction correction in minutes of angle happens to be very nearly equal to the natural tangent of the zenith distance of the object (or cot h); a table of tangents may therefore be used in an emergency for obtaining approximate corrections for refraction. More accurate corrections may be taken from Table VII, p. 500, which gives refractions for 50° Fahrenheit and 29.6 inches barometric pressure. Table VIII gives coefficients to apply to the above for different temperatures and pressures. This correction applies to all objects and is always subtracted from an observed altitude.

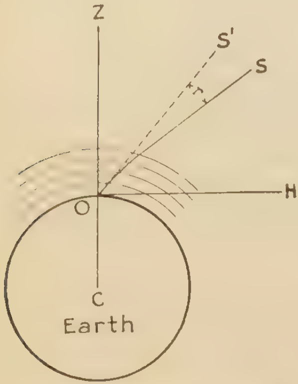


FIG. 2-8. ATMOSPHERIC REFRACTION.

EXAMPLE. Suppose that the measured altitude is 30° and the true altitude is to be found. The nat cot h (or tan of zenith dist. 60°) is 1'.73, or 1' 44''. From Table VII the refraction correction is found to be 1' 41'', which is a more accurate value. The true altitude is therefore 30° − 1' 41'' = 29° 58' 19''.

For an altitude of 15° the error of this approximate formula (cot h) is 10'', and for 10° it is 21''. From these comparisons it will be seen that even for approximate work the formula cannot be relied upon for altitudes lower than about 15°.

2-25. Parallax. Parallax is an apparent displacement of a body on the celestial sphere due to the fact that the observer is on the earth's surface instead of at its center, the common reference point for all observations. It is only appreciable for bodies within the solar system, the parallax of the stars being too small to measure. For the sun it is about 9'' at its maxi-

mum and for the moon about a degree. In Fig. 2-9, the point S is observed from O to have a zenith distance ZOS. If it had been measured at the center of the earth, the zenith distance

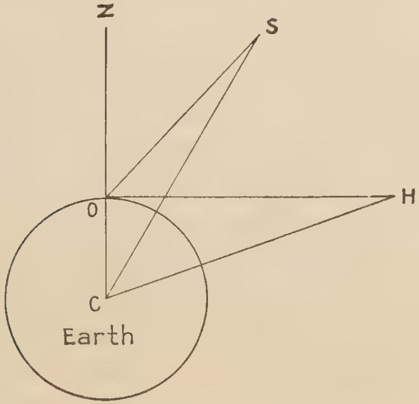


FIG. 2-9. PARALLAX IN ALTITUDE.

would have been ZCS. The difference between the two is equal to OSC, the parallax correction.

Since $$\sin S : \sin SOC = OC : CS$$

or $$\sin S = \frac{OC}{CS} \sin ZOS$$

it is evident that the sine of the parallax correction varies as the sine of the zenith distance, or as the cosine of the altitude. The ratio $OC : CS$ (or $OC : CH$) is the sine of the parallax of the body when it is on the horizon, that is, $\sin H$. H is called the "horizontal parallax." For such small angles as S and H the sines may be replaced by the angles themselves. Therefore we may write

$$S = H \cos h$$

Values of the horizontal parallax H may be found in the Ephemeris. The parallax correction for any observed altitude may therefore be computed by multiplying the horizontal parallax

by the cosine of the observed altitude. The correction should be added to the observed altitude. See Table VIII, p. 501.

EXAMPLE. The observed altitude of the sun is 60°, the horizontal parallax being 8″.8. The correction is 8″.8 × cos 60° = 4″.4. The true altitude is therefore 60° 00″ 04″.4.

2-26. Dip. The horizon which is visible to an observer at sea is a small circle below the true horizon. When the altitude of a body above the sea horizon is measured, as with the sextant, the true altitude is obtained by subtracting the angle of *dip*. This may be found approximately from the formula

$$\text{Dip (in minutes)} = \sqrt{\text{height of the observer's eye (in feet)}}$$

For example, if the eye of the observer were 64 feet above the surface of the sea, the visible horizon would be nearly 8 minutes of angle below the true horizon. The measured altitude should therefore be diminished by 8′. In order to allow for refraction this radical should be multiplied by a factor which is slightly less than unity, giving $58''.8\sqrt{h}$. Tables based on this more accurate formula therefore give slightly smaller values.

2-27. Semidiameter. In making observations upon the sun or the moon the angle should be measured to one edge (or limb) of the body and the observed angle reduced to the center by adding, or subtracting, the apparent semidiameter. This quan-

FIG. 2-10. ROELOFS SOLAR PRISM ATTACHMENT.

tity may be found in the Nautical Almanac. For the sun it is about 16′, but varies about 15″ either way. For the moon its mean value is nearly the same but its actual value is more variable.

Certain solar prism attachments permit observations to be made directly on the sun's center. One designed by R. Roelofs is mounted in front of the objective lens of a theodolite. It produces four green images of the sun which partly overlap to form a bright "cross," as shown diagrammatically in Fig. 2-10. By centering the cross hairs on the symmetrical target the center of the sun is sighted directly. When sighting the ground azimuth mark, the attachment unit may be swung to one side on a hinge without removing it from the instrument.

2-28. Hints on Observing. In observations of the character here treated the stability of the instrument is of great importance. The support should be firm and care should be taken that the instrument is not disturbed during the observations. If it is necessary to set the tripod in soft ground, it will be advisable to drive pegs several inches into the ground and to set the tripod legs in notches cut in the tops of the pegs. In order that the instrument may have time to settle into a stable position and also in order that it may have time to assume the temperature of the surrounding air it should be set in position a half hour or so previous to the time of the intended observations.

Careful attention should be paid to all adjustments of the instrument, especially if it is impossible to eliminate the instrumental errors by the method of observing. But when the best results are desired the observations should be conducted, if possible, in such a way as to eliminate any remaining error, even if the adjustments have been carefully made. In measuring altitudes the index correction should not be neglected, but should be determined each time the instrument is turned in azimuth.

The prismatic eyepiece is a necessary attachment to the transit when high altitudes are to be measured. By screwing this attachment to the eyepiece tube, altitudes as high as 70 to 75 degrees may be measured. It should be remembered that the prism inverts the image in the vertical direction but does not

affect it in the horizontal direction; i.e., if this prism is attached to a transit having an inverting eyepiece, objects will appear right side up but the left and right sides will remain interchanged.

In making observations at night it is necessary to illuminate the field of view in order to make the cross hairs visible. This is usually done by means of a ring shaped diagonal mirror placed in the shade tube in front of the objective. If a flashlight or a lantern is held at one side of the objective, the light is reflected into the telescope tube and gives a bright field, against which the cross hairs are visible. If no such reflector is provided with the instrument, a good substitute may be made by fastening a piece of tracing cloth or paper in front of the objective and cutting a hole about half an inch in diameter to admit light from the star. The light used to illuminate the field should be held so that it will not shine directly into the observer's eyes.

The transit should be kept accurately leveled. If it is provided with a striding level this may be used to keep the cross axis level, or to read its inclination.

2-29. Observations for Time by Transit of a Star across the Meridian. The simplest way of accurately determining the error of a watch by observation with an engineer's transit is to set the instrument in the plane of the meridian * and to observe the time when some southern star called a *time star* crosses the vertical hair. The star selected for this observation should be near the equator so that its apparent motion is rapid. Stars near the pole move too slowly to permit an accurate observation. The true time of transit of the star may be calculated as follows. The local sidereal time equals the right ascension of the star observed (see equation (2-17)); hence if only the sidereal time were desired the right ascension of the star as shown in the Nautical Almanac would be the sidereal time of the observation, and a comparison of this with the observed watch time would give the error of the watch on local sidereal time. But if mean time is desired, it would be necessary to convert this sidereal time into the corresponding mean solar time, by equation (2-20).

* For methods of determining the direction of the meridian also see Arts. 2-42 to 2-48 and Vol. I, Chapter VIII.

2-30. Preparation for the Observation. A list of stars with approximate times of transit and approximate altitudes should be made ready beforehand. A *star chart* is helpful in selecting stars and in identifying them in the sky. The Nautical Almanac, for example, contains charts showing the relative positions of northern, southern and equatorial stars as seen from the earth. The relative magnitude of these stars is also indicated.

The transit should be set up and leveled some time before the beginning of the observations. The altitude may be computed roughly from an estimated latitude. Before the time of transit of the first star the vertical hair is sighted on the meridian mark, the telescope inclined at the computed altitude and the focus set for infinite distance by sighting a distant object. In the ordinary telescope the star will appear in the east edge of the field about 2 minutes before it passes the cross-hair. The illumination should be regulated so that the star and the cross-hair can be seen with about equal distinctness. When the star passes the vertical cross-hair the watch is read. If it is desired to measure the altitude also, this may be done after the star has passed the meridian. From this altitude a closer value of the latitude may be computed.

EXAMPLE. Suppose that on Jan. 10, 1960, in longitude 71° 06′ west, the star β Orionis (Rigel) is observed at the instant of crossing the meridian, the observed watch time being $21^h\ 37^m\ 02^s.5$ Eastern Standard Time (Zone $+5^h$). From the Ephemeris the right ascension of the star on Jan. 10 is $5^h\ 12^m\ 38^s.2$, which is consequently the local sidereal time at the instant of observation. To reduce this latter quantity to the corresponding instant of Eastern Standard Time, it is necessary to convert this sidereal time to Greenwich sidereal time by adding the longitude in units of time ($4^h\ 44^m\ 24^s$). By subtracting the hour angle of the vernal equinox for 0^h G.C.T. from the Greenwich sidereal time, the result will be the sidereal interval since midnight (approximate G.C.T.). From the Ephemeris we find the hour angle of the vernal equinox at 0^h G.C.T., on Jan. 11, 1960,* to be $7^h\ 18^m\ 03^s.9$. The sidereal

* Jan. 11, 1960, is used because the date at Greenwich is one day later at the instant of observation. This will be ascertained when the computation is completely made. It is often necessary to revise a computation when the wrong date has been assumed.

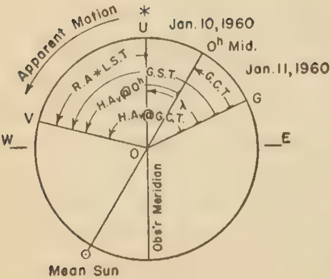


FIG. 2-11.

interval since midnight at Greenwich must be decreased by C' (Table V, p. 498) to obtain the mean time interval at Greenwich (G.C.T.). By applying the zone time with sign reversed to the G.C.T., the corresponding Eastern Standard Time and the watch error may be found. The tabular arrangement of the computation is as given below and the relations are illustrated in Fig. 2-11.

* Right ascension of β Orionis (Rigel) (Local sidereal time)	=	5^h	12^m	$38^s.2$
Longitude (λ)	=	+4	44	24 .0
Greenwich sidereal time (G.S.T.)	=	9	57	02 .2
H.A.$_v$ at 0^h G.C.T., Jan. 11, 1960	=	−7	18	03 .9
Sid. Int. since midnight Greenwich (approx. G.C.T.)	=	2	38	58 .3
Correction C'			−0	26 .0
G.C.T. Jan. 11, 1960	=	2	38	32 .3
Jan. 10, 1960	=	26	38	32 .3
Zone ($+5^h$) time correction	=	−5	00	00 .0
Eastern Standard Time (Jan. 10, 1960)	=	21	38	32 .3
Watch reading	=	21	37	02 .5
Watch slow	=		1^m	$29^s.8$

The above computation may be carried out by the G.H.A. system as contained in the Nautical Almanac by first recognizing that the L.H.A. of the star Rigel at the time of observation is $0° 00'.0$ since the star is on the meridian. By increasing the L.H.A. by the longitude, the G.H.A. may be found. If the Sidereal Hour Angle (S.H.A.) of Rigel is subtracted from the

* Since the tabulation in the American Ephemeris is for the mean places of stars for Jan. 1 of the current year, it is necessary to determine corrections for the date of observation by using formulae given therein. Other Ephemerides give listings at more frequent intervals, from which data may be interpolated for intermediate dates. See Art. 2-12.

G.H.A. of the star, the G.H.A. of the vernal equinox will result. By finding the G.C.T. which corresponds to the G.H.A. of the vernal equinox, the Greenwich Civil Time of the observation may be found as below. See also Fig. 2-12.

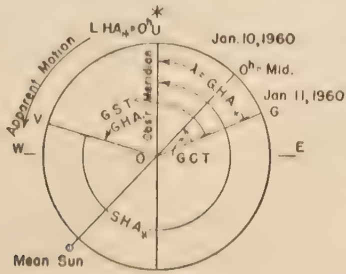


FIG. 2-12.

L.H.A. Rigel	=	0° 00′.0	
Longitude (λ)	=	+71 06 .0	
G.H.A. Rigel	=	71 06 .0	
Increase by	=	360 00 .0	
	=	431 06 .0	
S.H.A. Rigel	=	−281 50 .5	
S.S.T. = G.H.A.$_\Upsilon$	=	149 15 .5	
* G.H.A.$_\Upsilon$ at 2^h G.C.T.	=	139 35 .9	(Jan. 11)
Remainder	=	9 39 .6	
* Incre. G.H.A.$_\Upsilon$ for 38^m $32^s.0$	=	9 39 .6	
G.C.T. of observation	=	2^h 38^m 32^s	(Jan. 11)
Increase by 24^h	=	26 38 32	
Reduce to Zone $+5^h$	=	−5 00 00	
Zone $+5^h$ (E.S.T.) time	=	21 38 32	(Jan. 10)
Watch reading	=	21 37 02 .5	
Watch slow	=	1^m $29^s.5$	

2-31. Time by Transit of the Sun. The local apparent time may be found from the transit of the sun across the meridian, provided the direction of the meridian is known. About half an hour before noon, set up the transit so its line of sight swings in the

* Obtained from Ephemeris Table and Increment and Correction Table, respectively, in the Nautical Almanac.

plane of the meridian, and place the sun glass in position. When the sun's disc can be seen in the field, set the telescope so the middle horizontal hair is about at the middle of the disc. When the west edge is on the vertical hair note the time. When the east edge passes the hair, note the time again. The mean of these two is equivalent to an observation on the center.

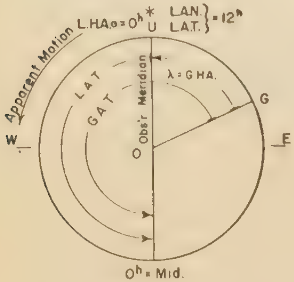


Fig. 2-13.

The true time of transit of the sun across the observer's meridian (Fig. 2-13) is 12^h, or Local Apparent Noon (L.A.N.). The longitude, if west of Greenwich, is subtracted to obtain Greenwich Apparent Time (G.A.T.). The equation of time is then found from an ephemeris for the instant when the observation was made. It is applied to the G.A.T. algebraically (sign is reversed) to obtain the true G.C.T. The Standard Time Zone +5 is subtracted to obtain the correct Eastern Standard Time of the observation. The difference between the correct E.S.T. and the mean watch reading is the watch error.

Example. Observed transit of sun over meridian, April 5, 1960, west edge $11^h 40^m 24^s$; east edge, $11^h 48^m 24^s$ Longitude $71° 06'$ ($4^h 44^m 24^s$) West.

Local Apparent Noon (L.A.N.)	=	$12^h 00^m 00^s.0$
Longitude (λ) West	=	+4 44 24 .0
Greenwich Apparent Time (G.A.T.)	=	16 44 24 .0
Equation of Time	=	−(−) 3 14 .7
Greenwich Civil Time (G.C.T.)	=	16 47 38 .7
Reduce to Zone 5^h (75th Mer.)	=	−5 00 00 .0
Eastern Standard Time (E.S.T.)	=	11 47 38 .7
Watch reading (mean)	=	11 47 26 .0
Watch slow	=	$12^s.7$

The above example may be solved by use of the Nautical Almanac by recognizing that the L.H.A. of the sun is 0° 00′.0

when crossing the meridian. The Greenwich Hour Angle is therefore equal to the longitude.

L.H.A. at time of observation	=	0° 00′.0	
Longitude (λ) West	=	71 06 .0	
G.H.A., sun's center	=	71 06 .0	
G.H.A. at sun's center at 16^h	=	59 11 .3	April 3
Remainder	=	11 54 .7	
Corr. sun's G.H.A. for $47^m\ 51^s.5$	=	11 54 .7	
G.C.T. of observation	=	$16^h\ 47^m\ 38^s.7$	
Reduce to Zone $+5^h$	=	−5 00 .0	
E.S.T. (Zone $+5^h$)	=	11 47 38 .7	
Watch reading	=	11 47 26 .0	
Watch slow		$12^s.7$	

2-32. Time by Altitude of the Sun. The apparent solar time may be found by measuring the sun's altitude at any time when the sun is not near the meridian, and solving the spherical triangle for the hour angle. The west hour angle of the sun is the local apparent time P.M. In order to increase the accuracy several measures of altitude are taken in succession and the mean values of the altitude and the mean watch reading treated like a single observation. If not more than 10 or 15 minutes elapse the error due to the curvature of the sun's path may be disregarded.

In measuring the altitudes place the sun glass in position, set the horizontal hair a little above the lower edge of the sun, if in the A.M., and when the sun's edge touches the hair read the watch and the vertical arc. In the P.M., set the hair below the lower edge or below the upper edge. The index error may be found by centering the telescope bubble and reading the vernier. It is advisable to examine the intervals between altitudes and between watch readings to see if they are nearly proportional. In this way mistakes may be detected.

The observed altitude must be corrected for index error, refraction, parallax, and also the semi diameter, unless this has

been eliminated by observing on the upper and lower edges and taking the mean. The declination is to be found for the Greenwich time of the instant of the observation. The watch must be near enough to correct time so that no error will result in the declination (say 2^m or 3^m). The latitude of the place must be known. The hour angle may then be computed by any formula for t. The value in degrees, minutes, and seconds must be converted into hours, minutes, and seconds. If the sun is west of the meridian, simply mark this P.M.; if the sun is east of the meridian, subtract this hour angle from 12^h and call it A.M.

To convert this apparent time into mean (civil) time subtract (algebraically) the equation of time. To convert into standard time add or subtract the difference in time between the place of observation and the standard meridian. The difference between the computed time and the mean watch reading is the error of the watch.

This observation is most accurate when the sun is about east or west of the observer.

Example

Observation on Sun for Time. Jan. 15, 1960; Lat. 42° 22′ N., Long. 76° 17′.4 W.

Lower limb	14° 41′	$9^h\ 20^m\ 06^s$ A.M.
Upper limb	16 08	9 27 58
Mean	15° 24′.5	9 24 02 E.S.T.
Refr. and Par'x.	−3.3	5
	h = 15° 21′.2	$14^h\ 24^m\ 02^s$ Gr. Civ. T.

$$\tan\frac{t}{2} = \sqrt{\left(\frac{\cos s \sin (s-h)}{\cos (s-p) \sin (s-\phi)}\right)}$$

in which

$$s = \tfrac{1}{2}(\phi + h + p)$$

L = 42° 22′	Decl. at 0^h = −21° 20′ 17″.2
h = 15 21 .2	$-645.2 \times \frac{14.4}{24} = -$ 6 27.1
p = 111 13 .8	Decl. = −21 13 50 .1
$2s$ = 168° 57′.1	Polar Dist. p = 111° 13′.8

$s =$	$84° \; 28'.5$	$\log \cos = 8.98354$	
$s - h =$	$69 \; 07.3$	$\log \sin = 9.97050$	Equa. of time at $0^h = -9^m \, 02^s.49$
$s - L =$	$42 \; 06.5$	$\log \csc = 0.17347$	$-21.56 \times \frac{14.4}{24} = - \quad 12.94$
$s - p =$	$-26 \; 45.3$	$\log \sec = 0.04918$	Equa. of time $= -9^m \, 15^s.43$
		$2)\underline{9.17669}$	

$$\log \tan \frac{t}{2} = 9.58834$$

$$\frac{t}{2} = 21° \; 11'.1$$

$$t = 42° \; 22'.2 = 2^h \, 49^m \, 28^s.8 \text{ (East)}$$

Local Apparent Time ($12^h - 2^h \, 49^m \, 28^s.8$)	$9^h \, 10^m \, 31^s.2$
Longitude (λ)	5 05 09 .6
G.A.T.	14 15 40 .8
Equation of time	−(−) 9 15 .4
G.C.T.	14 24 56 .2
Watch reading	14 24 02
Watch slow	$54^s.2$

The above time conversion may also be carried out through the use of the Nautical Almanac. Since t is east of the meridian, the L.H.A. will be equal to $360° - t$ or $317° \; 37'.8$.

Local Hour Angle	$317° \; 37'.8$
Longitude	76 17 .4
Greenwich Hour Angle	393 55 .2
Less 360°	33 55 .2
G.H.A. at 14^h G.C.T.	27 41 .1
Remainder	6 14 .1
Corr. in G.H.A. for $24^m \, 56^s.3$	6 14 .1
G.C.T. of observation	$14^h \, 24^m \, 56^s.3$
G.C.T. by watch	14 24 02
Watch slow	$54^s.3$

2-33. Time by the Measured Altitude of a Star. The determination of time from the altitude of a star is nearly the same as that described for the sun, except that the star is bisected with the horizontal hair. There is no correction for parallax or semidiameter. The corrections for index error and refraction should be applied. The triangle is solved for the hour angle, exactly as for the sun, but the hour angle so found is not the time. To obtain the local sidereal time, add the star's right ascension to the computed hour angle. If the star is east, its hour angle may be considered negative, or it may be considered as 24^h minus the calculated (east) hour angle. This sidereal time must be changed to standard time, or whatever time the watch reads. The star should be selected so that its bearing is approximately east or west at the time of the observation. A planet may be observed in exactly the same manner.

EXAMPLE. The altitude of Arcturus on April 13, 1960, was $40° 10'$ (star bearing to east) at $8^h 57^m 50^s$ P.M., Zone +5, by the watch. The declination of Arcturus $19° 23' 23''$ and the right ascension of the star $14^h 13^m 50^s.1$. Latitude $42° 18'$ N., Longitude $71° 00'$ W.

Obs. alt.	$40° 10'$
Refrac.	$-1'.1$
$h =$	$40° 08'.9$

$$\sin \frac{t}{2} = \sqrt{\left(\frac{\cos s \sin (s-h)}{\cos \phi \sin p}\right)}$$

$$s = \tfrac{1}{2}(L + h + p)$$

$\phi =$	$42° 18'.0$	log sec = 0.13098
$h =$	$40\ \ 08\ .9$	
$p =$	$70\ \ 36\ .7$	log csc = 0.02536
$2s =$	$153\ \ 03\ .6$	
$s =$	$76\ \ 31\ .8$	log cos = 9.36724
$s - h =$	$36\ \ 22\ .9$	log sin = 9.77317
		2)9.29675
		$\log \sin \frac{t}{2} = 9.64837$

	$\frac{t}{2}$ =	26° 25′ 28″	
	t =	52° 50′ 56″	(East)
	=	$-3^h\ 31^m\ 23^s.7$	(East)
Right ascension star	=	14 13 51 .9	
Local sidereal time	=	10 42 28 .2	
Longitude (λ)	=	4 44 00 .0	
Greenwich sid. time	=	15 26 28 .2	
H.A.$_\Upsilon$ at 0^h G.C.T.	=	13 28 39 .9	(April 14)
Sid. int. since mid.	=	1 57 48 .3	
C′, Table V, p. 498	=	− 19 .3	
Greenwich Civil Time	=	1 57 29 .0	(April 14)
Add 24 hours	=	25 57 29 .0	(April 13)
Zone correction	=	−5 00 00 .0	
Zone + 5^h Time (E.S.T.)	=	20 57 29 .0	
Watch reading + 12^h	=	20 57 50 .0	
Watch fast	=	$21^s.0$	

The above time conversion may be performed by use of the Nautical Almanac by first recognizing that the L.H.A. of the star (since t is east) is $360° - t$, or $307° 09'.1$, and obtaining S.H.A. of the star from this Almanac.

L.H.A. $360° - t$	=	307° 09′.1	
Longitude (λ)	=	71 00 .0	
G.H.A. of star	=	378 09 .1	
S.H.A. of Arcturus	=	146 32 .0	
G.H.A. of vernal equinox	=	231 37 .1	
G.H.A.$_\Upsilon$ for 1^h G.C.T.	=	217 12 .4	(April 14)
Remainder	=	14 24 .7	
Correction for $57^m\ 29^s.3$	=	14 24 .7	
Greenwich Civil Time	=	$1^h\ 57^m\ 29^s.3$	(April 14)
Add 24 hours	=	25 57 29 .3	(April 13)
Zone correction	=	−5 00 00 .0	
Zone + 5^h Time of observ.	=	20 57 29 .3	
Watch reading	=	20 57 50 .0	
Watch fast	=	$20^s.7$	

2-34. Methods of Determining Longitude. Rough determinations of longitude, sufficiently accurate however for correcting the quantities given in the Ephemeris, may be made by means of a watch or a chronometer. If the error of the watch on the mean solar time of any meridian is obtained by one of the methods described, and the watch then carried to a second meridian and the observations repeated, the difference of the errors at the two places is the difference in longitude, provided the watch has run correctly on mean solar time during the interval between the observations.

The longitude could be found in exactly the same way if sidereal time were observed except that it would be necessary to use a sidereal chronometer or else allow for the error of the watch on the sidereal rate.

The measurement of the difference of longitude of two places depends upon the measurement of the difference of their local times at the same instant. The two methods to be given here are: 1. that in which the government time signals are used, the result being a comparison of local time with Eastern Standard Time; and 2. that in which a timepiece is carried from one place to another.

2-35. Longitude by Time Signals. The local time may be found by any of the methods given in the preceding articles on time determination. Immediately before or after the time observation (preferably both) the watch or chronometer is compared with the Standard time by noting its reading when time signals are received by radio. The United States Naval Observatory broadcasts time signals on various frequencies at different times during the day from Annapolis, Maryland; Mare Island, California; Pearl Harbor, Hawaii; and Balboa, Canal Zone. The National Bureau of Standards broadcasts from Beltsville, Maryland and Maui, Hawaii, pulses that can be heard as clicks on ordinary radio phone receivers every second except the 59th of each minute. Voice announcements are also made at intervals. When transmitted, these signals begin five minutes before the hour and end on the hour. For example, the time signals from Annapolis are transmitted on most even hours, Greenwich Civil Time on the frequencies of 121.95, 5870, 9425, 13575, 17050.4 and

23650 kilocycles. The Beltsville frequencies are 2.5, 5, 10, 15, 20 and 25 Mc.

The pattern of the signals is such that each second is sounded beginning at $55^m\ 00^s$ of the preceding hour. Signals are sent out for each second during this period with the following exceptions.

Minute	Second										
	50	51	52	53	54	55	56	57	58	59	60
55	—		—	—	—	—					—
56	—	—		—	—	—					—
57	—	—	—		—	—					—
58	—	—	—	—		—					—
59	—										——

FIG. 2-14. TIME SIGNALS.

No signal is given for the 29th second of each minute. The 56th to 59th seconds, inclusive, of each minute are omitted; the beat for the 51st second of the 55th minute, the 52nd second of the 56th minute, the 53rd second of the 57th minute, the 54th second of the 58th minute, and 51st to the 59th second of the 59th minute are omitted. The sequence of the signals during the last 10 seconds of the 55th to 59th minutes of the hour are shown by the dashes in Fig. 2-14; the time is the beginning of the dash (the end of the dash has no significance). A long dash at the beginning of the 60th second of the 59th minute ends the series.

EXAMPLE. From an observation on the sun the local apparent time is determined. From this the local mean time is found to be $4^h\ 33^m\ 43^s.9$; the watch read $4^h\ 18^m\ 13^s.8$ at time of observation. The watch is therefore $15^m\ 30^s.1$ slow of local time. At noon by radio the watch is found to be $6^s.0$ fast of Eastern Standard Time.

Correction to Loc. Time		$+15^m\ 30^s.1$
Correction to E.S.T.		$-(-)\ 6\ .0$
Diff. in longitude	=	$15^m\ 36^s.1$
	=	$3^\circ\ 54'\ 01''.5$
Long. = $75^\circ - 3^\circ\ 54'.0$	=	$71^\circ\ 06'.0$ West.

2-36. Longitude by Transportation of Timepiece. In this method the local time at the first station is determined by any of the methods described. The instrument and watch are then taken to the second station and the observations repeated. The difference in watch errors is the time difference corresponding to the difference in longitude. If the rate of the watch is not known the result will be subject to this error. If a return trip to first station can be made and another observation made, then the average rate of the watch becomes known and the longitude difference may be corrected.

EXAMPLE. At the eastern station the watch is found by observation to be $15^m\,40^s$ slow on local time. At the western station the watch is $14^m\,10^s$ slow on local time. The watch gains 8^s per day. Western station observed two days later than eastern station. Watch error at western station corrected for rate is $14^m\,26^s$. Difference in longitude $= 15^m\,40^s - 14^m\,26^s = 1^m\,14^s = 18'\,30''$.

2-37. Latitude by Altitude of Sun at Noon. The altitude may be measured either by placing the line of sight of the transit in the plane of the meridian and observing the altitude when the sun reaches this plane, or it may be taken as the maximum value of the altitude. If the time is known closely so that the instant of local apparent noon may be calculated, this may be used to indicate the instant when the altitude is to be observed.

If observing by means of the maximum altitude the horizontal hair is set on the lower edge of the sun's disc a quarter of an hour or more before local noon. As the sun rises its motion is followed with the cross hair until the altitude no longer increases. The altitude is then read and the index error determined.

The altitude is corrected for index error, refraction, parallax, and semi-diameter. The declination must be taken out for the Greenwich time corresponding to the instant of local noon. The latitude is then found by equation (2-1).

EXAMPLE. The observed maximum altitude of the sun at noon, Jan. 1, 1960 $= 24°\,21'$. Longitude $= 71°\,06'$ W. Sun bearing south.

Obs'd. Alt.	24° 21′	Loc. Ap. Time	12^h
Refr. par.	−2 .0	Long.	4 44^m 24^s
	24 19 .0	Gr. Ap. T.	16^h 44^m 24^s
Semi-diam.	+16 .3	Equa. of time	−(−) 3 22
h	24 35 .3	Gr. Civ. Time	16^h 47^m 46^s
D	−23 02 .5	Decl. at 0^h	−23° 05′ 45″.5
Co-Lat.	47 37 .8	$+277.7 \times \frac{16.8}{24}$	+3 14 .4
Lat.	42° 22 .2 N		
		Decl.	−23° 02′ 31″.1

2-38. Latitude by Meridian Altitude of a Star. The latitude may be found by measuring the maximum (or meridian) altitude of a star in exactly the same manner as for the sun. No parallax or semi-diameter corrections are needed, but the index error and the refraction must be applied.

EXAMPLE. The observed meridian altitude of α *Hydrae* (Alphard) on March 16, 1960 was 39° 24′, star bearing south.

Observed Altitude	=	39° 24′
Refraction	= −	1 11″
Corrected Altitude	=	39 22 49
Declination − (−)	=	+8 29′ 21″
Co-Latitude	=	47 52 10
Latitude	=	42° 07′ 50″ N

2-39. Latitude by a Circumpolar Star at Time of Transit. Polaris is the best star to observe for this purpose because it is the brightest, but any star that can be positively identified may be used. Measure the altitude of the star when it is a maximum (or a minimum), which may be done by trial. The times of Culmination for Polaris may be found in special tables. (Table 9, p. 268, Vol. I.) The latitude is found by equa. (2-2), or equa. (2-3), using the corrected altitude for the purpose.

EXAMPLE. Observed altitude of Polaris at upper culmination = 43° 37′. Index correction = +30″. Sept. 30, 1960.

Obs'd. Alt.	$43° 37'$
Index corr.	$+30''$
	$43° 37' 30''$
Refraction	$-1' 01''$
h	$43° 36' 29''$
Polar dist.	$0\ 55\ 20$
Lat.	$42° 41' 09''$ N

2-40. Latitude by Altitude of Polaris at Any Hour Angle. If the watch error is known, the latitude may be found from the altitude of Polaris observed at any time. The observation consists merely in measuring one or more altitudes with the corresponding watch readings. If several altitudes are taken the mean value is used in the computation. From the mean watch reading we must compute the hour angle of Polaris; that is, we first find the local sidereal time, and then subtract from this the right ascension of Polaris, the result being the hour angle. The latitude is then computed from the formula

$$\phi = h - p \cos t + \tfrac{1}{2} \sin 1' \, p^2 \sin^2 t \tan h$$

$$(\log \tfrac{1}{2} \sin 1' = 6.1627 - 10)$$

p is in minutes of angle. Values of this correction may be found in the Ephemeris and in the Nautical Almanac.

Example. On Jan. 3, 1960, at a place in longitude $71° 05' 30''$ W, the corrected observed altitude of *Polaris* at $10^h 15^m 27^s$ P.M., E.S.T., $= 43° 28'.5$. I.C. $= -1'$. From the E.S.T. the local sidereal time is found to be $5^h 22^m 05^s.1$. The right ascension of Polaris is $1^h 56^m 42^s.0$. The hour angle of *Polaris* is therefore $3^h 25^m 23^s.1$ ($51° 20' 46''.5$). The calculation of the latitude is as follows:

Decl. $= +89° 04' 55''.6$ $\qquad p = 0° 55' 04''.4 = 55'.08$

$\log p$	$= 1.74099$	log const.	$= 6.1627$	obs. alt.	$= 43° 28'.5$
$\log \cos t$	$= 9.79562$	$2 \log p$	$= 3.4819$	I.C.	$= -1.0$
		$2 \log \sin t$	$= 9.7852$	refr.	$= -1.0$
$\log p \cos t$	$= 1.53661$	$\log \tan h$	$= 9.9764$		
$p \cos t$	$= +34.40$			corr. alt.	$= 43° 26'.5$
		log 2nd term	$= 9.4062$	$-p \cos t$	$= -\ 34.4$
		2nd term	$= 0.255$	2nd term	$= +\ 0.3$
				Latitude	$= 42° 52'.4$ N

2-41. Latitude by Circum-Meridian Altitudes. If the altitude of a star is taken when the star is a few minutes either east or west of the meridian, its altitude will be less than the maximum, but the latitude may still be found with considerable accuracy provided the error of the watch is known. The observation consists in taking a small number of altitudes and the corresponding times. These must be very close together to be worked up as a single observation. If they cover a long interval, say over 20 minutes, they should be worked out separately.

The altitude is corrected for index error and refraction. Then a correction is applied to reduce the altitude to what it would be at its maximum on the meridian. This is computed by the formula

$$C'' = 112.5t^2 \times \frac{\cos\phi\cos\delta}{\cos h}\sin 1''$$

$$(\log 112.5 \sin 1'' = 6.7367)$$

Here t is the hour angle of the star, h is the observed altitude, δ the declination. Since ϕ is required to be known before we can calculate this correction, we may use any approximate value. If the latitude is not known, we may first work up the observation just like a meridian observation and in this way obtain a fairly close value of ϕ. Then the correction is applied and the final value of ϕ obtained.

EXAMPLE. The altitude of γ *Ceti* was measured when the star was $3^m\ 14^s$ past the meridian, i.e., hour angle $= 3^m\ 14^s$, the altitude being $50°\ 33'$, star bearing south. Index corr. $-1'.4$. Approx. latitude $42°\ 29'$ N. Declination of γ *Ceti* $+ 3°\ 04'.1$ (1960). The correction computation is as follows:

$$\begin{aligned}
\log\cos\phi &= 9.8688\\
\log\cos\delta &= 9.9994\\
\log\sec h &= 0.1991\\
\log 112.5 \sin 1'' &= 6.7367\\
2\log t &= 4.5756\\
\hline
&\quad 1.3796\\
\text{correction}\quad 23''.97 &= 0'.4
\end{aligned}$$

The latitude is then:

Obs'd. altitude	=	50° 47′.0
Index corr.	=	−1 .4
		50 45 .6
Refraction	=	−0 .8
		50 44 .8
Red'n. to merid.	=	+0 .4
		50 45 .2
Declination	=	− +3 04 .1
Co-latitude	=	47 41 .1
Latitude (ϕ)	=	42° 18 .9 N

2-42. Observations for Azimuth by an Altitude of the Sun. The instrument is set up at a point on the line and pointed at a second mark on the line, the vernier reading zero. The upper clamp is loosened and a series of pointings made on the sun.* These are usually arranged so that the semidiameter, both horizontal and vertical, is eliminated. That is, we take an equal number of altitudes on the upper and lower edges and an equal number of horizontal angles on the left and right edges of the sun's disc. If we are observing in the afternoon (northern hemisphere) the sun is set first in the position shown in Fig. 2-15, then in the position shown in Fig. 2-16. For observations in the

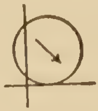


Fig. 2-15.

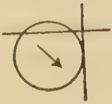


Fig. 2-16.

forenoon the other pair of quadrants may be used instead. For details regarding these pointings under various conditions reference is made to Vol. I, Chapter VIII. The watch is read at each pointing if its error is to be found, otherwise it may be read but once at the middle of the set of observations, or at the first and last altitudes.

The mean altitude, the mean horizontal angle, and the mean watch reading are to be treated like a single altitude and hori-

* When prism attachments are used (Art. 2-27), pointings may be made directly to the sun's center.

Example

Observation on Sun for Azimuth, May 24, 1960

Lat. 42° 29′.5 N. Long. 71° 07′.5 W.

	Hor. Circle vernier A	Vert. Circle	Watch (E.S.T.)
Mark	0° 00′		
⊙	67° 54′	43° 35′	$2^h\ 58^m\ 00^s$ P.M.
	68 11	43 20	2 59 21
	68 26	43 08	3 00 33
	69° 25′	43° 25′	$3^h\ 01^m\ 53^s$
	69 39	43 12	3 03 05
⊙	69 52	43 00	3 04 10
Mark	0° 00′		
Mean	68° 54′.5	43° 16′.7	$3^h\ 01^m\ 10^s.3$
refr. & par.		−0 .9	12^h
I.C.		+1 .0	5^h
		h 43° 16′.8	G.C.T. $20^h\ 01^m\ 10^s.3$

$$\cos Z_n = \frac{\sin\delta - \sin\phi \sin h}{\cos\phi \cos h}$$

nat $\sin\delta$	0.35639	
log $\sin\phi$		9.82961
log $\sin h$		9.83605
log $\sin L \sin h$		9.66566
$\sin\phi \sin h$	0.46308	
numerator	−0.10669	
log num.	= 9.02812 n	
log $\sec\phi$	= 0.13220	
log $\sec h$	= 0.13767	
log $\cos Z_n$	= 9.29799 n	
Z_n	= 101° 27′.3 (West)	
Hor. Angle	= 68 54 .5	
	170° 21′.8	
Azimuth of Mark	= S 9° 38′.2 W. (Fig. 2-17.)	

Decl. at 0^h = +20° 43′ 32″.0

$+661.3 \times \dfrac{20.02}{24}$ = + 9 11 .6

Decl. = δ = +20° 52′ 43″.6

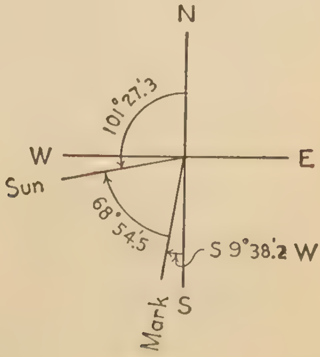


Fig. 2-17.

zontal angle taken to the **center** of the sun, with the corresponding watch reading. The altitude is to be corrected for index error, refraction and parallax. The azimuth is then calculated by a formula giving the azimuth, or angle at the zenith. This azimuth is to be combined with the measured horizontal angle to obtain the azimuth of the line.

2-43. Azimuth by Altitude of Sun — Southern Hemisphere. In making observations on the sun for azimuth in the southern hemisphere (sun bearing north), the pointings (with an erecting telescope) are taken on the left and lower limbs and on the right and upper limbs in the forenoon; and on the right and lower, and the left and upper limbs in the afternoon.

When making the calculations (all latitudes south of the equator) either of two procedures may be followed.

(1) The latitude, ϕ, may be considered negative and the same formulas used as in the northern hemisphere.

(2) The latitude, ϕ, may be considered positive and the algebraic sign of the declination, δ, reversed. In this case the azimuth obtained by formulas for Z_n will be from the south instead of from the north.

Example. At a place in latitude 33° 01′ S, longitude 71° 39′ W, an observation is made on the sun for azimuth on Aug. 4, 1960, at station A to determine the azimuth of the line AB. In calculating the azimuth, the usual procedure in the northern hemisphere is followed and the latitude is taken as negative.

	Hor. Circle		Vert. Circle		Watch (Zone $+5^h$)
Sta. B	0° 00′				
$\odot$	75 15		24° 22′		$2^h\ 52^m\ 46^s$ P.M.
$\odot$	74 52		23 03		2 57 48
Sta. B	0 00				
Mean	75° 03′.5		23° 42′.5		$2^h\ 55^m\ 17^s$
		refr. & par.	−2 .1		12
			h 23° 40′.6		5
				G.C.T.	$19^h\ 55^m\ 17^s$

nat sin δ	0.29376		Decl. at 0^h	$= +17° 18' 16''.7$
log sin ϕ		9.73630 n	$-961.8 \times \frac{19.92}{24}$	$= - \quad 13 \; 18 \; .3$
log sin h		9.60377	Decl. $= \delta$	$= +17° 04' 58''.4$
log sin ϕ sin h		9.34007 n		
sin ϕ sin h	0.21882			
numerator	0.51258			
log num.	9.70976			
log sec ϕ	0.07649			
log sec h	0.03812			
log cos Z_n	9.82437			

Z_n = 48° 08'.1 (West)
Hor. Angle = 75 03 .5

Azimuth of B = N 123° 12'.6 W
= S 56° 47'.4 W (Fig. 2-18)

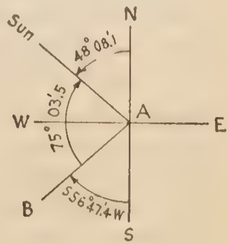


FIG. 2-18.

2-44. Azimuth by Altitude of a Star. The azimuth of a line may be found by observing a star, the process differing from the preceding in that the pointings on the star are made at the intersection of the horizontal and vertical cross hairs, and the correction for parallax is omitted. It is unnecessary to read the watch since the star's declination will not change appreciably during the course of the observation.

Example

Corrected altitude of *Spica* = 28° 36'.8 (star bearing to east). Latitude = 39° 14'.2 N. Declination of star = −10° 57'.3. May 15, 1900.

$$\cos Z_n = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

log sin δ	= 9.27884 n		
log sec ϕ	= 0.11096	log tan ϕ	= 9.91203
log sec h	= 0.05657	log tan h	= 9.73681
	9.44637 n		9.64884
1st Term	= −0.27949		
2nd Term	= −0.44550		
cos Z_n	= −0.72499		
Z_n	= 136° 28'.1 (East)		
Bearing of star	= S 43° 31'.9 E		

2-45. Azimuth by Polaris at Greatest Elongation. The approximate time of elongation * may be readily found in special tables. (See Table 9, p. 268, Vol. I.) A half hour or so before this time set up the transit over one end of the line to be sighted and level it carefully. Find the star and sight the vertical hair on it. As the star moves almost vertically (upward for eastern, downward for western elongation), it will require but a slight motion of the tangent screw to keep the vertical hair on the star. Follow it until it seems to move vertically, which should be about the time given by the tables. Lower the telescope and set a mark in line with the cross hair. Reverse the telescope, re-level if necessary, sight the star again and then set another point alongside the first. The point halfway between these two should be the true point in the vertical plane of the star at elongation. Then lay off the azimuth of the star, as calculated by equation (2-12), either with the transit or else by a calculated perpendicular offset. This gives a point exactly north of the instrument. From this the azimuth of any line may be obtained by angular measurement.

If preferred, we may measure directly from the star to a mark on the line to be observed, first with the telescope direct, then again after reversal, the instrument being set at 0° at the first sight.

EXAMPLE. *Polaris* is observed at its greatest western elongation on April 23, 1960, in latitude 45° 00.0 N. The declination of the star is 89° 04′ 38″. A point is set beneath the star at a distance of 630.0 feet from the instrument. Find the azimuth of the star at elongation and also the perpendicular offset to set a point on the meridian.

This problem can be solved directly by Formula (2-12). However, since p and Z (and usually the angle of offset for the true north) are quite small, use may be made of the approximation that the sine (or tangent) of small angles is equal to the number of seconds of angle times the sine (or tangent) of one second (0.000004848). Substituting p for δ in Formula (2-12), and applying the above relation, $Z'' = p'' \sec \phi$, where Z and p are in sec-

* This may be calculated in a direct manner as follows. Solve equation (2-13) to obtain the hour angle, t. Then solve equation (2-16) for the sidereal time S. Convert this into Standard Time as explained in Art. 2-23.

onds. The offset to the true north at any distance d may be expressed $(Z'' \tan 1'')d$. The solution follows.

$$
\begin{array}{ll}
p & = 0^\circ\ 55'\ 22'' = 3322'' \\
\log p & = 3.52140 \\
\log \sec \phi & = 0.15052 \\
\hline
\log Z & = 3.67192 \quad Z = 4698'' = 1^\circ\ 18'\ 18'' \text{ W. of N.} \\
\log \tan 1'' & = 4.68556 \\
\log 630.0 & = 2.79934 \\
\hline
\log \text{offset} & = 1.15682 \\
\text{offset} & = 14.349 \text{ ft. to the right.}
\end{array}
$$

2-46. Observations for Azimuth on a Circumpolar Star at any Hour. In determining the azimuth of a line of a triangulation net the process would consist in first obtaining the angle between an azimuth mark and a star, and from this angle calculating the azimuth of the mark. This azimuth combined with the angle between the mark and the triangulation signal would give the desired azimuth. The method of observing the angle between the mark and the star is similar to that of measuring a horizontal angle in triangulation work, except that since the star is continually changing its azimuth it is necessary to note the time of each pointing upon the star. If, however, the triangulation is being carried on at night, as is now frequently done, the azimuth observation is made by pointing on the star in its turn as though it were one of the series of signal lights at the triangulation stations, no special azimuth mark being necessary. The only difference between the sight on the star and the sight on a signal lamp is that the chronometer time of the former must be accurately noted and that readings of the striding level and readings of the star's altitude must be taken.

2-47. The Azimuth Mark. If an azimuth mark is to be used, it may be marked by an illuminated tripod target (Fig. 1-14), or it may consist of an electric light placed inside a box so that the light may shine through an aperture set accurately on the line. A target is painted on the side of the box toward the observer for use in the daytime. The size of the aperture used will depend upon the distance to the mark and the character of the light used as well as upon the power of the telescope. The exact size of the aperture is not of great importance since the light is centered by

the observer between two vertical lines in the telescope which are set 25″ to 35″ apart. For short distances the aperture should be stopped down so that the subtended angle is, say, about 3 seconds, that is, about one inch per mile. When observing on short lines the brilliancy of the light may be cut down either by reducing the intensity of the light itself or by increasing the illumination of the field of view of the telescope, and this seems to give more satisfactory results than varying the diameter of the aperture itself. The mark must be placed far enough from the instrument so that it will not be necessary to alter the focus in changing from the star to the mark. Any change in the focus may disturb the line of sight. A distance of a mile will ordinarily be sufficient; it will sometimes be necessary, however, to use shorter distances, on account of the difficulty of placing the mark in a good position.

It should be remembered that the higher the sight line is above the ground the less the error due to lateral refraction.

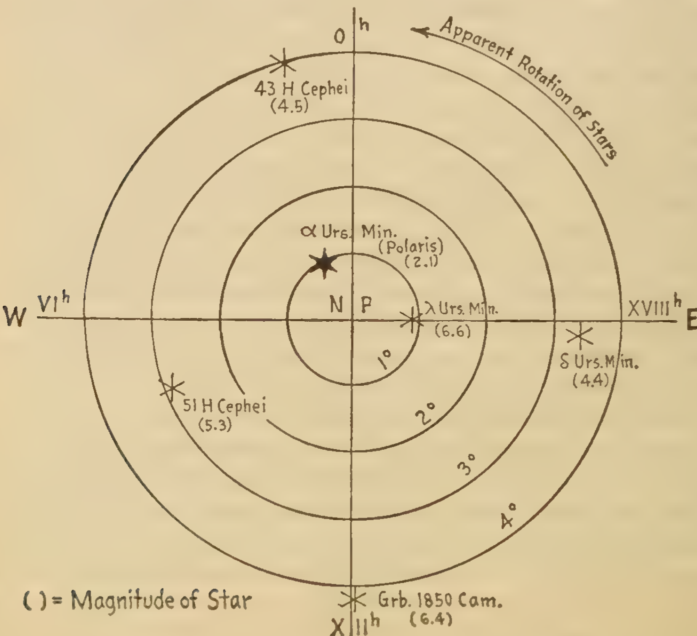


Fig. 2-19. Circumpolar Stars. (As viewed facing north)

2-48. Circumpolars. Since the star is continually changing its azimuth, and also since the latitude and the time are somewhat uncertain, it is advisable to use only close circumpolar stars for this observation. Those nearest the pole are shown in Fig. 2-19. *Polaris* is the only one, however, that is bright enough to be seen easily with the naked eye. By the aid of the diagram and using *Polaris* as a point of reference it is not difficult to identify the fainter stars with a telescope of moderate power.

2-49. The Observation. The details of the observation will vary, depending upon the type of instrument used. With either a direction or a repeating instrument the following precautions are important. Since the star is at a high altitude as compared with the (angular) altitude of ordinary triangulation points, it is of unusual importance to keep the instrument leveled and, by means of a striding level, to measure any small deviation of the horizontal axis from the true horizontal position while the telescope is pointing in the direction of the star. The instrument should be firmly mounted and should be protected from unequal heating of its parts, as from the body of the observer or from the lamp used for illuminating the cross hairs or for reading the circle. The instrument should be handled with the greatest care, the clamps and tangent screws being used in such a way as to avoid any lateral pressure, which would tend to disturb the instrument. In short, all of the precautions which need to be taken in refined triangulation work will apply here. The angles should be measured as quickly as is consistent with careful and accurate work. The longer time there is between pointings the greater the opportunity for the instrument to shift its position and so introduce errors into the results.

2-50. The Astrolabe. The astrolabe is an instrument (or an attachment that can be applied to a theodolite telescope) to obtain fixed vertical angles of high precision at 60° altitude. It is useful for making observations for time, longitude and latitude where an altitude of this amount is required for accurate work. The vertical angle will be as accurate as the forward angle between the faces of the prism.

There are two general types, *prismatic* and *pendulum*, both utilizing a direct and reflected ray from the celestial body in

such a manner that these rays form images on the image plane. These images change position as the altitude of the body varies and eventually merge into coincidence when the altitude is the same as the precise vertical angle between the prism faces. The prismatic type uses a mercury basin to form the horizontal surface of reflection; the pendulum type uses an optical flat plate suspended in the same manner as in the self-leveling level (Art. 3-8). A prismatic type attachment for a theodolite is shown in Fig. 2-20. The mercury reflection surface is under the prism, and a removable wind screen for the mercury surface is at the left.

FIG. 2-20. ASTROLABE ATTACHMENT FOR THEODOLITE.

2-51. Observations with a Direction Instrument. When observing with a direction instrument it will be advisable to use several different positions of the circle in order to eliminate errors of graduation. The number of positions used will depend upon the size and quality of the instrument and upon the precision required in the result. In each position of the circle the observations will include (1) a pointing on the mark, with readings of the microscopes, (2) a pointing on the star, with readings of the chronometer, the striding level and the microscopes; (3) (after reversal of the telescope) a pointing on the star with readings of chronometer, striding level and microscopes, and (4) a pointing on the mark with readings of the microscopes.

In order to give the bubble of the striding level time to come to rest, and in order to give the recorder time to read the chronometer and then record the reading it will be advisable to per-

form the work in the following order: — After completing the pointing and reading on the mark sight the telescope toward the star and bring the star's image near to the center of the field of view. Clamp the horizontal circle and call to the recorder to be ready. Place the striding level on the axis, perfect the pointing on the star, and call "tip" to the recorder when this pointing is made. Read the two ends of the bubble of the striding level but do not call off these readings until the recorder has had time to make his record of the observed time. Reverse the striding level, and prepare to read the first micrometer. As soon as the recorder calls "ready" the first (direct) level readings are given him, and then the readings of the micrometers in their proper order. The second (reversed) readings of the striding level are then taken and called off to the recorder. This completes half the observation. Remove the striding level, reverse the telescope, bring the star to the center of the field, clamp the horizontal motion, call to the recorder to "stand by," place striding level in position, then perfect the pointing on the star, etc., and complete the readings exactly as described for the direct observation. Finally make another pointing on the mark. The altitude of the star should be read to the nearest minute after each pointing, or at least twice during the observations (Art. 2-55).

The preceding program tends to decrease the total time taken for the observation on the star and hence makes the curvature correction (Art. 2-54) small or negligible.

If the azimuth mark is far above or below the horizon, then level readings should also be taken when the mark is sighted (Art. 2-55).

If the azimuth observation is being made in connection with triangulation at night on signal lamps, the above program would be modified accordingly. That is, the signals would be sighted in order, left to right, the star being sighted in its turn; after reversal of the telescope the signals would be sighted in the same order, the star again being sighted in its turn.

This method is illustrated in example on p. 161.

2-52. Observations with a Repeating Instrument. If a repeating instrument is used, the program may be similar to that frequently used in triangulation, i.e., six repetitions clockwise, left

to right telescope direct, followed by six clockwise repetitions, left to right telescope reversed of the explement. Pointing on the left hand object in each set should be made by the use of the lower clamp and tangent screw and that on the right hand object by use of the upper clamp and tangent screw. The striding level is read in both positions, at the beginning of the half set and at the end, while the telescope is pointing at the star. At each pointing on the star the time is noted. The circle is read as usual, only at the beginning and the end of the half set. In the second half set the pointings on the star would be made using the upper clamp and those on the mark using the lower clamp. In order to eliminate as far as possible any constant error of the clamps and tangent screws the plates should always be turned in a clockwise direction and the setting made with the tangent screws in the direction which compresses the spring, so as to insure a positive working of the tangent screw.

2-53. Calculating the Azimuth of the Star. The azimuth of the star is derived from the equation *

$$\tan Z = \frac{\sin t}{\cos \phi \tan \delta - \sin \phi \cos t} \qquad (2\text{-}14)$$

see Art. 2-18. From this formula it will be seen that the latitude and the star's hour angle must both be known. It will generally be necessary to determine these at the same time as the azimuth by special observations. (See Arts. 2-39-40.)

2-54. Curvature Correction. It would be a long process if we were to calculate the azimuth of the star separately for each pointing made upon it. However at times near elongation the star moves very slowly in azimuth and the small error involved in using the means of the times and angles can be largely over-

* Equation (2-14) may also be put in the form

$$\tan Z = \cot \delta \sec \phi \sin t \left(\frac{1}{1 - a}\right), \text{ where } a = \cot \delta \tan \phi \cos t$$

Values of $\frac{1}{1-a}$ will be found tabulated in Spec. Publ. No. 14. U.S.C. & G.S.

If extreme accuracy is not required in the computed azimuth, the following convenient formula may be used. $Z = p \sin t \sec h$, in which Z and p may be expressed either in minutes or in seconds of angle.

come by grouping the sights into several sets of intervals of not more than three minutes each. By using the average t of each set, the value of sin t can be found for that set, and the average of these sin t's substituted in Formula (2-14).

2-55. Level Correction. If the striding level shows any appreciable inclination of the axis the horizontal angle may be corrected by either of the following formulas:

$$C = \frac{d}{4}[(w + w') - (e + e')] \tan h,$$

when the level graduations are numbered in both directions from the middle, and

$$C = \frac{d}{4}[(w - w') + (e - e')] \tan h,$$

when the level graduations are numbered continuously in one direction. The terms e and w are the scale readings of the east and west ends of the bubble, and e' and w' the same when the level has been reversed; d is the angular value in seconds of one division of the striding level; and h is the altitude of the star, obtained during the observation. The primed letters in the second formula refer to readings where the level graduations increase toward the east.

The sign of the correction may be determined from the fact that if the west end of the axis is too high the telescope is turned too far to the left (west) when pointing at the star, and *vice versa*.

If the azimuth mark is placed far above or below the horizon it will be necessary to take readings of the striding level when pointing at the mark and to compute corrections to the circle readings as would be done for the pointings on the star. Ordinarily, however, the mark is not far from the true horizon so that this correction may be neglected provided the plate levels are sensitive and are carefully centered before each set is begun.

2-56. Diurnal Aberration. In very exact observations for azimuth, allowance should be made for the effect of diurnal

aberration; this is a slight apparent displacement of the star toward the east due to the motion of the observer about the earth's axis. The amount of this displacement depends upon the relation between the velocity of the observer and the velocity of light. The displacement occurs in the direction in which the observer is actually moving, which is always directly toward the east point of his horizon. The expression for the correction for diurnal aberration is

$$0''.319 \times \frac{\cos \phi \cos Z}{\cos h}$$

In ordinary work it is sufficient to take it as $0''.32$, since $\frac{\cos \phi \cos Z}{\cos h}$ is nearly equal to unity, for circumpolar stars.

The example on pp. 161 and 162 illustrates the notes and computations for an azimuth observation using a direction instrument.

2-57. Choice of Methods. In determining the azimuth of a line it is desirable to use a method which will permit of an observation being made at any time and with sufficient precision for the purpose in hand. In order that the star's azimuth may be computed at any time except at elongation it is necessary to know the local sidereal time of the observation, since this is needed in computing the star's hour angle. Since the time of the observation will always be somewhat in error, it is advisable to use only close circumpolar stars (preferably Polaris) for the azimuth observation, so that errors in the time will have the least possible effect on the hour angle. It is also necessary to know the latitude of the observer. In order that the computed azimuth of Polaris shall be correct within about $1''$, it will in general be necessary to know the time within about 1^s and the latitude to the nearest minute of arc. To obtain the latitude and the time with the required accuracy it will usually be necessary to make special observations for these quantities, for latitudes scaled from maps cannot always be relied upon, and the time obtained from any source except direct observations is uncertain, and even if this were reliable an ordinary watch can hardly be depended upon

Example

Record — Azimuth by Direction Method

Sta.: High Head; Date: Aug. 13, 1938; Inst. Hildebrandt; Observer: H. J. Shea; Lat. = 44° 46′ 34″.47 N.; Long. = 67° 22′ 18″.83 ($4^h\ 29^m\ 29^s.3$) W.

Position	Objects Observed	Tel. D or R	Mic.	°	′	Backward	Forward	Mean	Mean D and R	Remarks		
						″	″					
11	Dowling........	D	A	111	03	20	20					
			B			26	27	27.8				
		R	A	291	03	31	31					
			B			36	35	33.2	30.5			
										W		E
	Polaris.........	D	A	16	21	10	10			25.9		10.8
	$0^h\ 05^m\ 44^s.0$		B			07	08	08.8		9.4		24.6
	0 07 29 .0											
		R	A	196	21	30	30			16.5	+2.7	13.8
	$0^h\ 06^m\ 36^s.5$		B			31	32	30.8	19.8			
										24.4		09.2
										11.0		26.2
										13.4	−3.6	17.0
											2) −0.9	
											Av. −0.4	
12	Dowling.......	R	A	123	04	21	22					
			B			21	20	21.0				
		D	A	303	04	20	21					
			B			21	22	21.0	21.0			
	Polaris.........	R	A	28	22	17	17			10.8		25.9
	$0^h\ 12^m\ 42^s.0$		B			15	15	16.0		24.7		09.3
	0 14 53 .0											
		D	A	208	22	00	00			13.9	−2.7	16.6
	$0^h\ 13^m\ 47^s.5$		B			02	03	01.2	08.6			
										08.0		24.1
										26.7		11.3
										17.8	+5.0	12.8
											2) +2.3	
											Av. +1.2	

Note: Stride level reads continuously from one end; 1 div. = 9″. Chronometer Bond #541 is $2^m\ 18^s.2$ slow on Greenwich sidereal time.

COMPUTATION OF AZIMUTH, DIRECTION METHOD

Sta: High Head Mark: Dowling Date: Aug. 13, 1938

Date, 1938, position	Aug. 13 11	Aug. 13 12
Chronometer reading	$24^h\ 06^m\ 36^s.5$	$24^h\ 13^m\ 47^s.5$
Chronometer correction	+ 1 18 .2	+ 1 18 .2
G. Sid. Time	24 07 54 .7	24 15 04 .7
Longitude	4 29 29 .3	4 29 29 .3
Local Sid. Time	19 38 25 .4	19 45 35 .4
α of Polaris	1 42 42 .0	1 42 42 .0
t of Polaris (time)	17 55 43 .4	18 02 53 .4
(arc)	268° 55′ 51″.0	270° 43′ 21″.0
δ of Polaris	88 58 04 .4	
log cot δ	8.25565	8.25565
log tan ϕ	9.99661	9.99661
log cos t	8.27090n	8.10070
log a	6.52316n	6.35296
log cot δ	8.255651	8.255651
log sec ϕ	0.148826	0.148826
log sin t	9.999924n	9.999965n
log $\frac{1}{1-a}$	9.999855	0.000098
log (− tan A)	8.404256n	8.404540n
A = Azimuth of Polaris from north	1° 27′ 11″.0 E	1° 27′ 14″.5
Difference in time between D. and R.	$1^m\ 45^s$	$2^m\ 11^s$
Curvature correction	—	—
Altitude of Polaris = h	44° 45′	44° 47′
$\frac{d}{4}$ tan h = level factor	2 .230	2 .233
Inclination*	−0.4	+1.2
Level correction	−0″.9	+2″.7
Circle reads on Polaris	16° 21′ 19 .8	28° 22′ 08 .6
Corrected reading on Polaris	16 21 18 .9	28 22 11 .3
Circle reads on mark	111 03 30 .5	123 04 21 .0
Difference, mark — Polaris	94 42 11 .6	94 42 09 .7
Corrected azimuth of Polaris, from north†	1 27 11 .0	1 27 14 .5
	180	180
Azimuth of Dowling (Clockwise from south)	276° 09′ 22″.6	276° 09′ 24″.2

To the mean result from the above computation must be applied corrections for diurnal aberration and eccentricity (if any) of Mark.

Carry times and angles to tenths of seconds only.

* The values shown in this line are actually four times the inclination of the horizontal axis in terms of level divisions.

† Minus if west of north.

to run accurately for any considerable length of time. Furthermore, the computed local time would depend upon a longitude scaled from a map and hence be doubly uncertain. For these reasons the best results will be obtained when the local time and the latitude are determined at the same time as the azimuth itself.

The simplest way of finding the time would be to observe the transits of several stars across the meridian. But since the direction of the meridian may not be known with accuracy it will often be necessary to resort to some method which will give the necessary precision without requiring that the instrument be turned into the meridian plane. The methods described in Arts. 2-33 and 2-34 are especially adapted to such determinations.

PROBLEMS *

1. Make separate sketches showing the locations of the points for which spherical coordinates are given. Where necessary assume a latitude of 40° N and a longitude of 70° W.

   a. Azimuth (from south), 240°; altitude, 50°.
   b. Local hour angle 15^h; declination, +40°.
   c. Right ascension, 4^h; declination, −20°.
   d. Greenwich hour angle, 340°; declination, +10°.
   e. Sidereal hour angle, 80°; declination, −30°.

2. On May 4, 1960 at a place in latitude = 44° 47′.5 N, longitude = 67° 22′.2 W, it is desired to make observations for time by transit of stars across the meridian starting at $7^h\ 30^m$ P.M., Zone $+5^h$ time. If the sidereal time at 0^h G.C.T., May 5, 1960 = $14^h\ 50^m\ 05^s.6$ what would be the right ascension of a southern star which would cross the local meridian at $7^h\ 30^m$ P.M.?

   If it is desired to make observations on stars which cross the meridian at altitudes between 15° and 50°, what are the limits (disregarding refraction) in declination of stars meeting this requirement?

3. During the observations described in Problem 2, the star γ Corvi (right ascension = $12^h\ 13^m\ 46^s.7$) is observed to cross the meridian at a watch time of $8^h\ 51^m\ 45^s.5$ P.M., Zone $+5^h$, what is the error of the watch?

---

* Some of the following problems may be solved either through use of data from the American Ephemeris and Nautical Almanac or from the Nautical Almanac. In such problems sufficient information is given to solve the problem using data from either source.

The dates in the problems may be changed to a current year and the data obtained from either publication of the proper year.

4. Compute the error of the watch from the following observations on the sun, May 25, 1960.

Mark	Vernier A (Angle to right)	Altitude	Watch, E.S.T.
Mark	0° 00′		
	76 25	35° 28′	3h 42m 37s P.M.
	76 39	35 13	3 43 57
	76 49	35 02	3 44 55
	77 37	35 25	3 45 50
	77 49	35 12	3 46 55
	78 00	35 00	3 47 58
Mark	0 00		

Latitude = 42° 20′.5 N, longitude = 71° 07′.5 W. Index correction = 0′. Declination at 0^h G.C.T., May 25 = +20° 54′ 33″.3, May 26 = +21° 05′ 13″.1. Equation of time at 0^h G.C.T., May 25 = +3^m 12^s.30, May 26 = +3^m 06^s.28. Sun's G.H.A. at 20^h G.C.T., May 25 = 120° 46′.8.

5. The observed altitude of the star *ARCTURUS* (right ascension = 14^h 13^m 51^s.0, declination = +19° 23′ 15″.0, sidereal hour angle = 146° 32′.0) on April 9, 1960, is 38° 15′.5 (star east of meridian) at a watch time of 8^h 53^m 47^s.5 P.M., Zone +5^h, at a place in latitude 44° 47′.8 N, longitude 67° 22′.5 W. The sidereal time at 0^h G.C.T., April 10, 1960 = 13^h 12^m 53^s.7 and the G.H.A. of the vernal equinox at 1^h G.C.T., April 10, 1960 = 213° 15′.0. What is the error of the watch?

6. Find the latitudes from the following meridian observations:

Body	Date	Declination	Corrected Altitude	Bearing of Body
a. Sun	Dec. 21	−23° 26′.3	36° 25′	South
b. Sun	July 14	+21 43 .0	67 38	South
c. Sun	Dec. 21	−23 26 .3	67 42	North
d. Sun	July 14	+21 43 .0	79 57	North
e. Aldebaran	Dec. 1	+16 25 .8	52 10	South
f. Aldebaran	Dec. 1	+16 25 .8	52 10	North

7. At a place in longitude 67° 35′.5 W., the observed altitude of *POLARIS* is 46° 40′.5 (index correction = +0′.5) on September 4, 1960. The corrected time of the observation is 10^h 51^m 17^s P.M., E.S.T. For *POLARIS*, right ascension = 1^h 57^m 31^s.1, declination = +89° 04′ 33″.1. S.H.A. = 330° 37′.2. The sidereal time at 0^h G.C.T., Sept. 5, 1960 = 22^h 56^m 23^s.8. G.H.A. of the vernal equinox at 3^h G.C.T., Sept. 5, 1960 = 29° 13′.4. Compute the latitude.

8. From the data given in Problem 4, compute the bearing of the mark.

9. Compute the bearing of line AB from the following observation on the sun, Sept. 1, 1960. Instrument at station A.

	Vernier A (Angle to right)	Altitude	Watch (Zone $+3^h$)
Sta. B	$0^\circ\ 00'$		
	81 15	$34^\circ\ 17'$	$8^h\ 42^m\ 51^s$ A.M.
	81 01	34 33	8 44 06
	80 48	34 48	8 45 20
	79 13	35 15	8 50 29
	78 58	35 31	8 51 34
	78 44	35 45	8 52 49
Sta. B	0 00		

Latitude = $22^\circ\ 54'$ S, longitude = $43^\circ\ 09'$ W. Index correction = $+1'.0$. Declination at 0^h G.C.T., Sept. 1, 1960 = $+8^\circ\ 21'\ 55''.4$. Sept. 2 = $+8^\circ\ 00'\ 07''.8$.

10. Compute the Eastern Standard Time of western elongation of *POLARIS* on March 7, 1960. The right ascension of the star is $1^h\ 55^m\ 22^s.7$, declination is $+89^\circ\ 04'\ 51''.7$. Latitude = $42^\circ\ 21'.5$ N, longitude = $71^\circ\ 06'$ W. Sidereal time at 0^h G.C.T., March 7, 1960 = $10^h\ 58^m\ 50^s.9$, March 8 = $11^h\ 02^m\ 47^s.5$. The Sidereal Hour Angle of *POLARIS* = $331^\circ\ 09'.3$, and the G.H.A. of the vernal equinox at 1^h G.C.T., March 8, 1960 = $180^\circ\ 44'.3$. Compute also the azimuth of *POLARIS* at western elongation.

11. On October 29, 1960, *POLARIS* is sighted at $7^h\ 07^m\ 30^s$ P.M. (watch 30^s slow of Zone $+5^h$ time). The transit is at station A. The horizontal angle to the right from station B to the star is $98^\circ\ 35'$. The observed altitude of the star is $42^\circ\ 57'$ (index correction = $0'$). Latitude = $42^\circ\ 26'$ N, longitude = $71^\circ\ 03'$ W. For *POLARIS*, right ascension = $1^h\ 58^m\ 02^s.9$, declination = $+89^\circ\ 04'\ 52''.3$, S.H.A. = $330^\circ\ 29'.4$. The sidereal time at 0^h G.C.T., Oct. 30, 1960 = $2^h\ 33^m\ 14^s.2$. The G.H.A. of the vernal equinox at 0^h G.C.T., Oct. 30 = $38^\circ\ 18'.6$. By use of relation (2-15), compute the true bearing of line AB.

CHAPTER 3

VERTICAL CONTROL
PRECISE, TRIGONOMETRIC, AND BAROMETRIC LEVELING

3-1. Vertical Control. Precise leveling is used for establishing the elevations of the controlling bench marks which serve as starting points for all subsequent leveling. Trigonometric leveling, carried on simultaneously with triangulation, furnishes a rapid and inexpensive means of determining the elevations of the stations for such purposes as reduction to sea level, and correction of slope distances, such as obtained with electronic instruments, to horizontal measurements. It also furnishes a number of well-distributed points for use in topographical work. Barometric leveling yields but rough results as compared with the two preceding methods, but is very useful in reconnaissance work and in filling in details on small-scale maps.

3-2. Precise Leveling. Precise spirit leveling differs from ordinary leveling in that certain refinements are introduced into the methods of procedure and into the design of the instruments. Lines of precise levels have been run by the U. S. Coast and Geodetic Survey, the U. S. Geological Survey, the U. S. Army Engineers, the Mississippi River Commission, and others. In 1960, over 179,000 miles of first-order and 270,000 miles of second-order leveling had been completed in the United States. The standard elevations of all of the bench marks (about 387,000) are published in government reports. The required precision is indicated in Table II of Classification of Standards of Accuracy-Leveling (Art. 1-7).

3-3. Sources of Error. Some of the chief sources of error recognized in leveling work are

1. Settling of instrument or rod.
2. Unequal expansion or contraction of different parts of instrument caused by temperature changes.

3. Irregular refraction of the air near the surface of the ground.
4. Unequal lengths of backsight and foresight.
5. Selecting poor turning points.
6. Error in length of rod.
7. Change in length of rod due to change in temperature.
8. Bubble not centered at time of sighting.
9. Convergence of level surfaces.

FIG. 3-1. PRECISE LEVELING PARTY.

3-4. Eliminating Errors. The program of observations is planned so as to eliminate these errors as far as possible. Errors caused by settling of the tripod may be eliminated by taking the rod readings on alternate set-ups in such an order as to eliminate the settling provided it has a uniform rate. This is done by reading the backsight first at one set-up, and the foresight first at the next. In practice this procedure is accomplished by the use of two rods (Art. 3-11). These are in matched pairs, one re-

ferred to as No. 1 and the other as No. 2. These designations are retained throughout the work or until replaced with other rods. Rod No. 1 is always read first at each instrument set-up but is alternated from "back sight" to "front sight" with successive instrument set-ups.

Decreasing the interval of time between the backsight reading and the foresight reading will tend to diminish errors caused by temperature changes or by settling. The instrument should therefore be designed for rapid reading. The instrument must be protected from direct rays of the sun and from wind (Fig. 3-1).

Errors due to atmospheric refraction may be diminished by using a high tripod. Observations taken during the middle of the day are likely to be better than those taken in early morning or late afternoon because the refraction is less variable.

Errors caused by unequal length of sight are avoided by observing the stadia distances to the rod at each sight and keeping the differences in length within carefully specified limits. Any remaining difference can be allowed for in computing the results.

The rod is kept plumb by means of a spirit level on the back. The rod is also provided with a thermometer.

3-5. Instruments — Geodetic Level. The precise geodetic level used extensively by the U. S. Coast and Geodetic Survey for first order leveling is shown in Fig. 3-2. The telescope is made of iron-nickel alloy, or of invar, having a low coefficient of expansion. It has a magnifying power of 43 diameters, and in addition to the usual vertical and horizontal hairs it has two stadia hairs set at an interval corresponding to 1 in about 333. As the interval will vary in different levels the value should be determined in the field. The spirit level, which has an air chamber at one end for regulating the length of the bubble, has an angular value of $1.7''$ per division (of about $\frac{1}{16}$ inch). The level case holding the vial is set down into the telescope tube as low as possible. The instrument is first approximately leveled by means of the leveling screws and a circular bubble. The clamp holding the instrument to the tripod is then tightened and no further adjustments are made with the leveling screws. A micrometer screw of very fine pitch placed under the eyepiece serves to level the

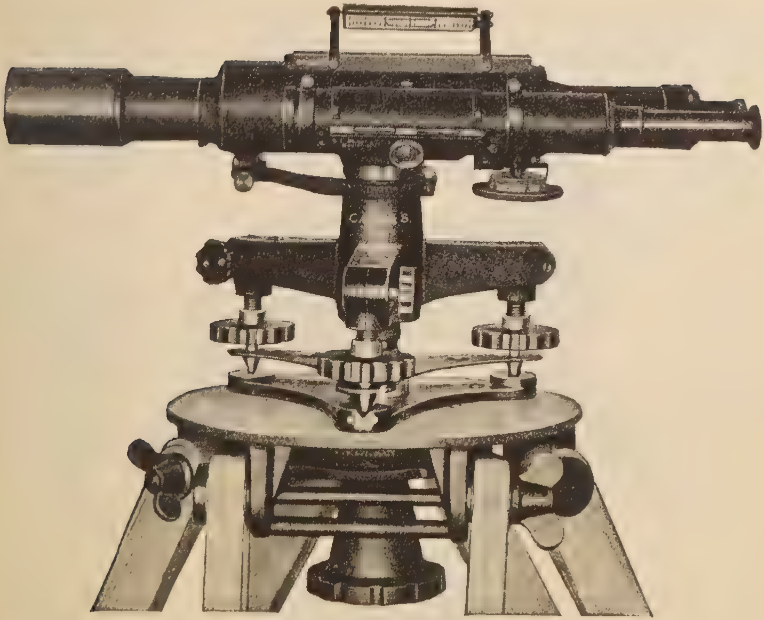


FIG. 3-2. THE U. S. COAST SURVEY PRECISE LEVEL.

instrument precisely by tilting the telescope about an axis over the centers.

The telescope is so designed that the observer sees the image of the rod and the cross-hairs with his right eye, and at the same time he sees the image of the bubble and scales, reflected by a mirror and prisms, with his left eye. The level is sometimes called a *prism level.*

This instrument has tripod legs of such a length that the observer may take readings while standing erect. In addition to being more comfortable for the observer, the high line of sight lessens the effect of refraction.

3-6. Other Types of Geodetic Levels. Several types of levels have been developed by both European and American manufacturers. They all have three leveling screw bases and micrometer screw adjustments for centering the bubble.

The precise tilting level (Fig. 3-3) has a spirit level equipped

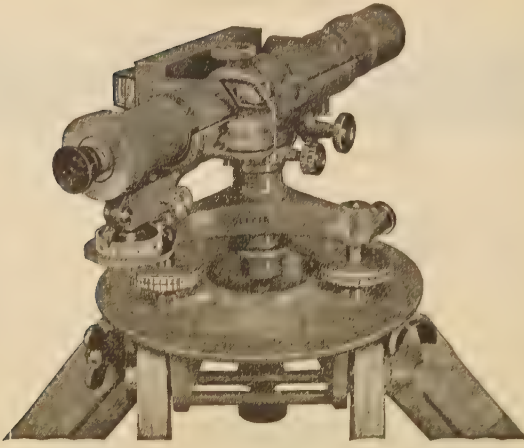


Fig. 3-3. Precise Tilting Level.
(Courtesy, C. L. Berger & Sons.)

with an air chamber mounted by brackets on the left side of the telescope. By means of a prismatic reading device attached above and to the left of the level vial casing (Fig. 3-4), the level bubble is made to appear split longitudinally and also transversely as shown in Fig. 3-5. The left view in Fig. 3-5 shows one end of the bubble before centering and the right view shows the bubble ends when the bubble is in the central position.

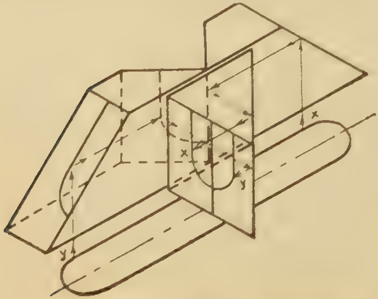
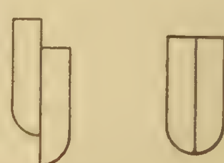


Fig. 3-4. Prismatic Reading Device.



Before Centering After Centering

3-5. Bubble as Seen in Prisms.

3-7. Precision Levels with Optical Reading Systems. There are several makes of precise tilting levels with prismatic devices for centering the bubble and an optical system for obtaining rod readings. The Wild NIII precision level shown in Fig. 3-6 is such a level with a prismatic bubble centering device similar in principle to that shown in Figs. 3-4 and 3-5. The rod readings,

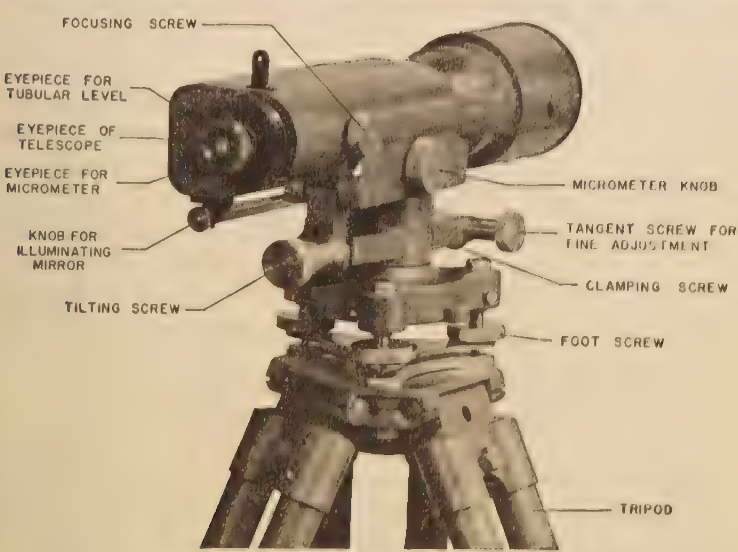


FIG. 3-6. WILD PRECISION LEVEL NIII.
(Courtesy, Henry Wild Surveying Instruments Supply Co.)

however, are obtained by an optical system in which a plane parallel glass plate is mounted in front of the objective lens in such a way that it may be tilted about a horizontal axis at right angles to the line of sight of the telescope (Fig. 3-7). By turning the micrometer knob on the right side of the telescope (Fig. 3-6), this plate may be tilted from the parallel position and the rays of light from the rod displaced, causing an apparent vertical movement of the rod graduations as viewed through the telescope. The amount of this apparent movement is recorded on a micrometer scale which is viewed through an eyepiece at the lower left of the telescope. In Fig. 3-7 the image of the rod is

shown as it appears through the telescope eyepiece. At the upper left the matched bubbles appear as seen through the upper left eyepiece, and at the lower left the view of the micrometer scale appears as seen through the lower left eyepiece. The rod is graduated in centimeters, and the micrometer scale is graduated in millimeters and tenths of millimeters. Hundredths of millimeters may be read from the scale by interpolation.

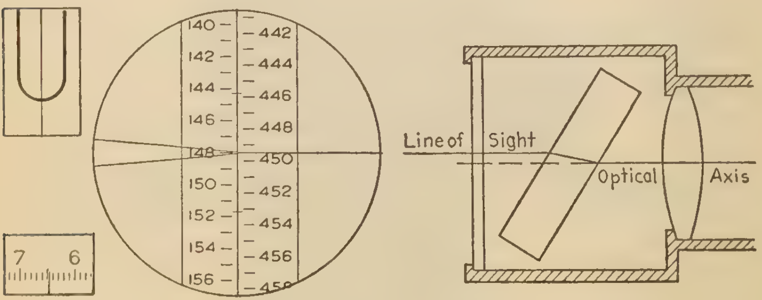


FIG. 3-7. READINGS OF WILD LEVEL (LEFT) — SCHEMATIC VIEW OF PLANE-PARALLEL GLASS PLATE (RIGHT).

In making a reading the instrument is carefully leveled using the tilting screw. The micrometer knob is then turned until the centimeter division reading on the rod scale is made to appear to coincide with the horizontal cross hair in the telescope. This coincidence is obtained by centering this division between the V-lines to the left of the horizontal half cross hair. In Fig. 3-7 the reading is 148 cm. at the coincidence plus 6.47 mm. read on the micrometer scale giving 148.647 cm. for the total reading.

The European style rod used with the Wild level (Fig. 3-7) has two scales graduated in centimeters, one on each side of the face of an invar metal band. This band is attached firmly to the base of the rod at one end and at the other end it is held by a spring which maintains a constant pressure on the band and permits the wooden part of the rod to change in length without affecting the length of the graduated band. The two sets of graduations are offset by an interval of about 4.5 mm. One scale is numbered from 4 to 300 and the other from 306 to 600 centi-

meters. At each sighting both scales are read and separate sets of notes are kept for each scale. Thus decidedly different readings are obtained from the same rod which permit a continuous check on the leveling and a prompt detection of errors.

The Wild level is light and compact and easily transported. It has a short focusing distance of about 6.5 feet which makes it useful for setting precise grades inside buildings as well as for running precise level circuits in the field.

3-8. Self-Leveling Level. In this type of level an optical system is employed to establish a precise level line of sight. The system replaces the sensitive spirit level bubble used in conventional types. See Vol. I, Art. 101a. The line of sight is reflected through a chain of three prisms within the telescope. The two end prisms are fixed while the middle one, known as the compensating prism, is suspended by wires from the roof of the telescope and is free to swing as a pendulum acting under the force of gravity. A line of sight coming from an object enters the first fixed prism, is reflected downward through the suspended prism, and then upward through the third (fixed) prism from which it is reflected to the cross hair and the eyepiece. So long as the pendulum prism hangs freely, a level line of sight is maintained regardless of whether or not the telescope tube is truly level.

The self-leveling feature permits rapid work and a degree of accuracy adequate for establishing level control for engineering works or topographic surveys.

For leveling of, say, second order precision, self-leveling instruments will give excellent results if carefully used. Such precautions are necessary as maintaining equal distances between foresights and backsights, the making of daily peg tests and adjustments if required and the use of precision leveling rods.

3-9. Geodetic Leveling Rods. These are one-piece rods of the self-reading type having either a flat or a T-shaped section. The rod shown in Fig. 3-8 consists of a hardened metal footpiece, an invar strip secured to this footpiece in such a manner that adjustments for wear of the footpiece can be made readily, and a wooden backing piece, also attached to the footpiece, to support the invar strip when under considerable tension but free to adjust for changes in temperature.

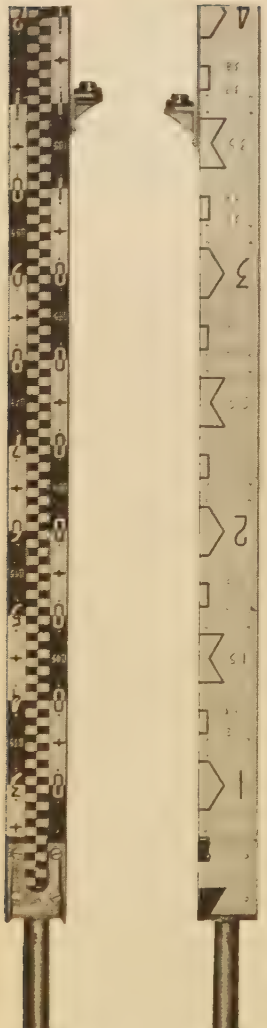


FIG. 3-8. U. S. COAST AND GEODETIC SURVEY PRECISE LEVELING ROD

The wooden backing is treated to prevent changes due to moisture. The sides are protected by strips of a special magnesium extrusion that overlap the edges of the segmental marking units of the front and back of the rod. A shallow, protective recess for the invar strip is routed out of the face of the rod.

The graduations on the invar strip consist of a checkered pattern, of alternate and staggered black and white rectangles of 1 cm. width painted on the strip in units of 1 meter between carefully calibrated registration markings. The graduations on the back (a rough scale of feet) and the black and white meter and decimeter identification and numbering system on the face are applied in the form of segments of thin sheet Bakelite engraving stock in units of one decimeter. The rod is read directly to centimeters and the millimeters are estimated. After the readings on the front of the rod have been made the rod is turned and readings on the back in feet are made for a check.

The rod is equipped with a small spherical level mounted at the side and provision for a thermometer to be mounted in the back such that the bulb can extend through the wood to touch the underside of the invar strip.

3-10. The Yard Rod. The use of a yard rod graduated in units, tenths and hundredths of yards offers some advantages in precise leveling. When

readings are noted on a three-wire instrument, the sum of the readings gives an average rod reading in feet. If a stadia interval of 0.3 per 100 is used, the total rod intercept multiplied by 1000 will give the rod distance in feet. These stadia distances should be used to insure that the backsight and foresight distances are kept nearly equal.

The yard rod shown in Fig. 3-9 consists of a T-section mahogany staff with a face $1\frac{1}{2}$ inches wide. The graduated ribbon 4 yards long is supported in recessed guides, the lower end being securely attached to the steel shoe and the upper end held in tension by a strong spring.

The full yard numerals are in red, such as the " 2 " above the " 9 " in middle section view (Fig. 3-9). The small dots under the tenth yard numerals are also in red, and denote the number of full yards to which the tenths refer, such as the " 2 " with two dots under it represents 2.20 yards. Other figures and markings are in black.

Fig. 3-9. Yard Rod. (Courtesy, Keuffel & Esser Co.)

3-11. Method of Observation. Regardless of the type of instrument used, two rods are usually used in precision leveling. If the level lines are being run where the turning points can be clearly marked, such as along a railroad, one rod always serves for a backsight and the other for a foresight. Should the levels be run along, say, a highway where the turns are less easily identified, the rod used as a foresight may be held in position to serve as a backsight for the next instrument setting to insure the preservation of the turn. Subse-

quently this rod is carried forward beyond the first instrument position and is set in place to serve as a foresight for the second instrument position. In the first case the level party moves along the line of levels as a unit; in the second, each rod serves alternately for a backsight, then a foresight.

The level is set up and leveled by means of a circular level. If the level is of the tilting-telescope type and has a cam action to lift the telescope off the point of the micrometer screw, the telescope is first lowered. In any case the level bubble is centered. This latter step is not necessary with self-leveling instruments.

The telescope is quickly pointed at the two rods to insure that the cross-hairs will fall on both rods. Also a gross inequality in foresight and backsight distances may be detected if one rod is in focus and the other quite out of focus.

In all cases, one of the rods is always read first. When the same rod is always used for a backsight it is read first at all setups; if the rods are alternately used for backsights and foresights, the same rod, whether it is rear or forward, is read first. If a three-wire method is used, the observer makes certain that the bubble is centered when reading each hair. The instrument is then pointed at the other rod and a similar set of readings are taken. The rodmen then read the rod thermometers. All of these readings are entered in the notebook by the recorder. If stadia intervals can be obtained, these should be found before the instrument is moved to avoid having an undesirable inequality in backsight and foresight distances.

If a scale of feet or meters is painted on the back of the rod, that scale is read immediately after reading the front, entered in the notebook and checked against the face reading. Sometimes this scale is graduated to represent three times the front reading.

3-12. Precautions in Using the Level. If the level is equipped with a cam screw, this is first turned to take the weight of the telescope off the micrometer screw. If the level bubble has an air chamber (see Vol. I, Art. 94, p. 74) by means of which the length of the bubble can be regulated, the telescope should be clamped in such a position that air will not escape from the bubble chamber when the level is being carried. A cloth cover should be placed over the telescope to protect it from the sun. The in-

strument should be carried in a nearly vertical position to avoid deranging its adjustment and all clamps should be tightened to prevent the instrument shaking on its tripod.

When making a new set-up, the instrument should be lowered gently onto the ground and the cloth cover removed after the umbrella is in position. The various provisions for preparing the instrument for observing are then repeated.

3-13. Accuracy — Length of Sight. The lines of levels are all run in a forward and in a backward direction, usually in short sections of 1 to 2 kilometers length. When possible the two runs are made during different parts of the day and under different atmospheric conditions. If the two differences of elevation of any section agree within the limits specified in Classification of Standards of Accuracy, Table III (Art. 1-7), the mean result is accepted as the final difference in elevation. If the discrepancy is greater than specified, the levels are repeated until a check is obtained. For first order leveling the differences must agree within $4^{mm}\sqrt{K}$, where K is the length of section in kilometers.

The closures specified in the Standards refer to loop closures, that is, a double (forward and back) line of levels. The distance K, however, is the actual distance between the two points between which the levels were run.

The lengths of the fore and backsights are derived from the stadia readings and enable the observer to see whether the fore or the backsight distances are too long. The instructions permit any length of sight to be taken up to 150 meters, but the above check on sections must be met. An average length of sight is less than 100 meters. The difference between the foresight distance and the backsight distance is not allowed to become more than 10 meters on any one set-up, and it must not exceed 20 meters for the total distance at any time.

3-14. Adjustment of the Level. The adjustment of the level is tested daily, by the following process. One rod is held at a turning point about 10 meters from the level and a reading taken. Then the other rod is held about 70 to 100 meters away and another reading taken. The level is then moved to a point about 10 meters from the second (distant) rod and the two readings repeated. The slope of the line of sight is then computed as

follows. The true difference of elevation from the first set-up is

$$(n_1 + Cs_1) - (d_1 + CS_1)$$

where n_1 is the near rod reading; s_1, the near stadia reading; d_1, the distant rod reading; S_1, the distant stadia reading; and C is a factor which, when multiplied by the stadia reading, gives the correction to the rod reading for the slope of the line of sight. From the second observation we have

$$(d_2 + CS_2) - (n_2 + Cs_2)$$

Equating these two and solving for C, we have

$$C = \frac{(n_1 + n_2) - (d_1 + d_2)}{(S_1 + S_2) - (s_1 + s_2)} \tag{3-1}$$

C is $+$ if the line of sight dips below the true horizon; $-$ if it sights upward.

Since there is an appreciable difference in the two rod distances it is necessary to correct the distant readings for curvature and refraction to obtain the correct value for C. The complete formulation may be stated as follows:

$$C = \frac{\left\{\begin{array}{l}\text{sum near rod readings} \\ \quad - (\text{sum distant rod readings} - \text{c. \& r. corrs.})\end{array}\right\}}{\text{sum distant rod readings} - \text{sum near rod readings}}$$

If C exceeds 0.010 the level must be adjusted. If it is less than 0.005 the adjustment should not be disturbed. If it is between the two, the observer may use his judgment but is advised not to adjust. The adjustment is made by moving the level case, the middle hair first being sighted at the correct rod reading on the distant rod. That is, if C is $-$ the level is centered and a reading taken. Then the telescope is lowered by an amount equal to CS and the level is then adjusted.

In the typical set of readings illustrated on p. 179, the station letters refer to instrument stations, not to the rod points themselves as in ordinary level notes. The three thread readings are recorded in a group. To the right is the mean of the three. Readings are taken to millimeters and the mean is carried to tenths. A difference of 3^{mm} is allowed in the two spaces between hairs. These intervals are recorded in the column to the

DETERMINATION OF C

The stadia constant for this instrument is 1 in 348.

(Left-hand page)					(Right-hand page)				
Number of Station	Thread Reading, backsight (near rod)	Mean	Thread Intervals Diff.	Sum	Rod	Thread Reading, foresight (far rod)	Mean	Thread Intervals Diff.	Sum
A	1542		14			0566		104	
	1528	1528.3	13		W	0462	0461.7	105	
	1515			27		0357			209
B	1301		13			2462		105	
	1288	1288.3	12		V	2357	2357.0	105	
	1276			52		2252			419
		+2816.6					2818.7		−52
		−2817.9			Corr. for curv. and refr.				367
		−1.3					−0.8		
							2817.9		

$$C = \frac{-1.3}{367} = -0.004$$

right of the mean. If the two differ by 1^{mm}, then the mean will be 0.3^{mm} greater or less than the middle reading. If they differ by 2^{mm} the mean will be 0.7^{mm} greater or less than the middle thread reading. The mean is quickly computed in this manner. For example, in the first backsight from A the half intervals are 14 and 13, which differ by 1^{mm}: the correction to 1528 is therefore 0.3^{mm}, giving 1528.3 for the mean. The sum of the half intervals gives the whole interval, 27^{mm}, a measure of the distance.

The thread readings are entered as read on the near or the far rod, respectively, regardless of whether the reading is a backsight or a foresight. The sum of the far rod readings is corrected for curvature and refraction. In this particular instrument the distance corresponding to an interval of 209^{mm} or 210^{mm} is a little over 73 meters. For this distance the curvature and refraction correction is 0.4^{mm}, that is, 0.8^{mm} for the double distance. (Table I, p. 490.) The rod reading is always too great by the amount of this correction. The value of C is therefore

$$C = \frac{2816.6 - 2817.9}{419 - 52} = \frac{-1.3}{367} = -0.004$$

The instrument is therefore in excellent adjustment. The line of sight is pointing upward about 0.8^{mm} in 73 meters.

The notes for a line of levels are kept in the form shown below. These are similar to those shown in the preceding table except that in the last column there is given the total stadia distance from the start. In this last column the two numbers should not differ by more than the amount corresponding to a distance of 20 meters, that is, about 60^{mm}. The stadia interval in common use is such that 1^{mm} corresponds nearly to one foot of distance. This gives a ready means of adjusting the rod distance. If the

SPIRIT LEVELING

(Left-hand page)

Date: August 29, 1957.
Sun: *C.* Forward. ~~Backward~~
(Strike out one word.)

(Right-hand page)

From B.M.: 68. To B.M.: *G*
Wind: S.T.
Stadia constant 1 in 348.

No. of Station	Thread Reading, back-sight	Mean	Thread Interval	Sum of Intervals	Rod and Temp.	Thread Reading, fore-sight	Mean	Thread Interval	Sum of Intervals
43	0674		99		V	2683		99	
	0773	0773.0	99		38	2782	2782.3	100	
	0872		198			2882		199	
	0925		106		W	2415		103	
44	1031	1030.3	104		35	2518	2518.0	103	
	1135		210	408		2621		206	405
	0484		98		V	2510		96	
45	0582	0582.3	99		35	2606	2606.0	96	
	0681		197	605		2702		192	597
	0398		97		W	2859		96	
46	0495	0495.0	97		34	2955	2954.7	95	
	0592		194	799		3050		191	788
	1027		26		V	1006		29	
47	1053	1053.3	27		34	1035	1034.7	28	
	1080		53	852		1063		57	845
							−11895.7		
		3933.9					+3933.9		
							−7961.8		
							2:25 P.M.		

foresight sum is 19^{mm} greater than the backsight sum, the front rodman should move his turning point toward the instrument about 19 feet to balance the distances.

3-15. Calculating the Difference in Elevation. The difference in elevation is obtained by adding the two columns of means and subtracting the smaller from the larger. This difference is then corrected for 1. curvature and refraction if any pair of sights is sufficiently unequal to require it; 2. error of adjustment of level ($= C \times$ diff. in sums of intervals);* 3. length of rod; 4. temperature of rod. In addition, there may be an *orthometric correction* (see Art. 3-20).

The difference in elevation in these notes is -7961.8^{mm}, showing that B.M. G is that number of millimeters lower than No. 68. The sums of the intervals show that the backsights are 7^{mm} longer. This corresponds to about 2.4 meters of distance. The correction for this error would be $-7 \times .004 = -0.03^{mm}$, a negligible quantity.

The correction for curvature and refraction may be taken from Table I, p. 490. More extensive tables will be found in Special Publication No. 239, Coast and Geodetic Survey.

3-16. Lines of Levels. Lines of first-order levels are often run on railroad tracks both for speed and for convenient grades. Distances to rods may be readily controlled by counting rails. Various turning points have been used; the top of a rail, the top of a rail spike, and an ordinary wire nail driven into a tie. Steel pins about a foot long are always carried, to be used when a turn must be taken on soft ground, as when trains are passing, or when on a highway.

When there is much variation in rod reading caused by variable refraction, especially near the ground, it is found that better results are obtained if the lowest (least) reading is adopted rather than the mean.

---

* This C factor is used to correct a series of elevations as follows: considering the backsight distances as positive and the foresight distances as negative, the arithmetic difference of the two for any particular station will produce a + or − sign, which multiplied algebraically by the sign of C will indicate the amount and direction of the correction to be applied to the elevation in question.

3-17. Crossing a River. When a river is to be crossed, the procedure is modified as follows. Two targets are placed on the rod, one above and one below the level reading. The middle hair is made to bisect the upper target and the micrometer drum is read. Then the lower target is bisected and the micrometer again read. The micrometer is also read for the central position of the bubble. These are repeated several times. These readings, together with the two target readings on the rod, furnish sufficient data for computing the rod reading of the middle hair for a level reading of the bubble. Usually observations are made simultaneously by two observers on opposite sides of the river. The observers then exchange places and repeat the whole operation.

3-18. Bench Marks. The bench mark now principally used is a bronze tablet marked with the number of the bench mark, Fig. 3-10. These are set in concrete posts. But in cities, readings

FIG. 3-10. COAST AND GEODETIC SURVEY BENCH MARK.

are taken on all of the principal bench marks passed by the line of levels. The distance between bench marks on the main line of levels should not exceed 15 kilometers; every 100 kilometers of line should have at least 20 bench marks. A good average distance between marks is 2.5 kilometers.

3-19. Datum. The datum for geodetic levels is mean sea level, as derived from tidal observations. This is considered to be sufficiently well determined by a series of "annual means." The heights of the tide are recorded on an automatic gage. The vertical motion of a float is communicated to the recording drum on which the record paper is fastened; the motion is reduced by

passing the cord over a series of pulleys so that a rise of 10 to 20 feet will make a record but a few inches high. The drum is driven at a uniform rate by means of a clock mechanism. The position of the float is referred by levels to a staff gage, and both are referred to some permanent bench mark nearby. (For one style of gage see Art. 7-26.)

It was formerly assumed that a line of levels must necessarily follow the curve of sea level and should check exactly on all tidal bench marks. It is now known that this is not the case. Elaborate investigations show that the Pacific Ocean is about two feet higher than the Atlantic, and that both oceans are higher in the northern than in the southern portions, assuming the levels to be correct. For instance, there appears to be about 16 inches difference between Portland, Me., and St. Augustine, Fla. For use on ordinary engineering work, it is the custom to adjust the levels to the local tidal bench marks when computing standard elevations of government bench marks. In other words, over small areas this variation in sea level may safely be neglected.

3-20. Orthometric Correction. On account of the spheroidal form of the earth and the action of centrifugal force, level surfaces at different elevations are not exactly parallel. A lake at a high elevation in latitude 45° N would be nearer to sea level at its northern end than it would at its southern end. This would cause an apparent error in a circuit of levels which rose from sea level to the south end of the lake, then followed the lake surface to the northern end, then dropped to sea level again, and finally followed the ocean level to the starting point. The amount of drop in the level surface may be computed roughly by the expression

$$-0.005288 \sin 2\phi \cdot h \cdot \Delta\phi' \cdot \text{arc } 1' \qquad (3\text{-}2)$$

$$(\text{arc } 1' = 0.0002909)$$

in which ϕ is the latitude, h is the elevation above sea level, and $\Delta\phi'$ the difference in latitude in minutes of angle (+ when going north). Accurate formulas and tables will be found in Coast Survey Spec. Publ. No. 140. As an illustration, Lake Michigan (eleva. 177^{m}) at Chicago is 0.02^{m} higher than it is at Milwaukee.

On the lines of levels connecting San Diego with Seattle the corrections aggregated more than one meter.

3-21. Level Circuits. When establishing level control, the line of levels usually begins and ends on previously set bench marks. At least two bench marks at either end should be recovered and checked before beginning or completing the project. By this procedure, the disturbance of any bench mark may be determined. The frequency with which new bench marks are established will depend on the nature of the project. For intensive use, such as for a large highway project, the bench marks should be set closer than for a less intensive use. If the bench marks are about two miles apart, it is good practice to establish temporary bench marks at somewhat more frequent intervals. Thus, if the line must be double-run, or checked for error, the sectional units available for comparison are not excessively long.

When covering an area it is common practice to establish a level net composed of a number of closed circuits. These circuits have common contacts at critical bench marks often referred to as junction points. Before beginning a project, the routes to be followed should be planned from a map of the region and, if necessary, examined in more detail in the field.

As soon as the fieldwork is completed the levels should be adjusted and elevations established for the bench marks. Appendix C, p. 527 contains a simple method of adjusting a level net. For more detailed methods, consult Special Publication No. 240 of U. S. Coast and Geodetic Survey, "Manual of Leveling Computation and Adjustment."

3-22. Trigonometric Leveling. The elevations of triangulation stations are usually established by observing the vertical angles between stations and computing the differences in elevation trigonometrically. These angles are taken at the time the station is occupied for the purpose of measuring the horizontal angles. The elevations of certain points in the survey, for example the ends of the base-line, are established by direct leveling from tide water or from a known bench mark. From the elevations of these points the elevations of the triangulation stations may be found by means of the differences in height derived from the vertical angles and the lengths of the triangle sides.

In measuring the vertical angles the instrument should be placed, if possible, over the center mark of the station; if it has to be placed to one side, its position should be located by azimuth and distance from the center mark. In either case its height above the station should be measured. A definite point on the signal at the distant station should be selected for sighting the cross-hair when measuring the vertical angle. From the known dimensions of the distant signal the height of the point sighted above the station mark may be obtained (see Art. 1-21).

In the most exact work the vertical angles are measured with a special vertical circle instrument; this may be either a repeating instrument or a direction instrument read by microscopes. In work of a less precise character fair results can be obtained by using the vertical circle of a theodolite in which the verniers read to 20″ or to 10″. With this instrument only single measurements of an angle can be made. The best result which such an instrument can give will be obtained by taking the average of several measurements, half of them with the telescope direct and the other half with the telescope reversed. Attention should of course be given to the index correction.

The chief difficulty in obtaining accurate results in this kind of work arises from the uncertainty of the angle of refraction, i.e., the angular deviation of the line of sight on account of the refraction of the air. This angle not only varies with the temperature and the atmospheric pressure but also varies with the locality. It may be nearly eliminated by taking simultaneous observations between two stations, so that the atmospheric conditions for the two observations may be assumed to be the same. If the two observations are made at different times, or if an angle is taken at only one of the stations, then the mean value of the refraction correction must be used in computing the difference in elevation, which introduces a very uncertain factor into the results. The best time to observe vertical angles is during the middle of the day, as the refraction is then much less variable than in the morning or evening.

3-23. Refraction Coefficient. The curve of the earth's surface along the line of sight is nearly circular. The curve of the path of the ray of light between stations is also circular but with a

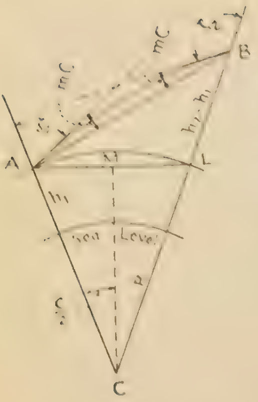


Fig. 3-11. Trigonometric Leveling.

radius approximately seven times as long as the former. The angle of refraction therefore bears a nearly constant ratio to the central angle, this ratio being known as the coefficient of refraction, m.

If r = the angle of refraction and C the angle at the center of the earth, then

$$m = \frac{r}{C}$$

and
$$r = mC \qquad (3\text{-}3)$$

3-24. Simultaneous Observations. Let A and B, Fig. 3-11, be two stations at which zenith distances ζ_1 and ζ_2 are measured; the elevation of $A(= h_1)$ is known and the elevation of $B(= h_2)$ is to be calculated. The zenith distances as observed must be reduced to the station mark by allowing for the height of the instrument and of the point sighted.* The ray of light between A and B is curved by passing through the atmosphere, and it is assumed that the amount of bending is the same at the two stations and consequently that each zenith distance is too small by the same amount (mC). The difference between observed zenith distances is the same as the difference between true zenith distances.

The difference in elevation, $h_2 - h_1$, may be found by solving the triangle ALB. From the law of sines

$$h_2 - h_1 = AL\frac{\sin BAL}{\sin ABL} \qquad (a)$$

* The formula for the reduction to the station mark is

$$c'' = -\frac{i - o}{K \text{ arc } 1''}$$

where c is the value of the angle in seconds to reduce the observed zenith distance to the zenith distance at the station mark, i is the height of telescope above the station mark, o is the height of object sighted above the station mark, K is the distance between the stations.

This reduction is made only in the case of reciprocal observations.

In the isosceles triangle ALC,

$$AL = 2(R + h_1)\sin\frac{C}{2} \tag{b}$$

In triangle ALB, $BAL = ALC - ABL$

$$= \left(90^\circ - \frac{C}{2}\right) - (180^\circ - \zeta_2 - mC) = -90^\circ - \frac{C}{2} + \zeta_2 + mC \tag{c}$$

$$BAL = 180^\circ - \left[\zeta_1 + mC + 90^\circ - \frac{C}{2}\right] = 90^\circ - \zeta_1 - mC + \frac{C}{2} \tag{d}$$

The mean of (c) and (d) gives

$$BAL = \frac{\zeta_2 - \zeta_1}{2} \tag{e}$$

In triangle ABC, $ABC = 180^\circ - [C + (180^\circ - \zeta_1 - mC)]$

$$= -C + \zeta_1 + mC \tag{f}$$

Also $$ABC = 180^\circ - \zeta_2 - mC \tag{g}$$

The mean of (f) and (g) gives

$$ABC = 90^\circ - \left(\frac{C}{2} + \frac{\zeta_2 - \zeta_1}{2}\right) \tag{h}$$

Substituting (b), (e), and (h) in (a)

$$h_2 - h_1 = 2(R + h_1)\sin\frac{C}{2}\,\frac{\sin\left(\dfrac{\zeta_2 - \zeta_1}{2}\right)}{\cos\left(\dfrac{C}{2} + \dfrac{\zeta_2 - \zeta_1}{2}\right)} \tag{3-4}$$

The value of R used in this formula should, strictly speaking, be taken for the latitude of the place of observation and for the azimuth of the sight, since the radius of curvature of the spheroid is different for different latitudes and for different sections through the same point.* An average value of R, sufficiently accurate for short lines, is given as follows:

* For exact values of R for different latitudes and different azimuths see U. S. Coast and Geodetic Survey Special Publication No. 120.

R (in feet)	20,926,000	R (in meters)	6,378,200
Log R (in feet)	7.32068	Log R (in meters)	6.80470

The following problems are solved using natural functions in a form that can be easily computed on a calculating machine. Simplified solutions are available utilizing logarithms and constants tabulated in U. S. Coast and Geodetic Survey Special Publication No. 247, "Manual of Geodetic Triangulation."

EXAMPLE. The zenith distance of Mt. Bache observed from Santa Cruz = 87° 35′ 01″.1; zenith distance of Santa Cruz from Mt. Bache = 92° 35′ 34″.2. The distance is 23931.6 meters. The elevation of Santa Cruz = 108.87 meters.

$$\begin{aligned} R &= 6{,}378{,}200 \text{ meters} \\ h_1 &= 109 \\ R + h_1 &= 6{,}378{,}309 \\ 2(R + h_1) &= 12{,}756{,}618 \end{aligned}$$

$$\sin\frac{C}{2} = \frac{K}{2(R+h_1)} = \frac{23{,}931.6}{12{,}756{,}618} = 0.001876 \qquad \text{Formula } (b)$$

$$\frac{C}{2} = 0^\circ\, 06'\, 27''.0$$

$$\begin{aligned} \delta_2 &= 92^\circ\, 35'\, 34''.2 \\ \delta_1 &= 87\;\; 35\;\; 01\;\; .1 \\ \delta_2 - \delta_1 &= 5^\circ\, 00'\, 33''.1 \\ \frac{\delta_2 - \delta_1}{2} &= 2^\circ\, 30'\, 16''.6 \\ +\frac{C}{2} &= 0\;\; 06\;\; 27\;\; .0 \\ \frac{\delta_2 - \delta_1}{2} + \frac{C}{2} &= 2^\circ\, 36'\, 43''.6 \end{aligned}$$

$$h_2 - h_1 = 2(R + h_1)\sin 0^\circ\, 06'\, 27''.0 \left(\frac{\sin 2^\circ\, 30'\, 16''.6}{\cos 2^\circ\, 36'\, 43''.6}\right) \qquad \text{Formula (3-4)}$$

$$= 12{,}756{,}618 \times 0.001876 \times \frac{0.043700}{0.998961}$$

$$\begin{aligned} h_2 - h_1 &= 1046.9 \\ +h_1 &= 108.9 \\ \text{Elev. Mt. Bache} &= 1155.8 \text{ meters} \end{aligned}$$

3-25. Determining the Value of *m*. Whenever reciprocal observations have been made the corresponding value of the coefficient m may be calculated. The simplest formula for obtaining m is

$$0.5 - m = \frac{(\zeta_1 + \zeta_2 - 180^\circ) R \sin 1''}{2K} \tag{3-5}$$

In the example on p. 188 $\zeta_1 = 87^\circ\ 35'\ 01''.1$, $\zeta_2 = 92^\circ\ 35'\ 34''.2$, $K = 23931.6$ meters. Using for R the value corresponding to latitude 37° N and azimuth 50°, the coefficient m is as follows.

$$\begin{aligned}
\zeta_1 &= 87^\circ\ 35'\ 01''.1 \\
\zeta_2 &= 92\ \ \ 35\ \ \ 34\ \ .2 \\
\text{sum} &= 180\ \ \ 10\ \ \ 35\ \ .3 \\
&\ 180 \\
\text{diff.} &= 0^\circ\ 10'\ 35''.3 = 635''.3
\end{aligned}$$

$$\begin{aligned}
R &= 6{,}374{,}600 \\
\sin 1'' &= 0.000004848 \\
2K &= 47{,}863
\end{aligned}$$

$$0.5 - m = \frac{635.3 \times 6{,}374{,}600 \times 0.000004848}{47{,}863} \qquad \text{Formula (3-5)}$$

$$\begin{aligned}
0.5 - m &= 0.4102 \\
m &= 0.0898
\end{aligned}$$

The value of m as found from a large number of observations is given in reports of the Coast and Geodetic Survey as follows:

Lines crossing the sea.	.078
Between primary stations (high elevation).	.071
In the interior of the country, about.	.065

In his *Geodesy*, Clarke gives for this coefficient

For rays crossing the sea.	.0809
For rays not crossing the sea.	.0750

These values are only averages. The actual value will vary with the atmospheric conditions. On some lines, as for example

across a lake, the station will be visible on some days, invisible on others, according to the amount of refraction affecting the line at the time. That the value of m varies during the day is shown by the following determinations made at Mt. Diablo, Cal.; 3^h A.M., .0893; 9^h A.M., .0812; 2^h P.M., .0640; 9^h P.M., .0827.

3-26. Observation at One Station Only. If the zenith distance is measured at one station only, then the value of m must be known in order to compute the difference in height. From equations (*d*) and (*e*) we have

$$BAL = \frac{\zeta_2 - \zeta_1}{2} = 90^\circ - \zeta_1 - mC + \frac{C}{2}$$

or

$$\frac{\zeta_2 - \zeta_1}{2} = 90^\circ - \zeta_1 + (0.5 - m)C \qquad (3\text{-}6)$$

EXAMPLE. Zenith distance of Mt. Blue observed from Farmington = 87° 07′ 18″.8. Distance = 15,519 meters; height of instrument = 2.20 meters; point sighted 4.40 meters above mark; elevation of Farmington = 181.20 meters. Assume that $m = 0.071$.

$$\begin{aligned} R &= 6{,}378{,}200 \text{ meters} \\ h_1 &= 181 \\ \hline R + h_1 &= 6{,}378{,}381 \\ 2(R + h_1) &= 12{,}756{,}762 \end{aligned}$$

$$\sin\frac{C}{2} = \frac{15{,}519}{12{,}756{,}762} = 0.00121653 \qquad \text{Formula } (b)$$

$$\frac{C}{2} = 0^\circ\ 04'\ 10''.9$$

$$C = 0^\circ\ 08'\ 21''.8$$

$$\begin{aligned} & 90^\circ\ 00'\ 00''.0 && \text{Formula (3-6)} \\ -\delta_1 &= 87\ \ 07\ \ 18\ .0 \\ -mC &= 0\ \ 00\ \ 35\ .6 = 0.071(0^\circ\ 08'\ 21''.8) \\ +\frac{C}{2} &= 0\ \ 04\ \ 10\ .9 \\ \hline \frac{\delta_2 - \delta_1}{2} &= 2^\circ\ 56'\ 16''.5 \\ +\frac{C}{2} &= 0\ \ 04\ \ 10\ .9 \\ \hline \frac{\delta_2 - \delta_1}{2} + \frac{C}{2} &= 3^\circ\ 00'\ 27''.4 \end{aligned}$$

$$h_2 - h_1 = 12{,}756{,}800 \times 0.00121653 \times \frac{0.0512539}{0.998623} \quad \text{Formula (3-4)}$$

$$\begin{aligned} h_2 - h_1 &= 796.58 \\ +h_1 &= 181.20 \\ \hline &\ 977.78 \\ \text{Red'n to Sta.} &= -2.20 \\ \hline \text{Elev. Mt. Blue} &= 975.58 \text{ meters} \end{aligned}$$

3-27. Rough Computations. A rough determination of the difference in height may be made by multiplying the horizontal distance to the station by the tangent of the vertical angle and applying a correction for curvature and refraction (Table I, p. 490). The curvature and refraction may also be found by the formula.

$$h = \frac{K^2}{1.7426} \tag{3-7}$$

where K is the distance in miles and h is the correction in feet.

This method of leveling is the one used in determining differences in elevation by means of the stadia or plane table, except that for very short sights or for rough work the curvature and refraction correction is omitted.

3-28. Barometric Leveling — The Barometer. The barometer is an instrument for measuring variations in the pressure of the air. Since this pressure varies with the height above the sea level the barometer may be used as a means of measuring differences in altitude. The atmospheric pressure varies also with changes of temperature and humidity, so that it is necessary in measuring difference in altitude with the barometer to determine the amount of these variations and to make proper allowance for them.

There are two kinds of barometers, (1) the *mercurial barometer* and (2) the *aneroid barometer*. The mercurial barometer records the atmospheric pressure by the height of a column of mercury in an evacuated glass tube. An inch in the height of the mercury column corresponds to a difference of about 900 feet in altitude. Mercurial barometers are little used in surveying because of the length of time required to take readings and the awkwardness of the instrument for transportation. Their use has been superseded by that of improved aneroid types.

Aneroid barometers contain a basic element which is a partially evacuated box or capsule with a flexible top which contracts and expands with changes in atmospheric pressure. The details of construction of these barometers differ particularly in the means by which the small movements of the basic element are magnified and transmitted to the recording dial. Aneroids are convenient for use in the field because of their small size and light weight.

The older barometers give results which may be in error by as much as 20 to 50 feet. Later models, called *altimeters* (Art. 3-31), have been perfected to give elevations with average errors of 2 to 5 feet and maximum errors of 5 to 10 feet. These refined instruments produce results which are sufficiently accurate for such surveying work as reconnaissance, contouring on small scale maps and establishing vertical control for aerial surveys.

3-29. The Aneroid Barometer. In principle the aneroid barometer consists of a hollow corrugated metal box *A* (Fig. 3-12) from which the air has been partially exhausted and which is so thin that it will change its form when the air pressure changes. The movement of the top of the box *B* is communicated to the

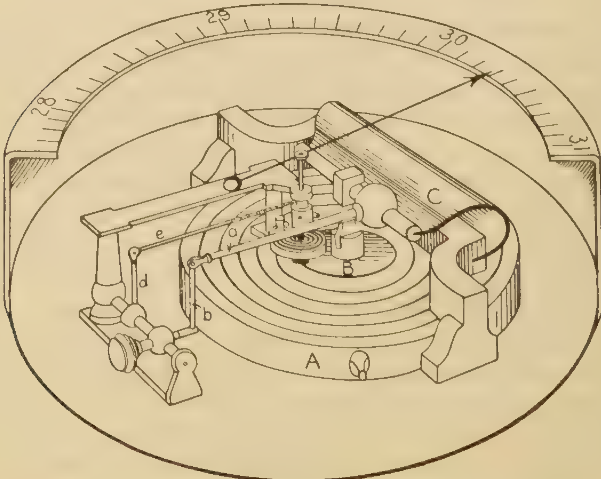


FIG. 3-12. MECHANISM OF THE ANEROID BAROMETER.

spring *C* and then through levers *a*, *b*, and *d*, thus moving the link *e*, at the end of which is a chain wound around the shaft on which the pointer is fastened.

The face of the dial in Fig. 3-12 reads in inches of mercury. A second dial is frequently added alongside of the inch scale but reading in feet of altitude. This scale is calibrated by comparing the aneroid with a standard mercurial barometer in an air chamber under different pressures. On the older barometers the zero of the altitude scale is usually arbitrarily placed opposite 31 inches. The height of the mercury column at sea level in Latitude 45° and at 0° C (32° F) is 76 mm. or 29.92 inches. On aneroids designed for surveying use, the zero reading of the altitude scale of feet is usually set at an arbitrary reading corresponding to an elevation of 500 or 1000 feet below sea level in order to avoid minus readings of the scale.

The scale of feet should not be movable with reference to the scale of inches of mercury, since the number of feet of altitude corresponding to one inch of mercury is different in different parts of the scale. On the back of the aneroid is an adjusting screw which regulates the pointing of the needle by altering the form of the corrugated box. This is used in adjusting the aneroid when comparing it with a standard mercury barometer. Most aneroids are marked "compensated," which means that the mechanism is so arranged, by the use of different metals, that changes of temperature of the instrument will not affect the reading of the needle. A thermometer is often placed on the dial which is used by the maker in adjusting the instrument for compensation. This compensation, however, is not always found to be perfect.

3-30. The Use of the Aneroid Barometer. Aneroid barometers of the type described above should be handled carefully in order to avoid disturbing the delicate mechanism and thus changing the relation between the reading of the pointer and the form of the metal box. When the instrument is to be read the case should be tapped lightly to be sure that the chain has not stuck and that the instrument has adjusted itself to the changed pressure. The barometer should not be heated either by the sun's rays or

by the body. It should stand a few minutes before it is read so as to allow it to come to the true reading. It may be held either horizontal or vertical, but should be held in the **same position at all stations,** as the reading is usually not the same in the two positions.

Altimeters of modern design do not require tapping nor is there any need to wait for the needle to reach the true reading. These instruments are not easily dearranged, but should be handled with reasonable care.

Since atmospheric pressure varies with temperature, it is necessary to apply a temperature correction to observed differences in elevation. For this purpose it is assumed that the mean of the observed temperatures at the two stations represents the mean temperature along the line. If the barometer is calibrated at 50° F, the observed difference in elevation should be multiplied by the factor

$$1 + \frac{t_a' + t_a - 100^\circ}{900} \tag{3-8}$$

where t_a' and t_a are the observed temperatures in Fahrenheit degrees. If the instrument is calibrated at another temperature, the last term in the numerator of the fraction should be twice the value of the calibration temperature used.

3-31. Altimeters. Barometers which are designed specifically for surveying use are called altimeters. These instruments are graduated directly in feet or meters of altitude. They are sensitive in readings but rugged in construction and therefore well adapted for use in the field. Two common types are the American Paulin System and the Wallace and Tiernan.

3-32. The Paulin System. In the American Paulin System the pressure or weight of the atmosphere on a sensitive diaphragm is balanced against the tension in a spring supporting that diaphragm. In the Paulin System the movement of the pointer on the dial is not transmitted through gears to the movement of the diaphragm, but the pointer is moved by the observer as he balances the weight of the atmosphere against the tension on the supporting spring of the diaphragm.

Fig. 3-13 shows the face of a Paulin System altimeter. At the top of the dial is the balance indicator *A* which must be centered exactly over the balance mark *D* when taking a reading. A mirror is provided behind the balance indicator to eliminate the effect of parallax in taking readings. The instrument may be

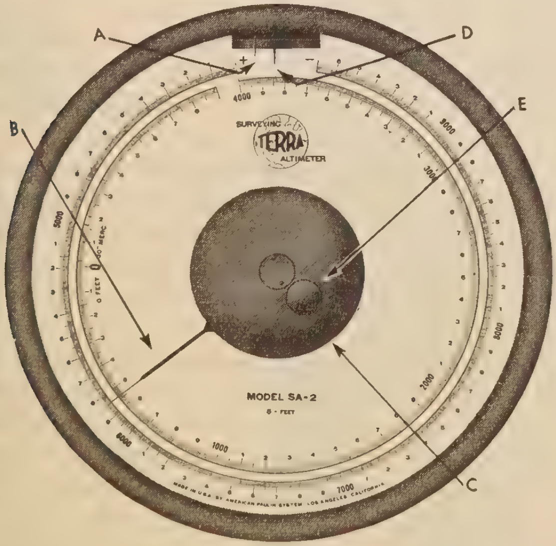


FIG. 3-13. FACE OF PAULIN SYSTEM ALTIMETER.
(Courtesy, American Paulin System.)

adjusted to read the actual elevation at a point of known elevation by rotating the pointer knob *C* until the balance indicator *A*, its image in the mirror and the balance mark *D* are all exactly in line. The knob *C* is then held firmly in place and the pointer *B* is moved to the desired reading by turning the reset control screw *E*.

If the instrument is taken to a higher elevation the balance indicator *A* will move towards the minus sign. At a lower elevation it will move towards the plus sign. The pointer *B*, however, does not change position with a change in altitude. To obtain the elevation in feet of a new position the instrument

must be re-balanced; i.e., D and A must be brought together by turning knob C which, in turn, moves the pointer B to the reading of the elevation at the new location. Since altimeters are usually employed to measure differences in elevations rather than to obtain actual elevations, it is not essential that the field readings correspond exactly to the true elevations at any point.

The altimeter shown in Fig. 3-13 is graduated in intervals of 2 feet and has a range of from minus 760 to plus 3600 feet. Other models read in intervals of 5 feet from minus 900 to plus 9700 feet, and in 10-foot intervals from minus 500 to plus 14,500 feet. Dials graduated in meters are also available.

The Paulin System has developed a portable *barograph* which provides a graphical record of pressure variations with time. Readings can be interpolated to the nearest foot. A recording thermometer supplies an accompanying record of temperatures. The barograph is well adapted for use at base stations where it eliminates the necessity for an observer.

3-33. Wallace and Tiernan Altimeter. This altimeter operates on the same principle as the barometer described in Art. 3-29. By improvements in design and construction a sensitive and reliable instrument has been developed which is much better adapted to surveying work than the older designs. The pressure element is an evacuated beryllium copper capsule. Deflections of the capsule are transmitted by a linkage to a geared sector which meshes with a pinion on the pointer shaft, mounted on a jeweled bearing. The pointer is read on a dial that is calibrated in feet or in meters. Any lag in the readings is prevented by a device which eliminates backlash between the sector and the pointer pinion. A desiccant is enclosed in the instrument to absorb moisture that might otherwise collect and alter the readings. The instrument is mounted in a shockproof metal case and provided with an outer case and strap for carrying it in the field. The manufacturer calibrates four instruments at the same time so that sets of 2, 3 or 4 " matched " altimeters may be purchased which will give identical readings under identical atmospheric conditions. This is an advantage when two or more altimeters are being used on the same job, such as in the single and two-base methods of altimetry (Arts. 3-36 and 3-38). An adjusting screw

is provided in the face of the dial so that the readings can be made to correspond exactly to bench mark elevations or to agree with other altimeters. A thermometer is supplied with each instrument and also an altimeter manual giving operating and maintenance information.

Two types of Wallace and Tiernan altimeters are available for barometric leveling: *precision* altimeters graduated in 5-foot intervals from minus 500 to plus 1500 feet, and in 1-meter intervals from minus 150 to plus 500 meters, and *sensitive* altimeters graduated in intervals of 10 feet from minus 1000 to plus 6000 feet or in 20-foot intervals from minus 1000 to plus 15,000 feet; also in meters by 2.5-meter intervals from minus 300 to plus 1800 meters and in 5-meter intervals from minus 300 to plus 4000 meters. The model shown in Fig. 3-14 is graduated in 10-foot intervals from zero to 3600 feet on the outside scale, and continuing from 3600 to 7000 feet on the inside scale. An indicator in the right-center of the dial shows which set of graduations should be read. Zero of the scale corresponds to 1000 feet below sea level at standard atmospheric conditions assumed by the manufacturer.

FIG. 3-14. WALLACE AND TIERNAN ALTIMETER.

The upper chart on the inside of the cover of the altimeter is for making temperature and humidity corrections. It is repro-

duced in Fig. 3-15. The curved lines bending upward from the horizontal represent relative humidities from 0 to 100 per cent. The lines slanting at 45° represent air temperatures from −40° F to +150° F. The vertical lines are correction factors to be applied to altitude differences. For example, if the air temperature is 90° F and the relative humidity is 40%, the chart is entered at the base with 90° and the sloping line corresponding to that temperature is followed until it intercepts the curved line representing 40% relative humidity. The correction (1.086) is read at this intersection by interpolating between the vertical lines. For an observed difference in altitude of 100 feet at 90° F and 40% relative humidity, the corrected difference is 100 × 1.086 = 108.6 feet.

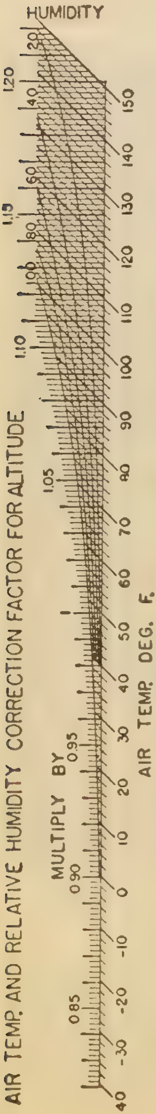


Fig. 3-15. Corrections to Altitude for Air Temperature and Relative Humidity.

The middle diagram on the inside of the cover of the altimeter is a conversion chart for changing feet of altitude into inches or millibars of mercury. The chart is based upon Smithsonian Meteorological Table 51, and applies at 10° C (50° F).

The lower chart on the cover is a temperature correction chart for the instrument. It gives corrections to be applied to altimeter readings for each degree of temperature above or below 75° F. These corrections are small, varying from zero to about 0.1 foot per degree F. They are usually neglected when only differences in elevation are required. They may become significant when absolute elevations are desired.

3-34. Measuring Difference in Elevation. Differences in elevation may be obtained in different ways depending upon the number of altimeters available. If only one is avail-

able it is carried with the observer and read at each station occupied. If two altimeters are used one is kept and read at a fixed station and the other taken to various observation stations. If three or more altimeters are available, one is left at a fixed station which is at a lower elevation than the points where elevations are desired and the other is placed at a fixed station higher than these points. The additional altimeters are taken to observation points between the two fixed stations. The use of one or two fixed base altimeters gives more accurate results than if only one is used.

3-35. Leveling with One Altimeter. If only one altimeter is used, it is taken to a starting point of known elevation and the altitude reading, time and the temperature recorded. The instrument is then taken to the various points where elevations are required and readings of altitude, time and temperature taken. The observer then returns to the starting point where the altimeter reading, time and temperature are again recorded. Due to changes in atmospheric conditions the first and last readings at the starting point will usually not agree and a correction must be applied to the intermediate readings. It is usually assumed that the change in the atmosphere has occurred uniformly with time, and the corrections are therefore computed in proportion to the elapsed time from the first observation. A closer determination of the nature of the change in atmospheric conditions during the survey may be obtained by taking two observations at each station, one upon arrival and another after waiting a short time. From the variations at the intermediate stations a curve may be plotted which will approximate that which would have been obtained if an altimeter had been left at the first station and read periodically during the survey. On this curve only the first and last points will be known, but the observations at the various stations will give the slope of the curve at different times during the survey. From this curve readings may be interpolated for the times of observation at the other stations.

The following tabulation illustrates the observations taken and the corrections necessary when only one altimeter is used. The survey starts at Station A with an altimeter reading of 1632

feet and returns to point A one hour later when the altimeter reads 1622 feet. The difference of 10 feet is the result of changes in atmospheric conditions during the survey. This discrepancy is distributed to the several altimeter readings in proportion to the elapsed time between each observation and the time of the first observation at A, assuming in this example that the change took place uniformly. For example, the correction at Station B is $\frac{8}{60} \times 10 = 1.3$ feet, which is added to the recorded reading of

DETERMINATION OF DIFFERENCES IN ELEVATION USING ONE ALTIMETER

Sta.	Altimeter Reading feet	Air Temperature °F	Time of Obser.	Correction for Atmo. Changes feet	Corrected Altimeter Reading	Diff. Altimeter Readings	Mean Air Temp. °F	Mean Air Temp. Corr.	Corrected Diff. in Elevation feet
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			A.M.						
A	1632	70	10:30	—	1632	+89	69	+3.4	+92
B	1720	68	10:38	+1.3	1721	+132	67.5	+4.6	+137
C	1850	67	10:50	+3.3	1853	−149	69	+5.7	−155
D	1698	71	11:04	+5.7	1704	−123	71.5	+5.3	−128
E	1574	72	11:14	+7.3	1581	+51	72.5	+2.3	+53
A	1622	73	11:30	+10.0	1632				

1720 feet in Col. (2) giving the corrected reading of 1721 feet in Col. (6).

The altimeter used in this illustration was calibrated at 50° F. For any other temperature a correction must be applied to obtain the true difference in elevation. The corrections may be found from graphs or tables supplied with the instrument, Fig. 3-15, or by the use of Formula (3-8) in Art. 3-30. The correction is computed using the mean of the observed temperatures at the two ends of the line.

A rough rule for temperature correction is that the altimeter reading should be increased 0.2 feet for each 100 feet of observed elevation for each degree of temperature above 50° F. If the

observed temperature is below 50° F the correction is subtracted. Between Station A and B the average temperature was 69° F, Col. (8), for which the correction to be added to the difference in altimeter reading in Col. (7) is 3.4 feet, giving a corrected difference in elevation of 92 feet in Col. (10).

In localities where high humidity occurs with high temperatures better accuracy may be obtained by recording wet and dry bulb thermometer readings and applying a correction to altimeter differences for relative humidity. If weather reports of humidity are available these may be used.

3-36. Single Base Method Using Two Altimeters. In this method one altimeter is left at a fixed station (single base) and is read frequently to determine the variations in the readings caused by changes in atmospheric pressure. It is assumed that the variation in pressure and therefore in altitude reading at other stations in the vicinity is the same as that observed at the fixed station. Readings of the altimeter at the fixed station are taken often enough, usually every 5 or 10 minutes, to determine accurately a curve of reading variations during the day, or for the interval of time covered by the survey. The watch time and temperature of the air are recorded with each altimeter reading.

The second (moving) altimeter is read at the beginning of the observations at the fixed station and its reading compared with that of the fixed base altimeter. Care should be taken to place both altimeters at the same level. The difference between the readings of the two altimeters is an *index correction* to be applied to the readings of the moving altimeter to reduce them to the readings of the fixed altimeter. The moving altimeter is then carried to the second station and the altitude, temperature, and time read again. Both the altimeter and the detached thermometer should be kept in the shade. As many other stations may be included as desired. If a long stay is made at any station the readings should be taken upon arriving and just before leaving. This will check the variation curve given by the stationary altimeter. It is well to make the time of traveling between stations as short as possible. The altitude, time, and temperature are read again at the first station upon the return. The change in altitude due to weather conditions may now be esti-

mated and allowed for. This is done by interpolating a reading of the stationary altimeter corresponding to the instant at which the moving altimeter was read at the distant station. The difference between this interpolated reading of the fixed altimeter and the corresponding reading of the moving altimeter while at the distant station, corrected for index error, is caused by the change in elevation. This difference must be corrected for the temperature of the air as described in Art. 3-35. The result is the difference in elevation of the two points. It is well to make a second observation at important stations on the return trip, thereby obtaining two independent determinations of the difference in height.

The following tabulation illustrates the method of computing differences in elevations when two altimeters are used. The fixed base altimeter No. 1 and the temperature were recorded every 5 minutes at A during the survey. From these readings, other readings were interpolated and are entered in Col. (2) for the exact times at which the moving altimeter No. 2 was read at Stations B, C, D and E. The difference between the interpolated altimeter readings at A and at each of the other stations give the uncorrected heights that these several stations are above or below the known elevation of A. Using the means of the temperatures interpolated at A and observed at the field sta-

Determination of Differences in Elevation Using Two Altimeters

Sta	Reading* Altimeter No. 1 feet	Air Temp. at A °F	Sta.	Reading Altimeter No. 2 feet	Air Temp. °F	Diff. in Altimeter Readings feet	Mean Air Temp. °F	Mean Air Temp. Corr.	Diff. in Elev from A feet
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A	1278	66	A	1278	66	0	66	0	0
A	1279	67	B	1350	64	+71	65.5	+2.2	+73
A	1280	68	C	1510	64	+230	66	+7.4	+237
A	1281	69	D	1470	66	+190	67.5	+6.6	+197
A	1280	67	E	1252	68	−28	67.5	+1.0	−29

* Interpolated readings.

tions, corrections are computed or taken from charts and entered in Col. (9). These are then applied to the altimeter reading differences giving the corrected differences in elevation above or below A (Col. (10)). In this example the readings of the two altimeters were made to coincide at Sta. A. If this is not practical, the difference in readings of the two altimeters at A is recorded and this index correction applied to all readings of altimeter No. 2.

3-37. Leapfrog Method. A modification of the use of two altimeters is the *leapfrog method.** Two altimeters are read simultaneously at a base station. Altimeter No. 1 is left at the base station while altimeter No. 2 is taken to the first field station. The two altimeters are read simultaneously and the difference in readings after corrections gives the elevation of the first field station. Altimeter No. 1 is then taken from the original base station to the second field station, thus leapfrogging altimeter No. 2 which remains at the first field station. The altimeters are again read simultaneously. The difference in readings after correction gives the elevation of the second field station with respect to the first field station. The altimeters are then brought together at the second field station and read. After comparison, altimeter No. 2 is taken to the third field station for simultaneous reading with altimeter No. 1 and then altimeter No. 1 is advanced to the fourth field station for simultaneous reading, after which altimeter No. 2 is brought to the fourth field station for comparison, and so on. The survey may be speeded by employing additional altimeters and by comparing altimeters at every third or fourth station instead of at alternate stations.

The advantage of the leapfrog method is that the altimeters are always close together and therefore are affected by atmospheric conditions which are more nearly the same than in the single-base method where the moving altimeter is often taken a considerable distance away from the fixed base. In the leapfrog method the base station is, in effect, carried along with the survey.

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* See "Altimetry — Its Present-Day Techniques" by Lester E. Demler, the Tenth Anniversary Meeting of the American Congress on Surveying and Mapping, June 1950.

3-38. Two-Base Method of Altimetry. In this method at least three altimeters are used; one at a lower base of known elevation, one at a higher base of known elevation and one or more roving altimeters which are read at various field stations where elevations are desired between the upper and lower base elevations. The altimeters at the base stations are read simultaneously at frequent intervals of time so that continuous records of readings at these points are available for comparison with readings at the field stations at which the time of observation is also recorded.

From the altimeter readings at a given interval of time the differences in elevation are obtained between the upper and lower base stations and between the lower (or upper) base station and the field station. The difference in elevation between the lower and upper base stations will not agree with the known difference in the elevation of these stations because no adjustments have been made for temperature or humidity of the air. However, the elevation of the field station can be obtained quite closely from the uncorrected altimeter readings by assuming that true differences in elevation are proportional to differences observed with the altimeters. In Fig. 3-16, for example, the pre-

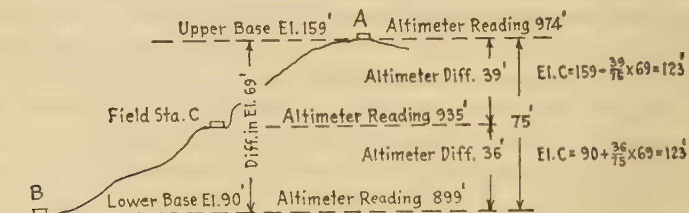

FIG. 3-16. TWO-BASE METHOD OF ALTIMETRY.

determined difference in elevation between upper base A and lower base B is 69 feet. The difference indicated by the uncorrected altimeter readings is 75 feet and the observed difference between the field station and the lower base is 36 feet. On the basis of the altimeter readings the elevation of Field Station C is $\frac{36}{75}$ of the total difference. Applying this fraction to the true difference of 69 feet gives $\frac{36}{75} \times 69 = 33$ feet as the corrected

difference between the field station and the lower base. The elevation of Field Station C is, therefore, 90 + 33 = 123 ft. The same method could have been used to determine the elevation of the field station from the upper base. One of the principal advantages of the two-base method is that the necessity for applying temperature and humidity corrections is eliminated.

The following tabulation illustrates a form of notes conveniently arranged for computing field station elevations. The notes correspond to the data in Fig. 3-16.

ALTIMETER COMPUTATION SHEET FOR TWO BASES

Upper Base Sta. (U.B.)	A	El. 159 ft.
Lower Base Sta. (L.B.)	B	El. 90
Difference of Elevation		69 ft.
Date: July 27, 1952		

Field Sta. (F.S.) / Time	Key to Computation	C / 2:04 P.M.	D / —	E / —
Reading F.S.	(1)	935		
Reading L.B.	(2)	899		
F.S. − L.B.	(3) = (1) − (2)	36		
Reading U.B.	(4)	974		
Reading L.B.	(5)	899		
U.B. − L.B.	(6) = (4) − (5)	75		
Diff. El. $\times \frac{(3)}{(6)}$	(7)	33		
El. L.B.	(8)	90		
El. F.S.	(9) = (7) + (8)	123		

For more precise results, *multiple base altimetry* may be employed,* in which several (usually four) base altimeters are placed in a quadrilateral pattern encompassing the area to be surveyed. The base altimeters are read at frequent intervals or recorded automatically on alticorders. From these corrected readings an equal pressure reference plane is established. Elevation differences are determined by comparing field readings with interpolated elevations on the equal pressure plane at the same horizontal positions.

* "Multiple Base Altimetry," W. F. Haring and A. H. Mears, Photogrammetric Engineering, December 1954.

3-39. Precautions in Use of Altimeter. For best results the altimeter survey should be made on days when the weather is stable and the barometer pressure is not subject to rapid variations due to changes in climatic conditions. Windy days with bright blue sky and detached clouds traveling rapidly overhead are to be avoided because the alternate patches of bright sunlight and shade cast over the ground cause variations in readings. Days with overcast sky, gentle winds and steady barometer pressures are best suited to barometric surveys. The best times for observations are two to four hours after sunrise or two to four hours before sunset. Midday readings should be avoided. The instrument should be shaded and protected from sudden jarring while being carried between stations.

On gusty days observations should not be made in the lea of an automobile or building; low pressure pockets will be found in such places which make the readings unreliable. The instruments should always be placed in the same position for reading, the horizontal position being most common. If possible base stations should not be located over 12 miles apart or differ by more than 250 feet in elevation.

PROBLEMS

1. Determine the value of C from the following observation:

Number of Station	Thread Reading, Near Rod	Thread Reading, Far Rod	
	1773	1800	
A	1755	1589	The stadia constant for this instrument is 1 in 330.
	1737	1379	
	1329	1703	
B	1318	1488	
	1306	1273	

2. From the following set of level notes run from B.M. A6 to B.M. 42 compute the difference of elevation:

(*a*) through using the mean of the thread readings.

(*b*) through using the difference of the sums of the thread readings.

Number of Station	Thread Reading, Backsight	Thread Reading, Foresight
1	1890 1654 1418	1870 1629 1388
2	1968 1731 1495	1844 1602 1360
3	1680 1444 1208	1652 1411 1170
4	1670 1438 1206	1691 1451 1211
5	1583 1345 1107	1851 1610 1369

The stadia constant for this instrument is 1 in 330.

3. At station C the observed zenith distance of station K is $90° 04' 21''.6$; telescope above station, 9.575^m; object above station, 1.92^m; difference of heights, $+7.655^m$. At station K, observed zenith distance to C is $90° 05' 57''.5$; telescope above station, 2.125^m; object above station, 9.37^m; difference of heights, -7.245^m. Log distance CK from triangulation, 4.29977. Elevation of C, 298.80^m. Azimuth C to K = $18° 30'$. Mean latitude of C and K = $34° 00'$. Compute the elevation of K from C and the value of m.

4. From the following observations taken with one altimeter determine the corrected differences in elevation.

Sta.	Altimeter Reading (feet)	Time of Observation	Air Temperature (°F)
		P.M.	
A	2572	2:15	65
B	2250	2:48	68
C	2480	3:20	66
D	2732	3:50	64
A	2580	4:30	60

5. Referring to Fig. C-1, p. 528, draw a similar sketch but with the following data substituted.

Distances: AB = 3 miles, BD = 5 miles, BE = 7 miles, EG = 7 miles, ED = 4 miles, DG = 6 miles, DC = 4 miles, GF = 5 miles, FC = 3 miles, and CA = 3 miles.

The following level circuits were run with elevation of point C known:

1. C = 360.25, A = 315.62, B = 280.42, D = 270.84, C = 360.45
2. C = 360.25, F = 290.80, G = 300.20, D = 270.68
3. D = 270.84, E = 285.60, B = 280.22
4. G = 300.20, E = 285.55

Find adjusted elevations for points A, B, D, E, F, and G.

6. From the following observations taken with two altimeters determine the corrected differences in elevation between the Base Station A and the other observation stations.

	Altimeter No. 1				Altimeter No. 2		
Sta.	Reading (feet)	Time	Air Temp. (°F)	Sta.	Reading (feet)	Time	Air Temp. (°F)
		A.M.				A.M.	
A	840	9:10	64	A	842	9:10	64
A	841	9:20	64	B	880	9:18	65
A	843	9:30	65	C	924	9:25	65
A	844	9:40	65	D	880	9:36	66
A	846	9:50	66	E	820	9:45	68
A	843	10:00	66	F	800	9:57	68
A	844	10:10	67	A	846	10:10	67

7. From the following observations taken with three altimeters determine the elevations above sea level of Field Stations (1), (2) and (3). The altimeter at the Lower Base Station is at elevation 1321, and the altimeter at the Upper Base Station is at elevation 1406; both referred to mean sea level.

Base Altimeter Readings			Roving Altimeter		
Time	Lower Base (feet)	Upper Base (feet)	Sta.	Time	Reading (feet)
A.M.				A.M.	
10:00	1820	1915	(1)	10:07	1872
10:05	1821	1917	(2)	10:18	1850
10:10	1822	1918	(3)	10:25	1836
10:15	1824	1921			
10:20	1823	1923			
10:25	1823	1925			
10:30	1821	1920			

CHAPTER 4

AERIAL PHOTOGRAMMETRY

4-1. Aerial Photogrammetry. Photogrammetric surveying is the technique of making maps from photographs of the terrain. In recent years this technique has almost entirely superseded ground surveying as the basic process for making maps of large areas. Although requiring much more expensive equipment and more personnel with a variety of skills, photogrammetry enables large areas to be mapped at lower cost per unit area than by conventional surveying, and in less time. The dividing line between the two is generally determined by the time available for the work, and the relative costs of meeting the requirements for the particular project.

The process of taking measurements from photographs is sometimes called *metric photogrammetry*, as distinguished from *photo-interpretation* which is the use of photographs to obtain qualitative information.

Photogrammetric surveys enable maps to be made of inaccessible terrain, and the infinite detail of a photograph makes it possible to make many kinds of maps in the comfort of an office and with precision instruments that cannot be used in the field. This does not eliminate the need for ground surveying, however, even on work done by photogrammetric techniques. Ground surveys are needed for control, for mapping areas obscured from the air by dense vegetation and areas where persistent bad weather prevents aerial photography, and for field identification of specific features.

Photogrammetric surveying consists of two major types, terrestrial and aerial. In terrestrial or *ground* photogrammetry, the camera is mounted on a tripod and the photographs are primarily in a horizontal direction. For aerial photogrammetry the photographs are taken vertically or nearly so.

Terrestrial photogrammetry is rarely used at this time except in a few special cases, such as where aerial photography is not possible because of steep-walled canyons or overhanging cliffs. Ground photogrammetry also has limited use in mapping small areas too costly to photograph by air, but where the user feels that it is justified by the inherent superiority of photographs over plane-table sketching for faithful representation of detail. Because of its low cost it has appeal to small engineering firms and as a training device for photogrammetry students. Since the basic principles of making an orthometric projection (a map) from perspective projections (photographs) are the same in both ground and aerial photography, this text will deal only with aerial photography.

4-2. Aerial Photography. The airplane is the most suitable of all aircraft to use when taking aerial photographs. It provides a convenient means for supporting the camera and photographer, and may be flown at approximately any desired altitude and azimuth.

Air photographs may be either *vertical*, *composite* or *oblique*. In taking vertical photographs, the optical axis of the camera is held vertical or nearly so, and the plate (or film) is horizontal.

Vertical views look like a map although the scales of the individual photographs are far from uniform. All photographs taken with the optical axis vertical are called " verticals "; they are really perspective views of the terrain produced on horizontal planes.

Composite photographs are taken with a multi-lens camera. One of the photographs usually is vertical and the others are inclined to the vertical; some multi-lens cameras have no vertical chamber. The inclined (or oblique) views of the composite set are for some purposes rectified (transformed into the plane of the vertical photograph) so that all the prints when used for plotting appear as if taken vertically.

In the *oblique* photographs the camera axis is sufficiently inclined to the vertical so that usually the horizon is shown (the film is always at right angles to the axis of the camera). Those obliques which include the horizon are called *high obliques* and

those that do not are called *low obliques*. Oblique views are more nearly like the ordinary pictures we are accustomed to see; therefore they give about the same appearance as terrestrial photographs. Maps are made directly from these oblique photographs by methods later explained.

Cameras used for taking vertical photographs are usually mounted in gimbals and sighted through a hole in the floor of the plane. The axes of the gimbals are supported on vibration absorbing mounts fastened securely to the structure of the airplane. By means of an attached level, the camera axis may be held vertical or nearly so at the instant of exposure. When side winds tend to blow the plane off its course, the plane is headed into the wind to counteract this effect. The angle between the longitudinal axis of the plane and the actual course is called the "angle of crab." In order that successive pictures will be directly in front of each other the camera must be rotated about its vertical axis counter to and in an amount equal to the angle of crab. If this is not done one picture will be offset on the next and they will appear when placed together like a flight of stairs.

All photographs are true perspectives of the ground viewed from the camera station. Vertical photographs appear more like a map than do obliques.

In taking aerial photographs, the plane is flown back and forth along straight parallel courses spaced at such a distance apart that pictures, taken at short intervals, will completely cover the area. (See Fig. 4-8.) If a map is to be prepared, a sufficient overlap between successive photographs is procured in the direction of flight to obtain stereoscopic coverage of the entire area. (See Art. 5-1.) This overlap is usually 60%.

For the production of maps from photographs a certain amount of ground surveying is necessary. The positions of several readily identified points, known as *ground control points*, must be determined horizontally and sometimes vertically, and these points must appear clearly on the photographs.

Aerial photographs may be taken for a number of purposes, as follows:

(1) to secure single vertical or oblique photographs for visual study;
(2) to cover an area with vertical prints which are not assembled but identified by an index map (Fig. 4-28);
(3) to obtain rough mosaics assembled from contact prints without correcting the scale of the prints or adjusting them to ground control;
(4) to obtain precise mosaics constructed of carefully matched portions of prints which have been enlarged or reduced as necessary to fit ground control (Fig. 4-29);
(5) to produce a planimetric map (Fig. 4-34);
(6) to produce a topographic map (Fig. 4-29).

The mapping of a region from photographs includes:

(1) the flying, which includes the navigation, or piloting necessary to fly to the place to be mapped and to cover the territory in the manner best suited for mapping purposes;
(2) exposing and developing the film and making the prints;
(3) determining the positions of the ground control points by ordinary ground surveying methods;
(4) constructing the map from the aerial photographs around the accurate plot of the ground control points; and
(5) additional ground work required to obtain necessary data not furnished by the photographs.

The surveyor is chiefly concerned here with the third, fourth and fifth of these operations.

4-3. Cameras.* Cameras for use in aerial photogrammetry are constructed to conform to rigid specifications. The material, construction and workmanship should be of the highest grade in order to obtain satisfactory photographs for use in plotting. Although film is used almost exclusively, some cameras are designed to use glass plates.

Since the terrain is usually photographed from altitudes varying from 1,000 to 50,000 feet, aerial cameras are of the fixed focus

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* For description of aerial cameras and elements of photogrammetric optics, see Manual of Photogrammetry, Second Ed., 1952, Chaps. II and III.

type. Filters are used to reduce the effect of haze and to improve the contrast.

Cameras for taking *verticals* are mounted on gimbals and provided with a level for leveling the focal plane. A stop watch is sometimes attached to the camera for timing the interval between exposures. By means of a tripping device the watch hands return to the zero setting the instant an exposure is made and automatically starts timing the next interval. Usually a separate vertical viewfinder accompanies all vertical cameras. As cameras used in taking oblique photographs are often pointed at different angles with the vertical, they are provided with view finders attached to the camera.

Some cameras are operated manually; others are automatic or semi-automatic. In automatic operation the camera is motor-driven, the power being supplied by a storage battery. An instrument called a "view-finder intervalometer," when set for overlap and regulated for speed of the plane, determines the interval between exposures, and operates the camera automatically. The photographer, however, has to keep the camera axis vertical and checks the ground speed through the viewfinder.

Specifications for aerial photography usually include statements defining the characteristics of the camera, lens, shutter, etc., that are to be used. The objective is to obtain clear, distortion-free photographs that will be best suited to the purpose of the survey. The essential parts of the camera are the lens, the shutter and the focal plane.

4-4. Lens. The most important features of the lens which influence the quality of picture produced are flatness of field, distortion, angle of coverage and speed. In the construction of the lens certain desirable features are sacrificed or compromised in order to obtain the sharpest picture possible in the plane of the negative. For example, the field of sharpest focus of a Metrogon lens (Art. 4-49) is like a concentric wave, low in the center and at the corners and high in between. When the flat film receives an image from this lens the definition of the image will only be very sharp along those concentric circles where the plane of the picture cuts the undulating field of sharpest focus (Fig. 4-1).

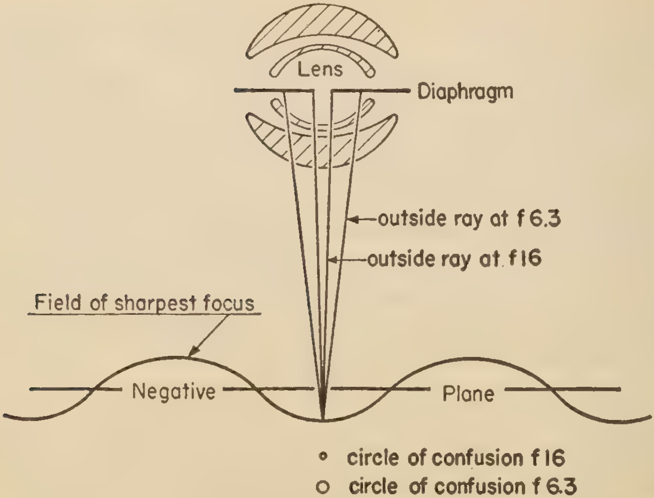

FIG. 4-1. EFFECT OF STOPPING DOWN A LENS.

Thus the definition of the picture is not uniform throughout the photograph.

The definition of a lens is evaluated mathematically by a resolving power test in which the *resolving power* is recorded as the maximum number of lines per millimeter that can be resolved (i.e., seen as separate lines) in the image taken at each 5 degrees of angle from the axis of the lens.

Nearly all lenses distort the image; i.e., certain areas of the picture are expanded or contracted in scale radially from the center of the picture. This often has the effect of picturing the sides of a square object as convex or concave instead of as straight lines.

Angle of coverage is the angle subtended at the camera lens by the width of the terrain covered in the photograph. In cameras used for aerial photography the angle will vary from 5° for some long-focus military cameras to 90° in the wide-angle Metrogon lens camera.

The speed of the lens refers to the amount of light which it admits and its ability to expose a negative under adverse lighting conditions or with an extremely fast shutter speed. In fast lenses

the diameter of the opening is large in relation to the focal length.

The *efficiency of a lens* is the amount of light which reaches the negative divided by the amount of light incident upon the lens. As a rough rule, 5% of light is lost at each air to glass surface. A lens that has four elements which are not cemented together would, therefore, lose about 40% of the light which it receives. This loss can be reduced by coating the surfaces of the lenses.

At the corners of the picture the light tends to fade out. The Metrogon lens, for example, delivers only about 5 per cent as much light to the corners as to the middle of the picture. To partly overcome this effect the filter is made denser in the middle portion which tends to equalize the illumination of the lens. Loss of light through the lens often results in poor image quality and low accuracy in the map.

By freeing a photograph of distortion the accuracy of the plotting is increased when using the simpler types of stereoscopic plotting machines or when certain graphical methods are employed. Frequently the stereoscopic plotting instruments have distorting optical systems which are designed to match the distortion of a particular type of lens (such as the Metrogon) and in this way eliminate the effect of the distortion in plotting.

Newer cameras, made by Wild of Switzerland and Zeiss of Germany, have lenses that are distortion-free for all practical purposes. Improved vignetting techniques have made it possible to admit more light all over the negative, thus improving resolution.

Any lens that is to be used in a camera for photogrammetric purposes should be tested by a qualified agency, such as the National Bureau of Standards. The characteristics of the lens usually requested for evaluation are distortion, resolving power and equivalent and calibrated focal lengths.

In cameras used in photogrammetric surveys the focal length does not have a definite mathematical value. This is so because moving the lens one way from the focal plane may increase the sharpness in the center and in the corners of the picture, whereas moving the lens in the opposite direction will sharpen the definition in the intermediate zone at the expense of that in the center

and corners. Thus the focal length setting is a matter of judgment on the part of the camera maker or tester.

The *back focal length* is the distance measured along the lens axis from the rear vertex of the lens to the plane of best average definition. The *equivalent focal length* is the distance so measured from the rear nodal point of the lens to the plane accepted as giving the best average definition over the entire field used in the aerial camera. The *calibrated focal length* is an adjusted value of the equivalent focal length so computed as to distribute the effect of lens distortion over the entire field; i.e., the maximum distortion in one direction is made equal to the maximum distortion in the opposite direction.

In general, the sharpest definition of objects on the photographs is obtained by stopping down the lens to the smallest possible opening that will photograph the subject clearly with the type of film emulsion and filter being used. The advantage of stopping down the lens is illustrated in Fig. 4-1. Since each point of the image is formed by a cone of rays with its base at the aperture of the lens, and since the undulating focal plane truncates this cone forming a circle of confusion, it will be seen that the smaller the base of the cone, the smaller will be the circle of confusion and the sharper the definition.

4-5. Shutters. Two types of shutters are commonly used in cameras: the between-the-lens shutter and the focal plane shutter. The between-the-lens shutter consists of leaves which open and close and which are placed between the front and rear elements of the lens. This type of shutter is essential in a camera to be used in a moving vehicle, such as an airplane, since it exposes all parts of the negative simultaneously (Fig. 4-2).

The focal plane shutter (Fig. 4-3) consists of a curtain with a slit in it situated immediately in front of the focal plane. In making an exposure the slit moves rapidly across the focal plane. Thus each increment of the negative is exposed at a different instant of time. If a camera with a focal plane shutter is used in an airplane, the picture will be distorted because the airplane is not in the same position over the ground when the first and last increments of the film are exposed.

The focal plane shutter is capable of faster shutter speeds than with the between-the-lens type and is more efficient in the transmission of light. It is often advantageous for photography to be used for visual purposes, but the pictures are not of sufficient sharpness for use in photogrammetric mapping. Focal plane shutters usually permit less expensive camera construction than do between-the-lens shutters.

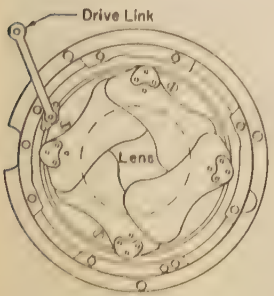

Fig. 4-2. Between the lens Shutter Partially Open.

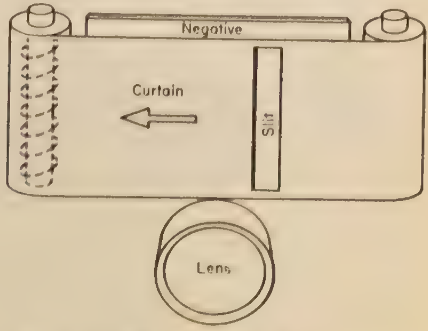

Fig. 4-3. Focal Plane Shutter.

4-6. Focal Plane. The negative film in a photogrammetric camera must be held flat in the focal plane. This is usually accomplished by applying a vacuum to the plate and thus pressing the film to it. The maximum variation from a true plane should not exceed 0.0005 inch. Collimating or fiducial marks placed in the focal plane are photographed on the negative as notches or other types of markings which may be used to locate the principal point of the photograph, and to detect stretch or shrinkage of the film. The film should be non-shrink topographic base to minimize such errors. The photographic emulsion commonly used is supersensitive panchromatic which is sensitive to all colors of the visible spectrum. The film comes from the manufacturers wound on spools similar to the film used in ordinary hand cameras.

4-7. Single-Lens Cameras. Fig. 4-4 illustrates the frame and cone of a single-lens camera that may be operated either manually or electrically. It has a between-the-lens shutter with speeds of from $\frac{1}{50}$ to $\frac{1}{300}$ second. The film used is 150 feet long by $9\frac{1}{2}$ inches wide producing 9×9-inch negatives. Different lengths of cone can be attached to the body of the camera giving focal lengths of 6, 12 or 24 inches.

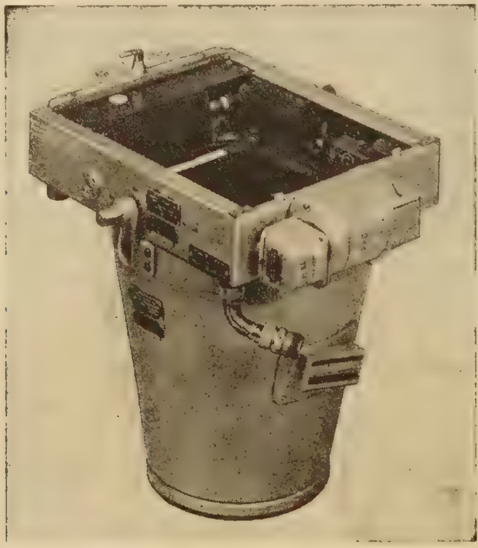

FIG. 4-4. SINGLE-LENS K-17 CAMERA (TOP REMOVED).
(Courtesy, Fairchild Camera and Instrument Corporation.)

A recording camera designed especially for aerial photogrammetric purposes, sometimes known as the "Cartographic Camera," is shown in Fig. 4-5. It is equipped with a 6-in. focal length, F6.3 Metrogon lens, and has a one-piece inner cone which includes both the lens mount and the focal plane frame. Shutter speeds range from $\frac{1}{10}$ to $\frac{1}{500}$ second. Fiducial marks, film shrinkage reference marks, altimeter, clock, exposure counter, data card, the lens serial number and the calibrated focal length of the lens are recorded on each negative. The film magazine provides for

FIG. 4-5. RECORDING CAMERA T-11.
(Courtesy, Fairchild Camera and Instrument Corp.)

a film capacity of 390 feet. Accessories shown in Fig. 4-5 include intervalometer (front center), vertical view finder (right center) and electrical connections.

Fig. 4-6 shows a fully automatic camera of European design. The negative size is 9 inches × 9 inches. Focal length is 3.46 inches. The lens of distortion-free design provides a high degree of resolving power over the entire field of the lens. The short focal length and large picture format permit extremely wide angle coverage at a small scale; it is therefore very suitable for photography where the objective is to cover as much terrain as possible from a single-lens photograph suitable for use in a stereoscopic plotting instrument.

In the RC-9 camera shown in Fig. 4-6, the eyepiece on the right is the viewfinder, the lever under the handle in the foreground permits changes in diaphragm opening. The cameraman

levels the camera in flight by means of the three hand screws, according to the bulls-eye bubble just in front of the viewfinder. Electrical connector to the intervalometer is at left rear. In

FIG. 4-6. WILD RC-9 SUPER-WIDE ANGLE CAMERA.
(Courtesy, Wild Heerbrugg Instruments, Inc.)

operation the film, shown across the bottom of the hinged magazine, is held flat by a vacuum system operating through perforations in the pressure plate behind the film. A portion of the azimuth ring for crab correction can be seen at right rear.

It is often desirable to take two aerial photographs simultaneously, each at an angle of 20° up from the vertical, on either side of the flight line (see Art. 4-11). While this can be achieved

with two separate cameras of the conventional vertical kind, rigidly related to each other in a fixed mount, efficiency of manufacture, installation and operation can be achieved by using a single camera assembly especially designed for the purpose.

4-8. Sonne Continuous Strip Aerial Camera. In this camera the film is drawn over a narrow slit in the focal plane at a velocity equal to that of the image as it crosses the focal plane. The resulting strip photograph gives a continuous picture of the ground. The aircraft is flown at very low altitude giving sharp prints to large scale, such as are particularly useful in highway condition surveys and for military reconnaissance. Dual strips may be exposed at constant angular separation producing pictures which may be viewed stereoscopically.

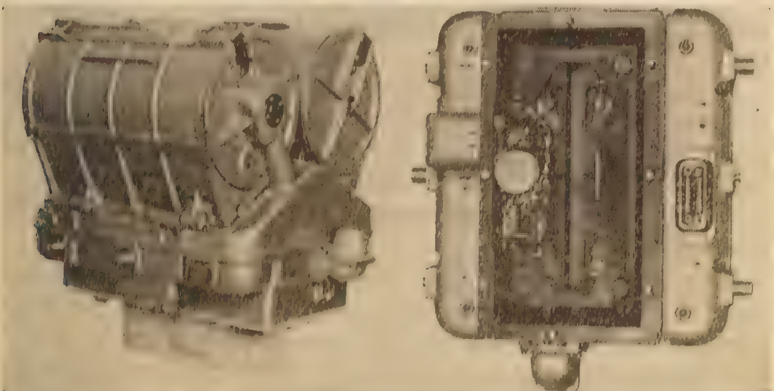

FIG. 4-7. SONNE CONTINUOUS STRIP CAMERA (LEFT)
VIEW OF SLIT MECHANISM FROM BELOW (RIGHT).
(Courtesy, Chicago Aerial Industries.)

4-9. Choice of Camera. In general, the characteristics required of a camera for stereoscopic mapping (Art. 5-1) are the opposite of those required for producing pictures for either visual inspection or for the construction of mosaics. For topographic mapping by stereoscopic methods a short focal length and wide angle camera is preferred because such a camera magnifies parallax (Art. 5-2), and it is from the measure of parallax that topographic maps are plotted. For example, a camera with a 6-inch

focal length is often used when topographic maps are desired, and one with a 12-inch focal length for most other uses where a minimum of relief displacement is desirable.

4-10. Viewfinder. The vertical viewfinder is the device used by the cameraman to determine the required time interval between exposures to give proper overlap. Essentially it is a fixed focus camera with a rectangular grid etched on a ground glass, in the focal plane of the lens. The cameraman uses a stop watch to time the passage of the image of a ground object between two specific transverse grid lines. This becomes the proper interval between exposures which he can then set on the intervalometer, an electrical device for triggering the camera at any desired time interval. Since the correct interval for a desired overlap will vary with changing ground speed and terrain clearance, the cameraman must continually check this interval throughout the flight.

In more advanced types of viewfinder the grid lines move along the ground glass by means of being drawn on a motor-driven endless belt of clear plastic; the cameraman adjusts the speed of the belt to make the image remain on a single transverse line as belt and image both move along. This automatically sets the intervalometer.

When there is a cross wind, the aircraft must be turned somewhat into the wind in order to stay on a desired course; the angle that the longitudinal axis of the aircraft makes with the actual direction of motion of the aircraft under these conditions is known as the angle of crab or drift angle. Whenever there is crab the ground object will not move along one of the longitudinal grid lines on the viewfinder but at an angle to it. By rotating the entire viewfinder until this crab effect is eliminated and by noting the angle of rotation, the cameraman determines the proper angle by which to rotate the camera itself. In some types of camera, rotation of the viewfinder will automatically rotate the camera through servo-motors. Elimination of crab makes the edges of the square photograph line up in the direction of flight.

4-11. The Photographic Mission. When a photographic mission is planned, the available maps of the region are studied in order to determine the most economical method of carrying on

the flight from the base of operations. A flight map is prepared on which are plotted the project boundaries and the flight lines, proper allowance being made for side lap (Fig. 4-8). An adequate flight map is a necessary aid to the pilot in keeping on his course by identifying ground features on the map.

The standard topographic maps, Art. 9-18, provide a basis for excellent flight maps. When good maps cannot be obtained the best map available, even if only a sketch, is used. It is sometimes necessary to fly an area for which no map exists. The pilot must then choose his flight lines and chart his course from his memory of identifiable points on the ground.

Before any exposures are made, a trial flight (dry run) is made over one or more flight lines in order to determine the angle of crab and the time interval between exposures. In general, the procedure is as follows. When the altimeter indicates that the predetermined altitude has been reached to give pictures of the desired scale the pilot brings the plane over a flight line and proceeds to fly over and along it on even keel. The cameraman corrects his camera for crab and sets the proper exposure interval on the intervalometer.

When this has been accomplished the photographer signals the pilot by flashing a signal light or by voice. The pilot now flies to a point about three exposures beyond the beginning of the survey, brings the plane "about" and when at the proper altitude and on course over the flight line, he signals the photographer to start making exposures. The same procedure is then duplicated in the opposite direction.

4-12. Flight Map. Fig. 4-8 shows a flight map covering an area of 15 minutes in latitude and longitude. The distance between flight lines was computed from the following data:

Type of camera	Single lens
Focal length of lens	$8\frac{1}{4}$ inches
Scale of photographs	$\frac{1}{20000}$
End lap	60%
Side lap	30%
Size of negative	9 × 9 inches
Altitude (See Art. 4-16)	13,900 ft.

FIG. 4-8 FLIGHT MAP. FLIGHT LINES ARE SHOWN AS DASH LINES.

It will be observed that seven flight lines are necessary to cover the area indicated. (See Art. 4-17.)

The pilot carries the flight map and by observing physical terrain features lying on or near the flight line shown in the map, he keeps the plane closely on the flight line.

4-13. Overlap. Where vertical photographs are to be used in the preparation of maps all methods of compilation require that the plumb points of the preceding and succeeding print be visible

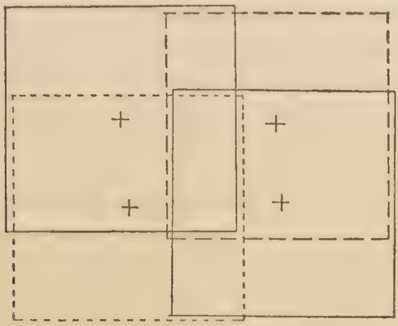

Fig. 4-9. Overlap, Showing 60% End Lap and 30% Side Lap.

in each photograph. (See Art. 4-23.) Pictures are therefore often taken with overlap of 60% in the direction of flight; this is sometimes called *forward overlap* and *end lap*. When photographs are taken with this overlap the entire area may be examined stereoscopically. This last feature is of great importance in interpreting the detail of the photographs.

In order that there may be no hiatus between adjacent strips in case flights deviate from the planned courses and to afford ties between adjacent strips in the mapping processes, photographs are taken with *side overlap* (usually simply termed *side lap*). The amount of side lap is usually 30% but not infrequently side laps of 60% are specified. (Fig. 4-9.) The amount of side lap will actually depend upon the quality of the flight map, the ability of the crew to navigate the aircraft (a good pilot will usually be able to hold a line within 15% of the width of the picture), the method of map compilation and the nature of the relief.

As end lap and side lap are increased, more individual photographs are required to cover a given area; this increases the amount of work both in the field and office.

The following six reasons may be given for providing for overlap:

1. In order to tie the different prints together accurately, it is desirable that the center of each photograph should appear on the edges of as many adjacent prints as possible.

2. The distortion caused by the lens, the distortion caused by tilt of the camera and the displacements caused by variations in elevation of terrestrial objects are all liable to be more pronounced in the outer parts than near the centers of photographs. In constructing maps these distortions can be overcome quite effectively by having the photographs overlap more than 50 per cent.

3. For viewing pairs of photographs stereoscopically only the overlapped portions can be used, so that at least a 50 per cent overlap should be allowed.

4. Since each portion of the territory is photographed at least two times and sometimes as many as nine times, an occasional defective exposure may be rejected and there still remain enough pictures to give all the details in this area. Thus a picture distorted by excessive tilt, or rendered useless by poor lighting, or by cloud shadows, may be thrown out without the necessity of taking a new picture or re-projecting the old one. In assembling mosaics, outer edges of prints can be discarded and only the "net areas" retained, thus eliminating parts containing major distortions.

5. The side lap is important in order to avoid a hiatus between adjacent strips in case the flights are not maintained straight and parallel.

6. An incidental benefit of overlap is the opportunity afforded of viewing objects from more than one angle, especially in stereoscopic examination. Sometimes an object will stand out clearly in one picture, but can scarcely be recognized in an adjacent one.

4-14. Relation between Scale of Print and Altitude of Camera. Strictly speaking, there is no such thing as the scale of a print because the ground usually has varying elevations; it is only

when the ground is perfectly level that the aerial print could have a definite and uniform scale. However, any elevation above sea-level may be selected as a datum and if the height is known at which a photograph was taken the scale of the print may be computed for the datum chosen.

There are two commonly used methods of expressing the scale of air photographs: (1) By the *Representative Fraction* (R. F.) and (2) By the *Scale Factor*. The R. F. is the usual fractional scale in which the numerator is unity; it is the ratio of a distance on the map to the corresponding distance on the ground and is expressed here as s. The scale factor is the usual expression of "so may feet to an inch," in which an inch on the map represents the specified number of feet on the ground and is expressed here as S.

If a distance D on the ground (datum) is represented by a distance d on the photograph, then the scale expressed as a fraction is $\frac{d}{D} = s$. From similar triangles (Fig. 4-10) it is seen that

$$\frac{d}{D} = \frac{f}{H} = s \tag{4-1}$$

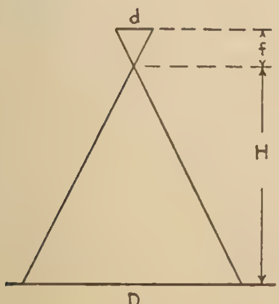

FIG. 4-10.

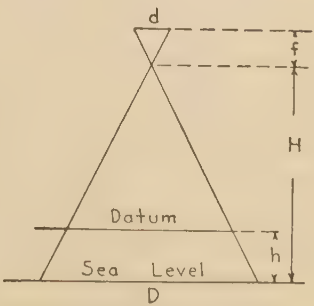

FIG. 4-11.

where f = the focal length of the lens and H is the elevation of the nodal point of the lens above the datum plane both expressed in the same units.

Another form of this equation is as follows (Fig. 4-11): If H is

the elevation of the camera and h is the elevation of the datum, then

$$s = \frac{f}{H - h} \tag{4-2}$$

From this relation we may find the altitude required to give a photograph the desired scale s, when the focal length of the lens is known.

The following relations between scale and altitude are developed from the above.

$$s = \frac{\text{focal length of the lens (inches)}}{\text{height of plane above datum (inches)}}$$

$$S = \frac{\text{height of plane above datum (feet)}}{\text{focal length of lens (inches)}}$$

$$H(\text{feet}) = \frac{f(\text{feet})}{s} = f(\text{inches}) \times S$$

4-15. Effect on the Scale of Variation in Height of Camera. If the height of the airplane above the datum were 100 feet and the focal length were 10 inches, the scale of the print would be 10 feet per inch. For a height of 200 feet, the scale would be 20 feet per inch, and for 300 feet, it would be 30 feet per inch. That is, a change of elevation of 100 feet always produces a change of scale of 10 feet per inch. For $f = 6$ inches, the variation in scale is $16\frac{2}{3}$ feet per inch for each change of 100 feet in the altitude.

4-16. Altitude of Plane. When preparing for a photogrammetric mission the pilot must be informed of the altitude at which the pictures are to be taken. Since the scale of the prints depends upon the average elevation of the plane above the ground, the flying altitude must be increased by the average height of the terrain, in order that the pilot may fly according to his altimeter which refers to mean sea level.

Using the specifications given in Art. 4-12 and assuming that the average altitude of the ground is 150 feet, the flying altitude is found as follows:

$$s = \frac{f}{H - h}$$

$$H = h + \frac{f}{s}$$

$$= 150 + \frac{8.25}{12}(20{,}000)$$

$$= 150 + 13{,}750 = 13{,}900 \text{ ft.}$$

4-17. Computing the Distance between the Flight Lines. The distance between flight lines may be computed as follows: Using the data given in Art. 4-12, the ground distance in miles corresponding to the width of the negative (9 inches) is

$$\frac{9 \times 20{,}000}{12 \times 5{,}280} = 2.84 \text{ miles}$$

If the side lap is to be 30%, then the distance between lines is

$$2.84 \times 70\% = 1.99 \text{ miles}$$

If the lines run north and south and the width of area to be photographed is 12.3 miles, the number of flight lines will be determined as follows:

Assume that the photographic coverage is required to extend 25% of the width of a picture beyond the boundaries of the survey on each side. Then 50% of the width of one picture will be allowed to cover this overlap. The number of flight lines required is therefore equal to the width of the area to be surveyed minus 50% of the width covered by one picture all divided by the distance between flight lines, plus one flight line, or

$$\frac{12.3 - 0.50\left(\dfrac{9 \times 20{,}000}{12 \times 5{,}280}\right)}{1.99} + 1 = 5.47 + 1 = 7 \text{ flight lines}$$

It will be observed in this case (Fig. 4-8) that the seven flight lines are spaced symmetrically with respect to the center meridian of the map.

4-18. Number of Photographs Necessary to Cover a Given Area. The number of exposures necessary to cover any known area may be estimated when the dimensions of the print, the scale of the picture, the amount of end lap and the number of flight lines are specified. The number of pictures equals the length of a flight line divided by the net advance of each picture along the flight line, multiplied by the number of flight lines plus a sufficient number to cover the start and finish of a line, usually taken as two exposures at each end. Using the specifications given in Art. 4-12, the pictures have an end lap of 60 per cent. The net advance will then be 40 per cent of the 9-inch dimension, Fig. 4-12. This will then be 3.6 inches advance, or about 6070 feet in ground distance. Since the distance along a flight line between the north and south boundaries of the area to be mapped (Fig. 4-8) is 15.5 miles, the theoretical number of prints per flight line is

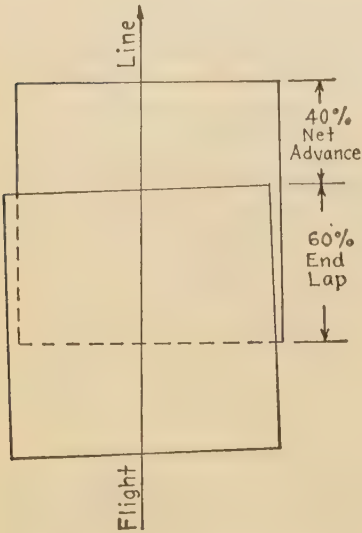

FIG. 4-12. OVERLAPPING PHOTOGRAPHS.

$$\frac{(15.5)(5280)}{6070} = 13.5, \text{ say } 14$$

Increasing this figure by 4 to allow for two extra prints at each end of the flight line, the total number of exposures per flight line is 18. Since there are seven flight lines, the total estimated number of exposures is 126. This figure is only approximate, and indicates the number of pictures which will ultimately be processed. The number of pictures actually exposed will usually run from 15 to 100% more. Some of this extra photog-

raphy will be due to reflights which might be required because of inexperience of the crew or poor quality of the flight maps; some will be due to irregular terrain or shape of area to be mapped.

With an end lap of 60 per cent and a side lap of 30 per cent, the net new coverage of any one photograph is 40 per cent of 70 per cent, or 28 per cent of the total area of the photograph. Thus a photograph covering 10 square miles contains only 2.8 square miles not already covered on other photographs.

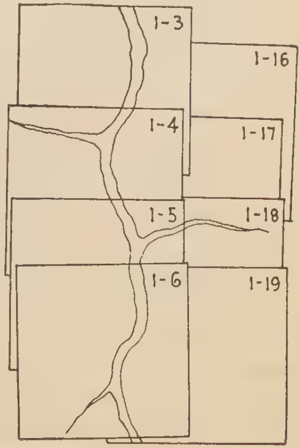

FIG. 4-13. PORTION OF INDEX MAP.

4-19. Index Map. After the film has been developed all the negatives are numbered consecutively in the order in which they were taken and an *index map*, Fig. 4-13, is prepared showing the relative positions of all the pictures taken during the flight. For this purpose it is convenient to use a templet. The templet is made by cutting an opening, to scale, in a piece of transparent celluloid corresponding to the area covered by each print. Where the plotting is done on a map the templet is oriented by comparing the data shown on the negative with those on the map. Where no map is available the ground control points are plotted to some convenient scale. The prints are fitted together roughly making what corresponds to one large photograph (mosaic). Then the position and orientation of the individual photographs are estimated and plotted by means of the templet on the index map sheet. It is not necessary, however, to prepare a line drawing of the index map since it can be obtained directly by photographing the assembled and numbered prints as illustrated in Fig. 4-28.

4-20. Errors and Distortions in Aerial Photographs. The errors and distortions in obtaining suitable photographs for mapping may be divided into two general groups:

(1) Errors of the camera, and distortion of film and print materials.

(2) Distortions arising from the conditions outside of the camera.

The errors to be considered in the first group are:

(*a*) camera construction and adjustment
(*b*) lens distortion
(*c*) film shrinkage
(*d*) paper shrinkage

Under the second group we have:

(*a*) changes of scale caused by the inability to maintain the plane at a constant altitude
(*b*) changes of scale caused by relief (hills and valleys)
(*c*) displacement of ground objects caused by relief
(*d*) displacement of objects caused by tilt.

4-21. Errors and Distortions in Cameras, Film and Print Paper. Cameras and lenses must be constructed to precise and rigid specifications, to minimize errors due to these sources (see Art. 4-3). Any distortions still remaining are generally known or can be carefully measured for any individual camera, and in many cases can be compensated by suitable lenses in precision optical devices which use the photographs (see Art. 5-14). The National Bureau of Standards will calibrate a lens and camera upon request, tabulating the amount of lens distortion and giving the precise focal length of the lens, a figure which is constantly used in various photogrammetric computations.

Since changes or distortions in film or printing paper used for mapping, due to temperature changes, handling, developing, and drying, must be kept at a minimum, only materials of the best quality should be used. Film known as supersensitive panchromatic topographic base is now obtainable; any changes in this film resulting from the above causes are usually within the limits allowable in plotting. Printing paper will always change, but, if the shrinkage is uniform, the effect of this change can be corrected; it is only when the change is not uniform that serious

errors are likely to remain in the resulting map. Waterproof printing paper may be obtained in which the shrinkage is practically uniform. For very precise work, however, any changes in the film or printing paper may be determined by comparing the distances between the reference marks on the film or paper with the distances between the corresponding marks on the camera. Where very accurate measurements are to be made the prints may be made on glass plates.

4-22. Scale Errors. When taking air photographs for mapping, the plane is frequently flown at a precomputed altitude in order that the scale of the photographs will be suitable for the scale of the map. The altitude is indicated on an altimeter, an instrument somewhat similar to an aneroid barometer. As altimeters are not precise instruments and because of meteorological conditions there will be variations in altitude which, of course, change the scale of the map. (See Art. 3-28.)

Another cause affecting the scale is the varying elevation of the ground. The scale near the summit of a hill would be larger than the scale near its base.

If a day is selected when the barometer is changing slowly and the wind is fairly uniform in direction and intensity, the flying altitude ought not to vary more than 5% from that specified.

4-23. The Vertical. The point on the ground vertically beneath the lens at the instant of exposure is called the *ground plumb point* or *nadir point* (sometimes called *vertical point*). Its image in the picture is called the *plate plumb point* or *plate nadir.* The principal point, which is the intersection of the optical axis with the plane of the photograph, is taken as the intersection of the diagonals of the picture, or the intersection of lines between the reference marks on its margins. When the optical axis is vertical the plate, or film, is horizontal and the principal point will then coincide with the plumb point. If the plate is tilted, which is usually the case, these points will not coincide. A point approximately midway between the principal point and the plumb point on the picture is called the *isocenter* (or *center of distortion*). It is located at the intersection of three planes; that of the tilted photograph, that of the horizontal photograph that would have been obtained had the camera axis been maintained

vertical at the instant of exposure, and that of a vertical plane containing the optical axis of the camera. It may be defined as that point lying on the line connecting the principal point and the plumb point which is at a distance from the former equal to the focal length of the photograph multiplied by the tangent of half the angle of tilt. (See Figs. 4-18 and 4-19.)

The position of the plumb point and isocenter could be located on the print provided the amount and direction of its tilt were known. This may be determined if the position and elevation of three or more ground control points appearing in the print are known. For methods of determining tilt see Art. 4-27.

Attempts have been made to determine tilt by photographing on each negative at the instant of exposure the image of the position of two level bubbles placed at right angles to each other. Experience has shown that these have not proved entirely satisfactory. For a discussion on tilt see Art. 4-26.

4-24. Displacement Caused by Relief. It is only when the ground is perfectly level that the truly vertical photograph will represent a projection in which all points are correctly located with respect to scale and direction. As the condition seldom, if ever, exists it is evident from a study of Fig. 4-14 that, because

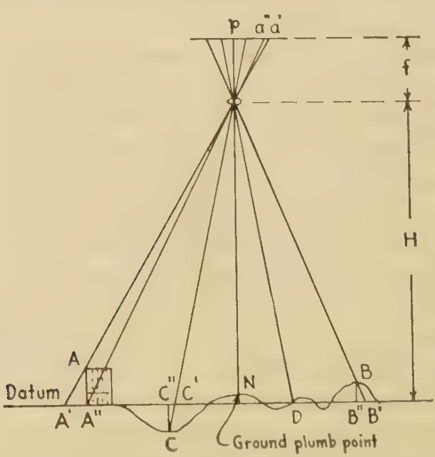

Fig. 4-14. Displacement Caused by Relief.

of perspective projection, any object that does not lie in the datum will be displaced. If the object photographed is above the datum plane, its position on the print will be farther out from the center than its true position on an orthographic projection. For example, the top corner of the building A (Fig. 4-14) will be shown on the datum at A' instead of A'' and on the print at a' instead of at a''. Since this is a displacement in a vertical plane the error on the print is **radial from the plate plumb point,** which in the case shown coincides with the principal point. This error is very noticeable on tall buildings near the margin of a picture, the buildings all appearing to be tipped away from the central part of the picture. All hills above the datum are similarly displaced outward. (B, Fig. 4-14.) A contour near the top of a hill above the datum will therefore show two errors: first, the contour is nearer the camera than is the datum, so it is shown on too large a scale; second, the whole contour is displaced outward because it is above the datum plane. Fig. 4-15 shows an orthographic projection of two contours and a road. The scales in this drawing are correct. Fig. 4-16 shows a photograph of the same hill from a point to the right of S. The contour CD is shown on too large a scale and is displaced to the left. The roadway near the summit is also displaced.

All points below the datum (valleys) are displaced inward, as at C, Fig. 4-14. Points in the datum are not displaced. The plate plumb point, which coincides with the principal point in a truly vertical print, is not displaced whether above, below or

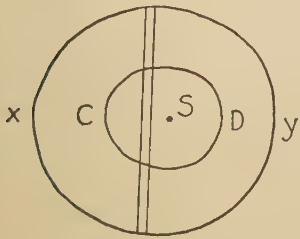

Fig. 4-15. Orthographic Projection of Hill and Roadway.

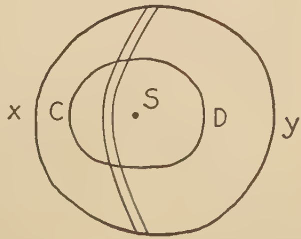

Fig. 4-16. Photograph of Same Hill Showing Scale Error and Lateral Displacement.

in the datum. It should be observed that the magnitude of displacement decreases with an increase of altitude at which the photograph is taken and increases the further the point is out radially from the plate plumb point.

4-25. Calculation of Relief Displacement. In Fig. 4-17 the camera station is O at an elevation H above the datum. N is the ground plumb point, or nadir. The summit of the hill A at

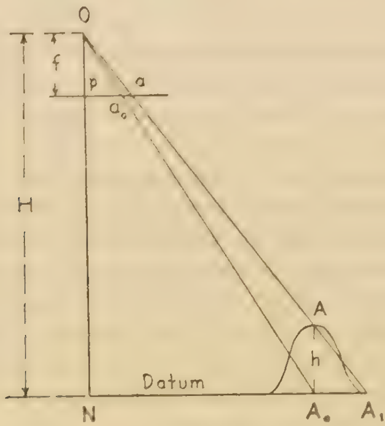

FIG. 4-17. RELIEF DISPLACEMENT.

an elevation h above the datum will appear on the datum at A_1 and on the print at a. A_0 is the orthographic projection of A on the datum. The displacement of A on the print is equal to a_0a and may be found as follows.

By proportion

$$\frac{h}{H} = \frac{A_0A_1}{NA_1}$$

but

$$\frac{A_0A_1}{NA_1} = \frac{a_0a}{pa}$$

Therefore

$$\frac{h}{H} = \frac{a_0a}{pa}$$

or

$$a_0a = pa\frac{h}{H}$$

Letting $a_0a = d$, displacement of image due to relief
$pa = r$, radial distance of image from principal point

then, the general relation for displacement due to relief is

$$d = \frac{rh}{H} \tag{4-3}$$

EXAMPLE. Assume the flight altitude $H = 10{,}000$ feet, the elevation h of a hill $A = 300$ feet. If the radial distance, r, is 3.084 inches, what is the displacement due to relief?

$$d = 3.084 \frac{300}{10{,}000} = 0.092 \text{ inches}$$

4-26. Displacement Caused by Tilt. Tilt is defined as the inclination of the optical axis to the vertical at the instant of exposure. As yet no practical means have been found for keeping the camera axis truly vertical when taking photographs from an airplane. The effect of tilt can best be studied by noting what occurs when a photograph is taken of a rectangle on the datum plane, the camera being vertically above the center of the rectangle. If the camera axis is vertical, the image will be rectangular and the scale will be uniform. If the axis is tilted the image will be trapezoidal; all points on one side of the axis of tilt will be displaced outward while those on the other side will be displaced inward. In Figs. 4-18 and 4-19 all points to the right of the axis of tilt are displaced outward; those to the left, inward. The scale on the outer edge will be too large while the scale on the inner edge will be too small.

In Fig. 4-18, O is the position of the nodal point of the lens; $ABCD$ represents a rectangle on the print, the distance On' being $= f$, the focal length of the lens. Point n is the photograph nadir, and p is the principal point of the tilted picture. The angle t is the angle of tilt, or angle between the true vertical On and the camera axis Op. iE is the tilt axis, or line of intersection of the horizontal print $ABCD$ and tilted print $A'B'C'D'$; the print is true to scale along this line only. The point i is called the isocenter and for small angles of tilt may be assumed to be midway between the points p and n. If M is a point midway

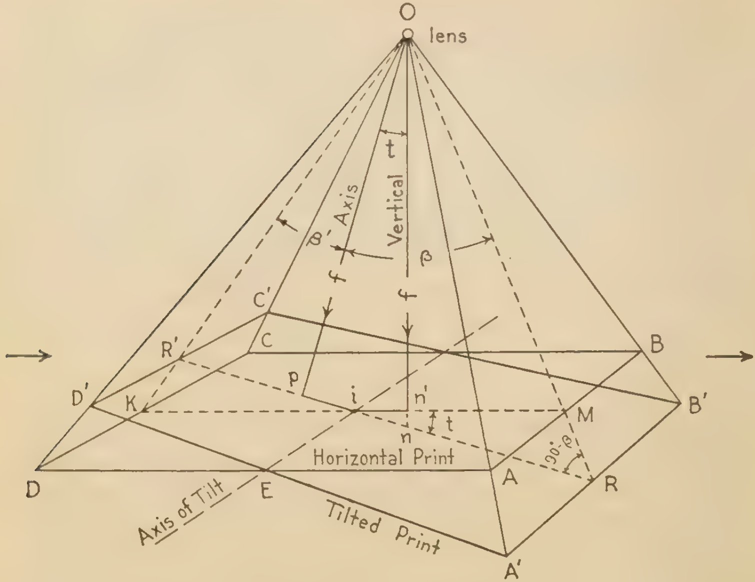

FIG. 4-18. EFFECT OF TILT.

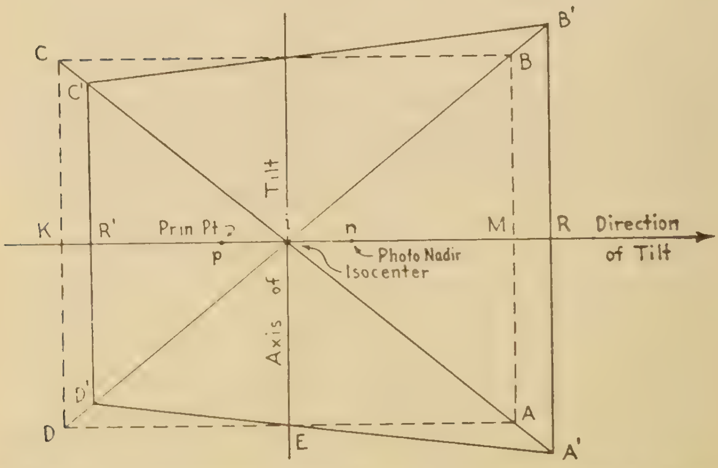

FIG. 4-19. PHOTOGRAPH OF HORIZONTAL RECTANGLE WHEN CAMERA IS TILTED.

between A and B, its image on the tilted print is R, midway between A' and B'. Let the angle $pOM = \beta$.

The relative positions of M and R may be found as follows:

$$iM = f \tan (\beta - t) + f \tan \frac{t}{2}$$

$$iR = f \tan \beta - f \tan \frac{t}{2}$$

The displacement of R (outward) on the print $= iR - iM =$

$$f \tan \beta - f \tan (\beta - t) - 2f \tan \frac{t}{2} \qquad (4\text{-}4)$$

Similarly the inward displacement of K (at R') is

$$f \tan (t + \beta') - f \tan \beta' - 2f \tan \frac{t}{2} \qquad (4\text{-}5)$$

If the point whose displacement is desired is not on the axis iR but is at some image point as A' (Fig. 4-18) its displacement may be computed as explained later. Before this is done, however, it will be shown that the displacement of all points in the tilted photograph are radial from the isocenter.

Fig. 4-20 is a section of Fig. 4-18 through the lens O and the line KiM. Point Q is the intersection of a horizontal line through the lens O and the line Ri produced.

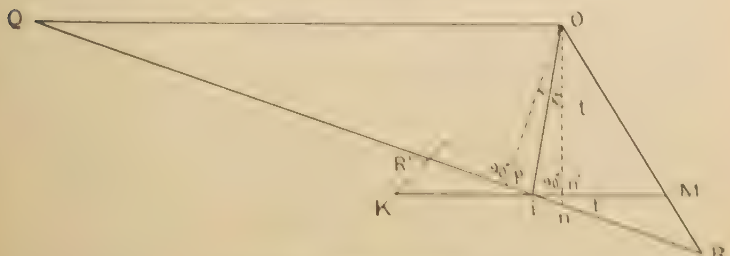

Fig. 4-20. Section of Fig. 4-18.

From Figs. 4-18 and 4-19, and observing that in isosceles triangle QOi, $Qi = QO$

$$\frac{RA'}{MA} = \frac{OR}{OM} = \frac{QR}{Qi} = \frac{QR}{QO} = \frac{iR}{iM}$$

or
$$\frac{RA'}{iR} = \frac{MA}{iM}$$

But
$$\tan RiA' = \frac{RA'}{iR} \quad \text{and} \quad \tan MiA = \frac{MA}{iM}$$

Therefore angle RiA' = angle MiA.

Hence it is seen that the angle RiA' in the tilted photograph between the principal axis Ri and the line iA' joining the isocenter with the image point, is equal to the angle MiA in the vertical photograph between the principal axis Mi and the line iA joining the isocenter with the image point, from which it follows, that the error in the position of a point caused by tilt is radial from the isocenter i.

If we place the angles MiA and $RiA' = \theta$, then

$$iA = \frac{iM}{\cos\theta} = \frac{f\tan(\beta - t) + f\tan\dfrac{t}{2}}{\cos\theta}$$

$$iA' = \frac{iR}{\cos\theta} = \frac{f\tan\beta - f\tan\dfrac{t}{2}}{\cos\theta}$$

Hence
$$iA' - iA = \frac{f\tan\beta - f\tan(\beta - t) - 2f\tan\dfrac{t}{2}}{\cos\theta} \tag{4-6}$$

that is, the outward displacement to any image point may be computed by formula (4-6).

In a similar manner the inward displacement may be computed by

$$\frac{f \tan (t + \beta') - f \tan \beta' - 2f \tan \frac{t}{2}}{\cos \theta} \tag{4-7}$$

EXAMPLE:

$$f = 8.25''; \beta = 30°; t = 3°; \theta = 0°$$

then the outward displacement along the principal axis =

$$8.25 \tan 30° - 8.25 \tan 27° - 2(8.25) \tan 1°.5 = 0.127''$$

which is the actual distance on the picture that the image is from its correct position.

For displacement where $\theta = 20°$

$$\frac{0.127}{\cos 20°} = \frac{0.127}{.93969} = 0.135''$$

4-27. Approximate Method for Determining Tilt.* The approximate tilt of a photograph may be determined directly from measurements made between identifiable points in adjacent photographs. In this method two components of tilt are determined separately and then combined. y-tilt is defined as tilt about an axis perpendicular to the line of flight, and x-tilt is tilt about the flight line axis.

The following determination of the y-tilt assumes that the elevation of the airplane is constant between two exposures, which is normally near enough to the case not to introduce any appreciable error in the solution. In Fig. 4-21, photographs I and II are consecutive pictures each containing images of ground points A, B, C and D. A and C are selected so that their images a_1 and c_1 in photograph I lie in or very near a perpendicular to the line of flight passing through the principal point of the photograph p_1. Similarly, B and D are selected so that their images b_2 and d_2

* For other methods of determining tilt see "Manual of Photogrammetry," American Society of Photogrammetry, Second Edition, pp. 336–345, and "Topographic Manual," Part II, Photogrammetry, Special Publication No. 249, U. S. Coast and Geodetic Survey, pp. 164–7 and 189–192.

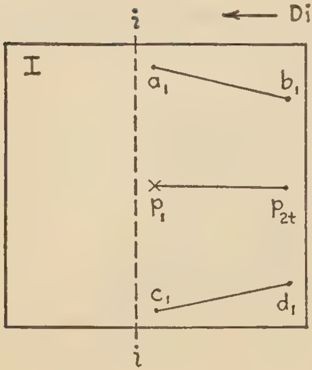

FIG. 4-21. IMAGES IN TILTED PHOTOGRAPHS.

are very nearly on a perpendicular to the line of flight through the principal point of photograph II. The dashed line *ii* in photograph I represents the true axis of y-tilt for this picture, which, for small angles of tilt, will lie parallel to and very close to line $a_1p_1c_1$. In this approximate solution it is assumed that the line $a_1p_1c_1$ coincides with *ii*. The length of a_1c_1 as it appears in picture I will be very nearly correct since it lies in the y-tilt axis where the scale is unaffected by y-tilt. If the picture has x-tilt, the overall distance a_1c_1 will still be substantially correct because the distances p_1a_1 and p_1c_1 were selected to be about of equal length so that the outward displacement on one side of the principal point will about balance the inward displacement on the opposite side of the principal point.

On photograph II the distance a_2c_2 is obviously less than the correct distance a_1c_1. Assume that a_1c_1 measures 7.0 inches from picture I and that a_2c_2 measures 6.9 inches and p_2p_{1t} measures 3.5 inches from photograph II. In Fig. 4-22 it is desired to locate

FIG. 4-22. DETERMINATION OF y-TILT FROM PHOTOGRAPH IMAGES.

point Q which is at the intersection of the tilted plane of photograph II about the y axis and a horizontal plane passing through the rear node of the camera lens at O. This point will be where the lines b_2a_2 and d_2c_2 intersect. The above measurements indicate that in a distance of 3.5 inches (p_2 to p_{1t}) these lines have converged 0.1 inch. Therefore, by proportion,

$$\frac{0.1}{7.0} = \frac{3.5}{p_2Q} \quad \text{or} \quad p_2Q = \frac{3.5 \times 7.0}{0.1} = 245.0 \text{ inches}$$

If the lens has a focal length of $8\frac{1}{4}$ inches, then

$$\frac{245.0}{8.25} = \text{cotan of the angle of tilt} = 29.7$$

and the y-tilt of photograph II $= 1° \; 56'$.

Similarly, if $b_2d_2 - b_1d_1$ measures 0.3 inch, $p_1p_{2t} = 3.6$ inches and $b_2d_2 = 6.8$ inches, then

$$\frac{0.3}{6.8} = \frac{3.6}{p_1Q}, \; p_1Q = 82 \text{ inches, cotan } t = \frac{82}{8.25} = 9.94,$$

and y-tilt in photograph I $= 5° \; 45'$.

From the example above for photograph II it will be seen that a convergence of 0.05 inch will result in a y-tilt of about 1°. With careful scaling and proper choice of points, y-tilt may be determined to about a $\frac{1}{4}°$ on 9×9-inch pictures taken with a lens of 8.25-inch focal length.

The determination of x-tilt by the method to be described below is more approximate than that outlined for determining y-tilt. The x-tilt is determined by studying the pattern of matched contact prints along a flight line. It may normally be assumed that in most pictures there will be very little tilt present, and that a line run through the average positions of the centers of the pictures, disregarding those which are obviously off-line, will coincide very closely with the mean center line of flight. Any picture which is substantially out of line, as is photograph No. 2

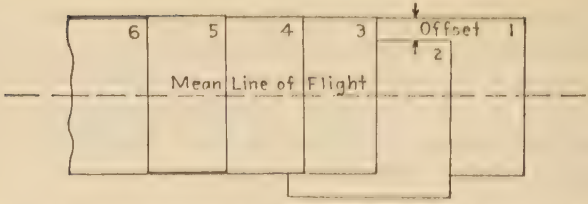

FIG. 4-23. DETERMINATION OF x TILT FROM OFFSET PHOTOGRAPHS.

in Fig. 4-23, is evidence that this picture is tilted. The offset of the tilted photograph from the mean line of flight is an approximate measure of the tangent of the angle of x-tilt. It is more practical to make the comparison of offsets with the mean line along the edges of the picture using the collimation marks as a guide. The offset of photograph No. 2 can readily be measured from the mean line of the edges of the pictures in the strip. Thus, if the offset is 1.0 inch, and the picture was taken with a camera with 8.25-inch focal length, the approximate tangent of the angle of tilt will be $\frac{1}{8.25} = .121$, giving an x-tilt of $6° 55'$. In order to obtain the best results by the foregoing method, the camera levels must be kept in adjustment. If the bubble in the x direction is out of adjustment by, say, 2° of angle, the determination of tilt by the above method will be with reference to a false datum at 2° slope instead of to the horizontal.

Having determined the x-tilt and y-tilt of the photograph the nadir may be located approximately by dropping a perpendicular to the mean flight line from the principal point, and then laying off the x ordinate of y-tilt ($f \tan t_y$) from this perpendicular along the flight line towards the large-scale side of the photograph. (See Fig. 4-24.) The isocenter is plotted midway between the principal point and the nadir,

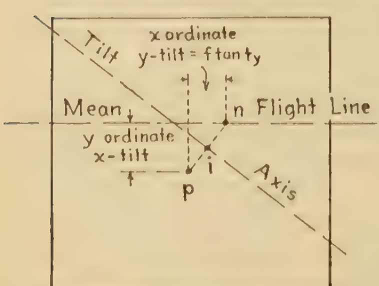

FIG. 4-24. LOCATION OF NADIR AND ISOCENTER ON TILTED PHOTOGRAPH.

which is very nearly correct for small angles of tilt. The axis of tilt of the photograph will pass through i and will be perpendicular to pin.

4-28. Combined Effect of Relief and Tilt. It was shown in Art. 4-24 that the displacement of a point caused by relief is radial from the plumb point, or nadir, and in Art. 4-26 that the displacement of a point caused by tilt is radial from the isocenter. This is illustrated in Fig. 4-25 which represents a plan view of

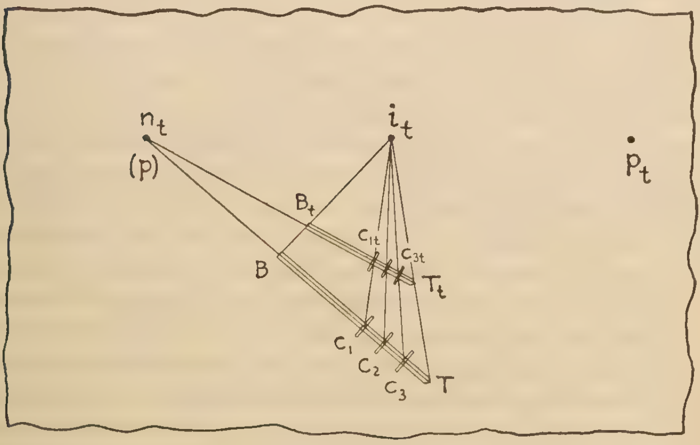

FIG. 4-25. EFFECT OF RELIEF AND TILT DISPLACEMENTS.

the superimposed images of objects traced from a tilted and an untilted photograph, each taken from the same point in space. The nadir of the tilted picture n_t and the principal point of the untilted picture p are assumed to coincide. The isocenter of the tilted picture is at i_t and the principal point of the tilted picture is at p_t. The image of a telegraph pole is shown as BT in the untilted photograph, and as B_tT_t in the tilted photograph. Displacements due to relief, caused by the height of the pole in this example, lie in the radial from the nadir in both the vertical and tilted pictures. Points C_1, C_2 and C_3 are images of cross trees of the pole in the untilted picture and C_{1t}, C_{2t} and C_{3t} are these images in the tilted photograph. For any given elevation,

such as represented by the images of the cross tree C_3 and C_{3t}, the relationship between the untilted and tilted images is defined by an isocenter radial — C_3C_{3t} in this case. Thus in a photograph of flat, horizontal ground of datum elevation the image in a tilted print and its position in a horizontal print both lie in the isocenter radial. If, however, the ground is at some elevation other than datum two steps must be taken in recovering the corrected position of an image. First a distance is set off along the nadir radial representing the height which the image point in question is above or below datum. Secondly from the position so determined a distance is set off on the isocenter radial corresponding to the tilt displacement of the image.

Referring to Fig. 4-25, the images of the top of the pole T and T_t are in the same datum. Cross tree C_3 is displaced a distance TC_3 in the untilted photograph due to its lower elevation. In the tilted photograph the image of C_3 is at C_{3t} on the isocenter radial, and its relation to the top of the pole has changed to T_tC_{3t}. In this example the correction for tilt has decreased the error due to relief. In other parts of the photograph the error due to tilt may tend to increase the outward relief displacement.

In Fig. 4-25 the degree of tilt and the separation of points p, i and n are exaggerated for illustrative purposes. Under good flying conditions the tilt seldom exceeds 3° and usually averages less than 1°. With values of tilt no greater than these and when the relief is not excessive the error is very small if both relief and tilt displacements are assumed radial from the principal point.* Such an assumption is commonly made in the radial control method of plotting maps (Art. 4-38).

4-29. Determining Height of Camera from Known Points in Vertical Photographs. If the length of a line appearing in a vertical photograph is known on the ground and the elevation of its two ends are known, the elevation of the camera may be found at the instant when the exposure was made.

Fig. 4-26 represents a plan view of a photograph taken from camera station O in Fig. 4-27. Points a, and b on the photograph

* See "Manual of Photogrammetry," American Society of Photogrammetry, Second Edition, pp. 415-416, for method of determining errors introduced by assuming that both tilt and relief displacements are radial from the principal point.

are the images of points A and B on the ground. The distance between A and B on the ground is D, which appears in the photograph as d_p. The elevation of A is h_a and of B it is h_b. In Fig. 4-26 the point a is where the image of A would be if A and B had been at the same level. Since A is higher than B, its image at a_i is displaced outward from a.

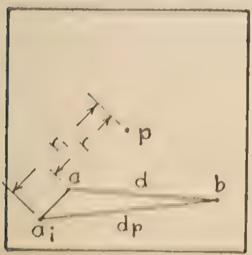

FIG. 4-26.

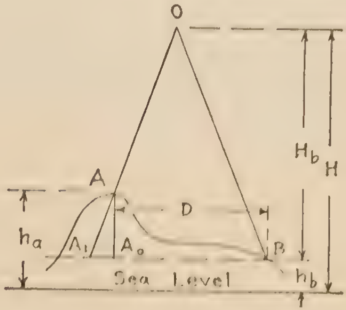

FIG. 4-27.

From Art. 4-14 we find that when there is no tilt a distance on the photograph is to the same distance on the ground as the focal length of the camera is to the height of the camera station above the ground where the line was measured. From Fig. 4-26

$$\frac{d}{D} = \frac{f}{H_b} \tag{4-8}$$

In this relation only D and f are known, but an approximate or preliminary value for the height of camera H_p can be obtained from d_p as follows:

$$H_p = \frac{fD}{d_p} \tag{4-9}$$

From Art. 4-25, the displacement of an image due to relief is to the distance of the image from the principal point as the height of the object above the datum is to the height of the camera station above the same datum. From Figs. 4-26 and 4-27,

$$\frac{r_1 - r}{r_1} = \frac{h_a - h_b}{H_b}$$

The distance r_1 may be scaled from the print and H_p may be used as a preliminary value for H_b, then

$$r_1 - r = \left(\frac{h_a - h_b}{H_p}\right) r_1 \qquad (4\text{-}10)$$

The displacement $(r_1 - r)$ is subtracted from r_1 giving the location of point a on the print. The distance $ab = d$ is then scaled from the print and a direct solution made for H_b in terms of d, D and f. $H_b + h_b = H$, the height of the camera above sea level.

EXAMPLE.

$h_a = 300$ ft.	$r_1 = 4$ in.
$h_b = 200$ ft.	$f = 6$ in.
$D = 4000$ ft.	$d_p = 5$ in.

Find elevation of camera station.

$$H_p = \frac{6 \times 4000}{5} = 4800 \text{ ft.}$$

$$r_1 - r = 4 \times \frac{300 - 200}{4800} = .083 \text{ in.}$$

$$r = 4.00 - .08 = 3.92 \text{ in.}$$

a is located on the print by laying off r from the principal point. The distance d is scaled from the print and found to be 4.8 inches. Then

$$H_b = \frac{6 \times 4000}{4.8} = 5000 \text{ ft.}$$

Elevation of Camera Station = 5000 + 200 = 5200 ft.

If the difference in elevation between A and B does not exceed 5 per cent of H, the above approximate solution will give results that are very nearly correct.

4-30. Ground Control. Determination of tip and tilt of a photograph is possible if at least three non-colinear points in the photograph have had their positions and elevations established.

Ground control is the term applied to those points identifiable in the photographs whose horizontal positions and elevations referred to a selected datum are known. Their locations and elevations are determined by ordinary ground methods of surveying. The amount of ground control may consist of many or few points according to the accuracy required and the mapping method to be employed.

In deciding how the control points should be located it should be observed that a minimum of three points known in position are necessary to fix independently the position of a vertical photograph from which a planimetric map is to be compiled. Where stereoscopic processes are to be used, at least two points known in position and elevation and a third point known in elevation are desirable in the area of common overlap. A fourth point of known elevation is also desirable for stereoscopic mapping to provide a check on the elevation of the other points and to assist in carrying elevations from one photograph to the next between control points, a process sometimes called "bridging."

The control should be positioned so as to form as strong a geometrical figure as can be drawn with either the single photo or the overlap. Thus in deciding how control should be located, consideration should be given to the proposed use of the photographs. It should be observed that triangulation stations are usually elevated points and that these are subject to large relief displacements in the pictures. Traverses, on the other hand, may be run along highways, in valleys, near to the level of the selected datum, and therefore be nearly free from this objection. Differences in relief are of minor importance in using stereoscopic instruments.

A control traverse with levels has the advantage that it may be run by an ordinary surveying party with the usual equipment, but a triangulation system of the order of accuracy required usually takes less time and is more economical provided the terrain is suitable for triangulation.

In taking levels, points should be selected which are easily identified in the pictures and which are placed so as to give the best determination of tilt. For plotting with stereoscopic machines, the points of known elevation should be chosen so that

they will fall near the four corners of the stereoscopic model (Art. 5-13).

Frequently the photographs are taken before the ground control has been established. When this is the case, the photographs are examined in the office and the most desirable locations for control points, so far as the construction of the map is concerned, are encircled on the prints. The photographs are then taken in the field and definite identifiable points found within the circled areas and tied in to the control survey.

In districts where there are no roads or other physical objects which are easily identified on the photographs, it may be advisable to lay on the ground man-made markers of light-colored cloth or other materials contrasting with the background and placed in the form of a cross or circle so that they will show up well in the photographs. They must be large enough to be easily found in the photographs. They should be firmly anchored to the ground and tied in promptly to the control survey before they become disturbed by animals, curious people or the weather.

4-31. Use of Electronic Devices for Establishing Ground Control. While conventional surveying instruments and techniques still provide the basis for most photogrammetric ground control, new electronic devices are also used extensively for this purpose. The saving in time and the ability to cover vast areas at comparatively low cost, particularly if the terrain is inaccessible, often far outweigh the high initial cost of the equipment.

Electronic distance measuring systems, such as the *Tellurometer* and the *Geodimeter* (Arts. 1-35–39), are useful tools for running traverses or triangulation for ground control, especially in rugged terrain and where sights are long, and a high degree of accuracy is required. Vertical angles must be measured to reduce the measured slope distance to the horizontal, and horizontal angles must be turned. Atmospheric pressure, temperature and humidity readings are also needed for applying corrections to instrument readings. The elimination of taping, however, is the outstanding advantage of the use of these instruments.

4-32. The Elevation Meter. The elevation meter, designed and used by the U. S. Geological Survey, is a device for rapid leveling along passable roads. The equipment is mounted on a

four-wheel-drive carryall truck provided with four-wheel steering. As the apparatus proceeds along the road, an electromagnetic field acts on a very sensitive pendulum in such a way as to generate an electrical signal whose strength is proportional to the sine of the angle of slope. Another electrical signal is generated by a revolution counter connected to a special fifth wheel, which measures the distance traveled. By means of an electronic integrator, the two signals are combined into a continuous and automatic record of the difference of elevation from an initial starting point. To eliminate dynamic effects, the vehicle must be stopped to take elevation readings. Proceeding at moderate speeds over readily passable roads, the elevation meter can produce about 50 to 100 miles of leveling per day, reliable within about 2 feet, depending on operating techniques and distances between controlling bench marks.

4-33. Ground Control by Shoran. In extensive aerial surveys, particularly in undeveloped territory where few ground control points are available, the position of the aircraft may be determined by an electronic measuring device known as *shoran* (SHOrt RAnge Navigation). A transmitter in an aircraft sends out a signal which is received by each of two ground stations, located at the ends of a known base line. The signals are amplified, and transmitted back to the aircraft. In the aircraft, the round-trip time of signals to each of the stations is measured, divided by two, and converted into miles and recorded on two counters to .01 mile and by interpolation to .001 mile (or about 50 ft.). These distances to each of the stations together with the known length of base line fix the location of the aircraft.

As the aircraft flies along any desired course, a motion picture record is made of the distances to each of the ground stations as shown on the counters. A single exposure of the movie camera records the two Shoran distances simultaneously with the taking of an aerial photograph, thus giving a Shoran fix for the plumb point of each photograph. In the laboratory the intersections of the distances from the two ground stations at each exposure are plotted to the map scale, thus charting the course of the aircraft.

The Shoran method may also be used to measure the distances between triangulation stations in establishing ground control.

Radar receivers and transmitters are set up at each station and the aircraft is flown along a course perpendicular to the line joining these two ground stations. As it approaches this line, the *sum* of the two Shoran distances becomes smaller and is a minimum as the aircraft crosses the line. Thereafter it increases again. That minimum sum, after a few refinements are made, is the distance between the two stations. In practice, a series of these sums are observed with the movie camera in the aircraft, both before and after the line crossing. Once the ground station radar units are set up, the aircraft can make many line-crossings with hardly more trouble than a single one, and thus obtain a large number of readings for accurately establishing the length of the triangulation line.

The line-crossing technique makes it possible to measure all the lines in a network instead of all the angles; hence the name *trilateration* has been given to the method. In remote areas, where no basic triangulation exists, it is necessary to perform such a trilateration operation for a network of stations before those stations can be used as the ends of known base lines to provide Shoran fixes for aerial photography. If the ground station locations are permanently monumented, they constitute the basis of a first order control network where none existed before.

The wavelength at which Shoran operates (about 300 mc) requires direct line-of-sight clearance between aircraft and ground station. Due to the curvature of the Earth, this limits the range at which Shoran may be used. The maximum distance depends on the ceiling of the aircraft, on the location of the ground station and on the intervening terrain. Hence it is customary to place ground stations on mountain tops wherever possible and to use aircraft with high ceilings. Under a favorable combination of these factors it is possible to " see " a ground station 250 miles away, and therefore lines as long as 500 miles may be measured by the trilateration technique. This makes it possible to " bridge " long gaps in conventional triangulation nets, such as stretches of ocean between islands and the mainland. The formula for maximum range possible with Shoran due to Earth curvature is

$$D = \sqrt{2(H - G)} + \sqrt{2(K - G)} \qquad (4\text{-}11)$$

where D is the maximum range in miles, H is the altitude of the aircraft in feet, K is the elevation of the ground station in feet, and G is the mean elevation of the intervening terrain.

4-34. Other Electronic Positioning Devices. A number of other electronic positioning systems exist which do not have the line-of-sight limitations of Shoran. Most of these are known as phase comparison devices. Three ground stations must be used, preferably close to water which helps conduct the ground wave. At the aircraft a receiver measures the phase difference between signals received from two of the stations. This represents the difference in distances to the two stations. Since the locus of constant difference is distance to two points is an hyperbola, the position of the aircraft must be on a certain hyperbola whose location appears on a chart printed for the purpose. Another receiver locates the aircraft on another hyperbola drawn with respect to the third station and one of the first two. The location of the fix is at the intersection of the two hyperbolas. Examples of the hyperbolic systems are *Loran*, *Lorac*, *Raydist* and *Decca*. While they have the advantage of not requiring line of sight they are less accurate than Shoran for ground control purposes. They are used for fixing position of sounding craft in hydrographic surveying (Art. 7-44).

Hiran is a version of Shoran, incorporating certain additional electronic features to increase its accuracy for geodetic work.

An electronic navigating-positioning technique which is independent of any ground stations utilizes the *Doppler* principle as applied to electronic transmission. Four radar beams in a square array are transmitted downward from the aircraft, each a few degrees off the vertical in a direction outward from the center of the square. The two which are inclined toward the forward direction of the aircraft will be reflected off the ground and received at the aircraft at a slightly different frequency from the two which are inclined toward the rear, due to the Doppler effect. Measuring this frequency gives a measure of the ground speed of the aircraft. Also, if two sides of the square are not parallel to the direction of flight, the relative frequency shift of the radar

beams on either side will detect and measure the misalignment, thus providing the value of the drift angle. The ground speed and drift angle, taken together with the compass heading, provide all the information needed for dead reckoning of position. A computer in the aircraft provides the position information relative to the initial starting point. Thus the Doppler system can be used for navigating flight lines and, by photographing the face of the computer, a record can be kept of the track for future recovery in the laboratory. Any dead reckoning system can acquire intolerable cumulative errors, but they can be kept at a minimum by frequent checks over known ground points.

4-35. Radar Altimetry for Vertical Control. The radar altimeter is an electrical device installed in an aircraft for sending out a radar signal which is reflected back to the aircraft receiver from the nearest ground surface. The time required for the signal to travel from the airplane to the ground and back is converted into feet of height that the aircraft is above the ground.

In order to use this method to determine ground elevations it is necessary to know the altitude of the aircraft above sea level when exposures are made. The ordinary barometric altimeter is subject to considerable influence aloft by variable weather conditions, to the extent that its reading can be off the true altitude by several hundred feet. To correct this for radar altimetry purposes, two techniques are used. Firstly, measurements are made in flight of the angle of crab, the indicated air speed and the free air temperature. Since the slope of the constant pressure surface along which the aircraft is flying depends on these elements, it is possible to compute that slope from the measurements. Secondly, the true elevation of the constant pressure surface along which the aircraft is flying is obtained at frequent intervals by flying over points of known elevation and adding to the known elevation the reading of the radar altimeter. Knowing the slope between these check points, it is possible to obtain a much more accurate map of the constant pressure surface than would be available from the barometric altimeter alone. Where points of known elevation are not numerous or conveniently spaced it is possible to fly a network of intersecting flights with the radar altimeter and relate the pressure surfaces to each other

at the intersection points, tying the entire network to a minimum of three points of known elevation.

A radar altimeter specifically designed for aerial survey work is the *Airborne Profile Recorder* (*APR*). It features a dish-like parabolic antenna which concentrates the beam in a 1.6-degree cone and a device for detecting and correcting for slight departures by the pilot from the constant pressure surface he was supposed to fly. The profile is drawn in the aircraft on a continuous strip chart at a paper speed generally twelve inches per minute (approximately four inches per mile) and a vertical scale of about one hundred feet to the inch. When carefully used, accuracies of five feet are attainable with the system.

4-36. Astronomical Control. For certain purposes positions obtained by observed latitude and longitude may be used to control the mapping. This avoids the necessity of traversing the entire distance between the stations. This method is subject, however, to considerable errors caused by the local deflection of the plumb line especially in mountainous country. It must, therefore, be regarded more as a method of controlling reconnaissance maps than for very accurate mapping. See Art. 2-34 to 41.

4-37. Photogrammetric Maps. The term *photogrammetric map* or *photo map* is usually applied to a photographic reproduction of a mosaic. The latter is a continuous representation of the ground obtained by piecing together individual photographs into a composite picture. A mosaic can be assembled to almost any desired degree of accuracy. The cost, however, increases as the standard of accuracy is raised.

Some of the advantages of mosaics are low cost, rapidity of reproduction, completeness of detail and ease of understanding. They are more readily understood by the non-technical man than engineers' plans. In estimating timber, in traffic studies, and in many other applications, a mosaic is often preferable to a line map.

The following terms may be used to define the different types of photographic maps.

Uncontrolled Mosaic. — A representation of the ground made by matching aerial photographs without reference to ground control points.

Controlled Mosaic. — A representation of the ground made from aerial photographs by bringing them to a uniform scale and fitting them to ground control stations. Sometimes called a *photo map*.

Planimetric Map. — A map showing the natural or cultural features (or both) in plan only, often called " line map."

Topographic Map. — Map made from data derived from aerial photographs showing contour lines and planimetric data.

An example of a rough uncontrolled mosaic is the photo-index map shown in Fig. 4-28. A portion of a controlled mosaic is shown in the upper view of Fig. 4-29, and a topographic map of

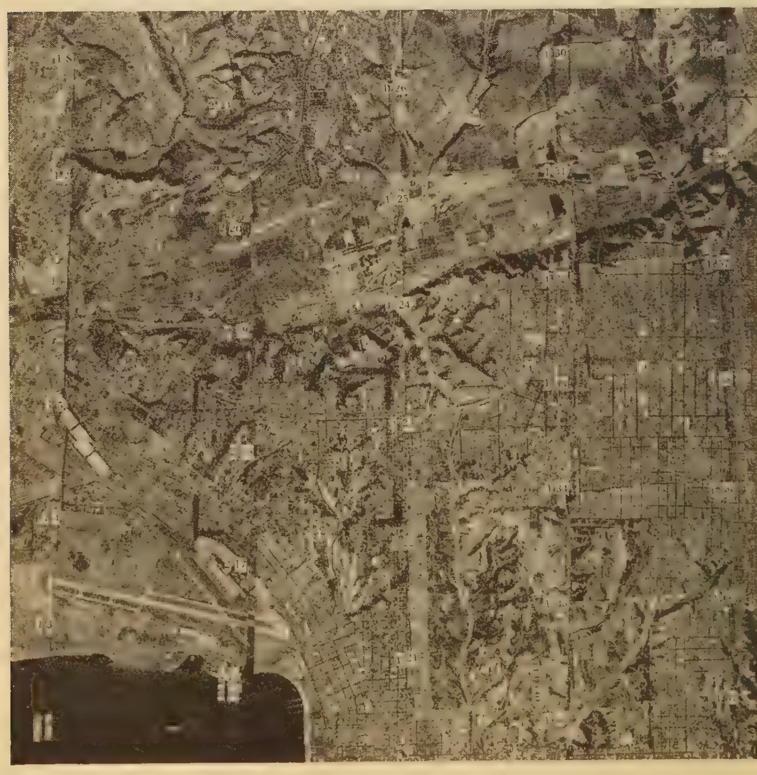

FIG. 4-28. MOSAIC PHOTO-INDEX MAP.
(Courtesy, Fairchild Aerial Surveys, Inc.)

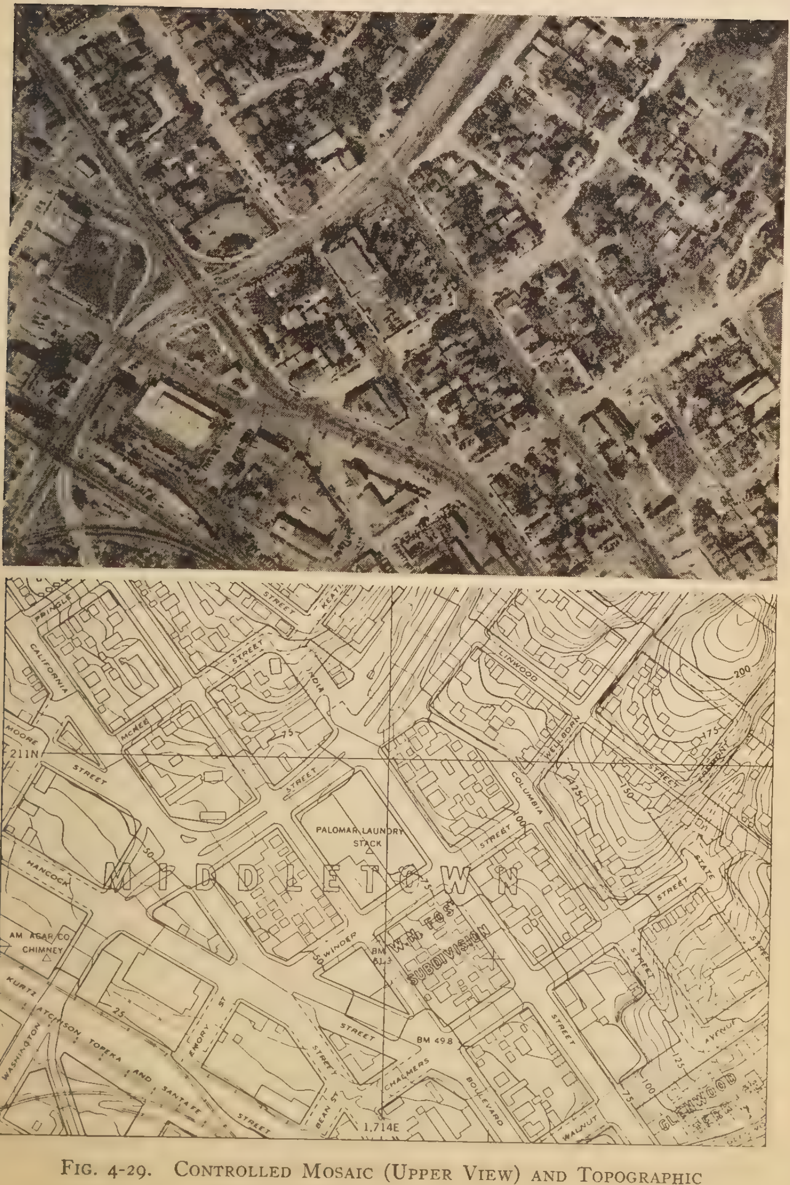

FIG. 4-29. CONTROLLED MOSAIC (UPPER VIEW) AND TOPOGRAPHIC MAP (LOWER VIEW) OF SAME AREA.

(Courtesy, Fairchild Aerial Surveys, Inc.)

the same area, derived by photogrammetry, is shown in the lower view of Fig. 4-29. A planimetric or line map is illustrated in Fig. 4-34.

4-38. Orienting Prints and Compiling Maps. The photographs may be oriented and line maps prepared by the application of graphical methods. Mechanical and optical devices greatly facilitate the work. Methods of compiling controlled mosaics and line maps are described in the following articles.

4-39. Graphical Radial Triangulation Method. This method is also commonly called *Radial Intersection Method*, *Radial Intersections* and *Radial Line Plot.* This method is based on the assumption that the photographs have such small angles of tilt that they may be assumed to be true verticals. Therefore, it is assumed that *angles measured at the principal point are true horizontal angles* regardless of change in scale or displacement of high points because such errors are radial from the photograph nadir, which, in vertical photographs, coincides with the principal point.

Suppose that on three consecutive overlapping pictures we have the three principal points p_1, p_2, p_3 (Fig. 4-30) and that

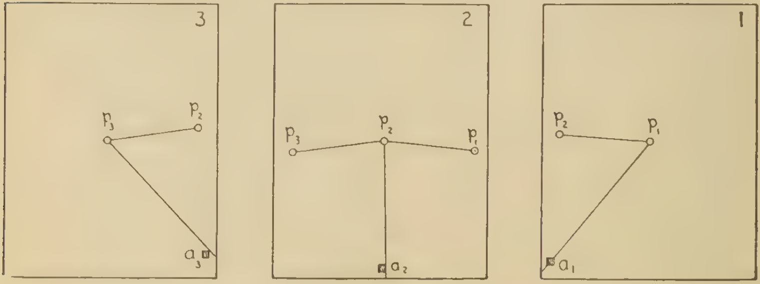

FIG. 4-30. RADIAL LINES.

from each principal point a radial is drawn on the corresponding print to the corner of a building a. By superposing the pictures so that the various images of each principal point are made to coincide (Fig. 4-31) it is seen that all three lines intersect in a common point but that this point does not coincide exactly with

any of the positions of the building on the pictures. The true position of the corner of the building is at the intersection of the three radial lines (Fig. 4-31). This error in position on the print may be due to difference in scale between the plot and the photographs because the airplane was not flown at the exact elevation intended, or it may be due to relief displacement of the ground on which the building stands or due to misinterpretation of the building on one of the photographs. If the three lines so drawn do not have a common intersection, or nearly so, an error in the work should be looked for; if this is not discovered, one of the pictures probably is distorted by tilt and should be rejected when it is identified. If the point appears on four or five prints, then four or five radial lines may be drawn; the greater the number of intersecting lines the better the check on the plotting, and the easier it is to decide which print is causing the error.

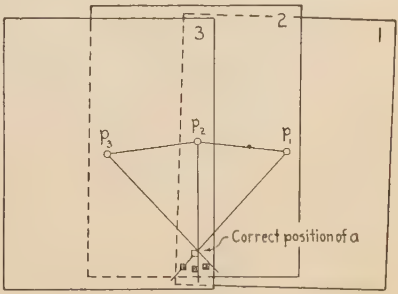

FIG. 4-31. INTERSECTION OF RADIALS.

In applying the principle of radial intersection, it is best to plan the flight so that the picture scale will be approximately the same scale at which it is desired to compile the radial plot. If the pictures have already been taken, then a scale should be selected which is close to the average scale of the pictures. The nominal scale for which the photographic mission was flown is usually a suitable scale, as the flight is generally designed to produce this mean scale. If the scale of the pictures is not known, it may be determined by scaling the longest distance that can be conveniently obtained from a carefully matched strip of pictures between two points which can also be identified on an accurate map of known scale. The scale of the radial plot should ordinarily be within plus or minus 10% of the mean scale of the pictures.

The map projection grid is laid out on a transparent tracing either by latitude and longitude or by plane coordinates and all

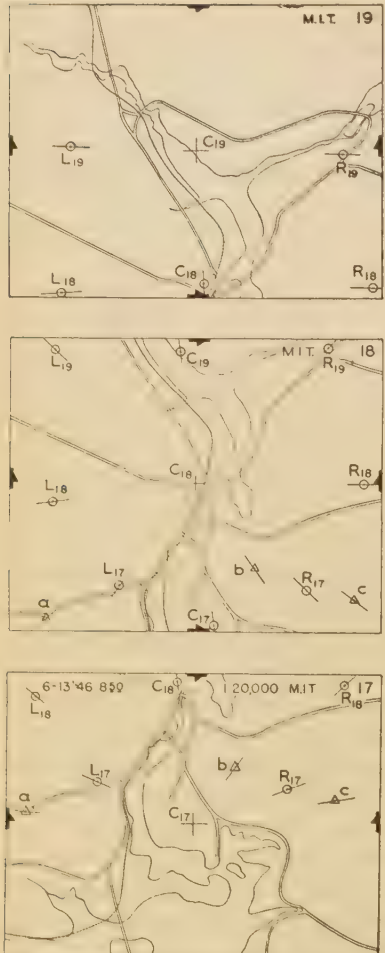

FIG. 4-32. CONTROL POINTS.

known control points are then plotted thereon. The center (or principal point) of each photograph, C_{17}, C_{18} and C_{19} (Fig. 4-32), is next identified and marked in all the succeeding and preceding photographs in which it appears. The viewing of the overlapping portions of the photographs in a stereoscope (Art. 5-6) will aid materially in identifying conjugate images of the principal points and other control points, and will assist in avoiding mistakes and improving the accuracy of the plot.

In the first two photographs of a flight, at least three points should appear, the ground coordinates of which are known. These points are called *ground control points* (*a*, *b* and *c* in Fig. 4-32).

For the purpose of extending the control, sharply defined images called *picture control points* are chosen. These points may be images of objects to be plotted on the map such as road intersections, or they may be any feature which shows clearly and sharply in the photographs. They should be so chosen that they will appear in three photographs along a given flight. To do this, they should be located to the right and left of the principal point when looking along the flight

line. The picture control point to the left of the principal point of Photograph 17 is labelled L_{17} and that to the right of the principal point, R_{17}; similarly, in Photograph 18, the left and right points are labelled L_{18} and R_{18}, etc.

Radial lines are now drawn from the principal point of each photograph to all the ground control points, picture control points and to conjugate images of the principal points of adjacent prints. These lines should be inked in with a color that will contrast with the tone of the print and be visible through the tracing on which the ground control points have been plotted.

A minimum of 9 points must be selected on each picture. These points should be the principal point of the picture and the conjugate images of the principal points of the two adjacent pictures in line of flight and points toward each side of the picture, substantially opposite the first three points. The result will be a rectangular pattern of points, with one point in the middle, one near the middle of each side, and one near each corner of the picture. In the first and last picture, of course, there will only be six points selected since the compilation of the map depends upon the utilization of points which appear on at least two pictures. If a planimetric map is to be compiled by the radial line method, points should be identified along the planimetric features, such as streams and roads, at sufficiently frequent intervals so that the location of other features may be interpolated between these points.

The orientation of the successive pictures is based on the principle of resection. Intersections of rays to picture control points indicate their orthographic projection. The tracing cloth solution of the three-point problem (Art. 7-41) is used for this purpose. The first picture is oriented under the tracing cloth so that the radial lines from the center to the ground control points fall through the plotted position of these points on the tracing. The center of the photograph is then correctly oriented and is so marked on the tracing. Radial lines are now drawn on the tracing to the selected picture control points and to the photograph centers shown on the print. This picture is removed and the second photograph oriented in the same manner.

The intersections of the radial lines as drawn on the map sheet

from C_{17} and C_{18} through L_{18} and R_{18} determine the map positions of these points. These two positions, together with the plotted position of C_{18} may now be used for resecting Photograph 19. As each photograph is oriented, three new picture control points are then located and serve as control for the subsequent photograph. The foregoing process is continued until a new set of

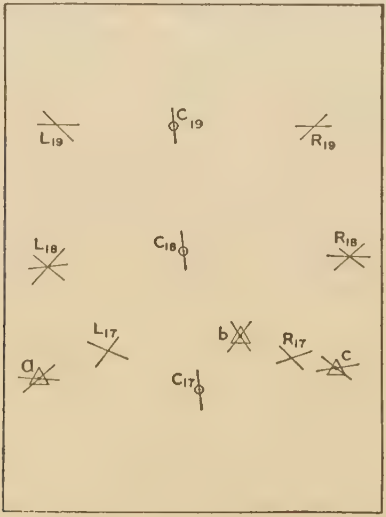

FIG. 4-33. ADJUSTED PLOT OF RADIAL TRIANGULATION.

ground control points has been reached, when a check is obtained. If the points on the tracing do not check with the ground control points then the positions of the picture control points on the map are adjusted to correct the errors in their positions. Fig. 4-33 is the adjusted plot of radial triangulation for the prints shown in Fig. 4-32. Using this plot to control the position of the prints, the final map is drawn as shown in Fig. 4-34. In practice Figs. 4-32, 4-33 and 4-34 would all be drawn to the same scale. For clarity, Fig. 4-34 is drawn to double the scale of Figs. 4-32 and 4-33.

In locating the successive prints, the radial line to its center and radials through the previously established ground control

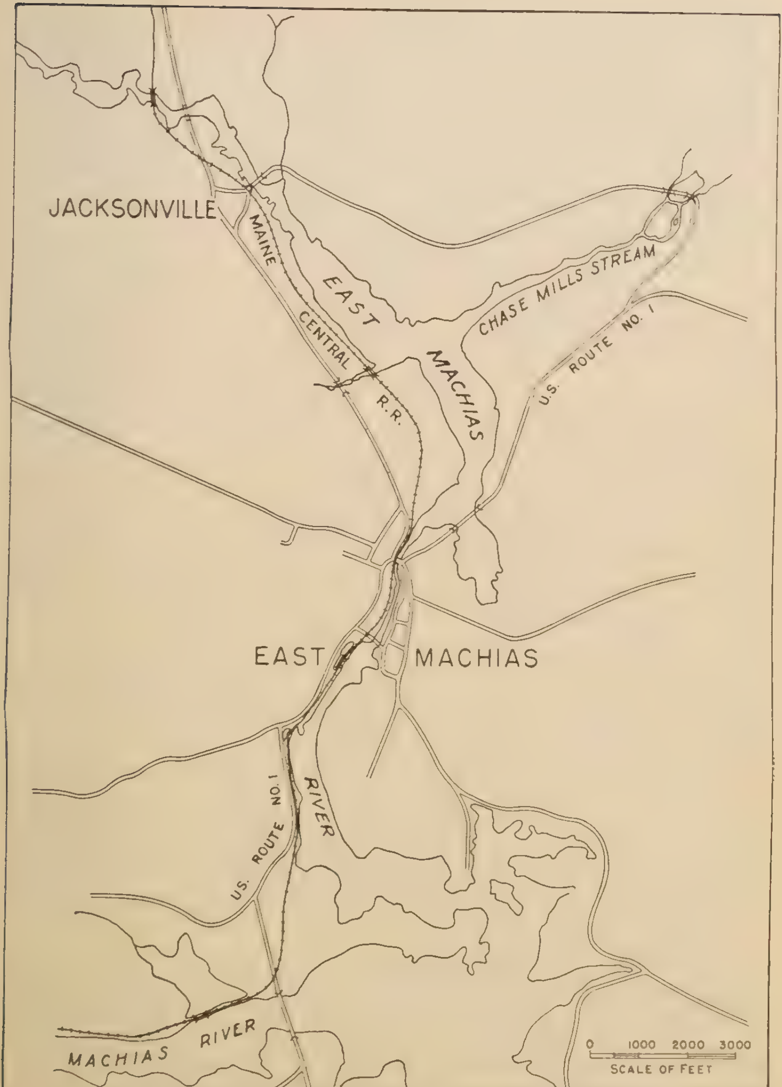

FIG. 4-34. MAP COMPILED FROM AERIAL PHOTOGRAPHS.

or picture control points are used. If these do not all fit, but no gross error is evident, each radial may be thrown out slightly, points on opposite edges of the print being on opposite sides of the radial line. Due to errors introduced by the assumptions that displacements caused by relief and small angles of tilt are radial from the principal point perfect checks cannot be expected, but the tracing may be adjusted so that these errors are equally divided and a compromise position adopted.

If prints alone are used there is no way of knowing the exact scale. Consequently, the scale chosen is that of the line on the first photograph joining the center of the print and the plotted position on this print of the principal point on the next print. In this case, the second print is oriented by placing its center at the point taken from the first print. This scale is later changed to that required for the finished map.

When an adjoining strip of photographs is to be located, additional radial lines will be traced to the points on the first strip. Since the locations of all elevated or depressed points are effected by displacement, it is essential to cut them in by radial lines in order to secure a true orthographic projection.

When the photographs are badly tilted, the assumption that the displacements due to relief radiate from the principal point is no longer valid. In areas of large variations in relief, the photograph nadir should be used instead of the principal point since the effect of displacement due to relief is greater than the effect of tilt. When the area has low relief and the photographs are tilted, the isocenter should be used as the effect of the displacement due to tilt is more important than the effect of relief (Art. 4-28). In moderate relief a point between the isocenter and the nadir may be used.

4-40. Hand Templet Method. The radial triangulation method described in the previous article has the disadvantages that radial lines are drawn directly on the map projection and that the map sheet must be shifted over the photograph. This complicates the drawing especially when the map sheet is large. Another important disadvantage is that in the event that a check on ground control stations is not realized after bridging by picture control stations, the adjustment is both tedious and unsatisfactory.

A superior method of preparing a radial line plot is to draw the radial lines for each photograph directly on a transparent sheet about the size of a print. Film base or cellulose acetate is excellent for this purpose because of its high degree of transparency. Fig. 4-35 is an illustration of a hand templet prepared in this manner for Photograph 17.

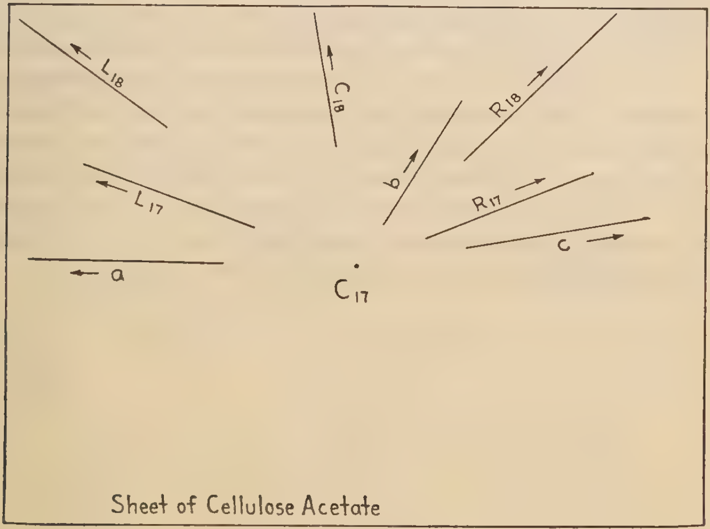

Fig. 4-35. Hand Templet for Photograph 17.

The laydown of the plot is done directly over the projection and the intersections denoting the positions of the picture control points are easily noted because of the transparency of templets. If small discrepancies exist when ground control points are reached in the process of plotting, adjustments are made by shifting the various templets by small amounts. The map positions of the picture control points are marked by pricking through the templets at the intersections of the radials.

4-41. Slotted Templet Method. The most satisfactory method for utilizing the principle of radial control is a mechanical assembly which automatically adjusts the control plot to conform to the ground control points. Templets are cut in card-

board or cellulose acetate for each picture in which the centers are indicated by circular holes and radials to ground and picture control points by slots radiating therefrom. These templets are then assembled by inserting pins or studs in the holes or slots and fitting each templet in turn over the pins. The studs have a vertical hole in their center through which a needle point may be inserted. For a detail of a stud see Fig. 4-36.

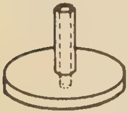

FIG. 4-36. STUD.

In preparing the templets the centers and the control points are identified on all the photographs as before. These locations are transferred to the cardboard or cellulose acetate by placing the photographs over the templet and pricking the points through with a needle. The templet cards are then taken to a special punch which first punches out the centers and thereafter the radial slots. Fig. 4-37 shows a completed templet for Photograph 17, Art. 4-39.

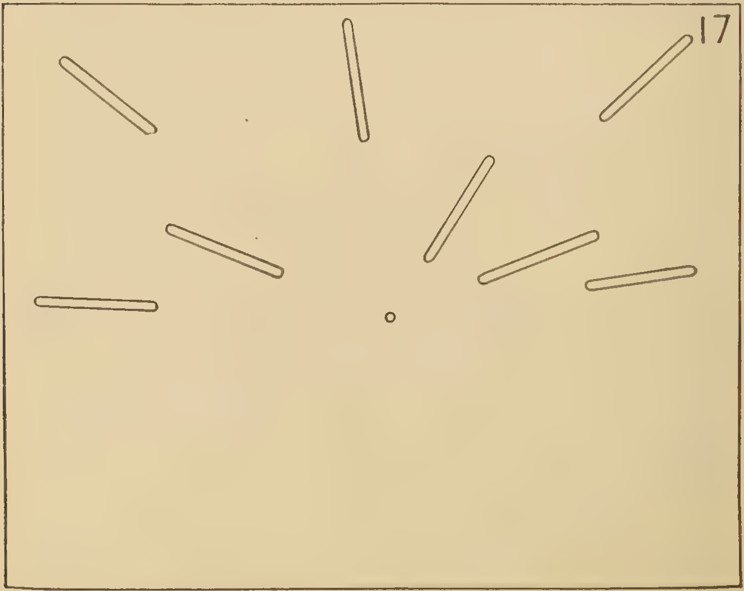

FIG. 4-37. SLOTTED TEMPLET FOR PHOTOGRAPH 17.

After the projection and control points have been plotted on the assembly board, studs are securely fastened at each control point. This may be done by inserting a needle point through the stud and fixing the needle point directly into the plotted position of the control point. Before laying down the first templet, a stud is inserted in the hole corresponding to the principal point and studs are inserted in the radial line slots to the various picture control points. Each templet is fitted in turn over the pins and the entire assembly adjusted to fit the rigid ground

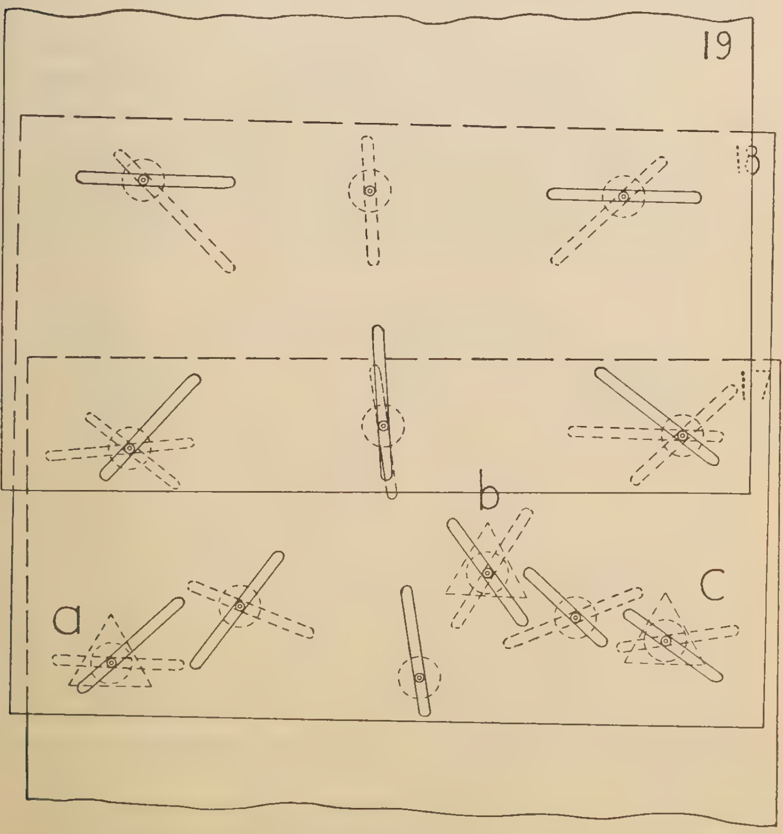

FIG. 4-38. SLOTTED TEMPLETS IN PLACE FOR PHOTOGRAPHS 17, 18 AND 19.

control studs. All other studs are free to shift through the tri-angulation system of the slots in such a way that their positions automatically conform to the scale established by the fixed ground control points. Fig. 4-38 is an illustration of a slotted templet laydown for the three prints discussed in Art. 4-39.

4-42. Spider Templet Plot. Another method of carrying out a radial triangulation plot by mechanical methods is through the use of spider templets. In this process, the radial lines are represented by a length of spring steel having a three-inch slot in one end and a series of holes $\frac{3}{4}$ inch apart in the remainder of its length.

In preparing a spider templet for a given print, pins are driven directly through the images of the ground and picture control points. Studs, similar to those used in the slotted templet method, are placed over all the pins except the principal point and the conjugate images of the principal points of adjacent photographs. A threaded stud is placed over the pin marking the principal point of the given photograph. A pin is driven

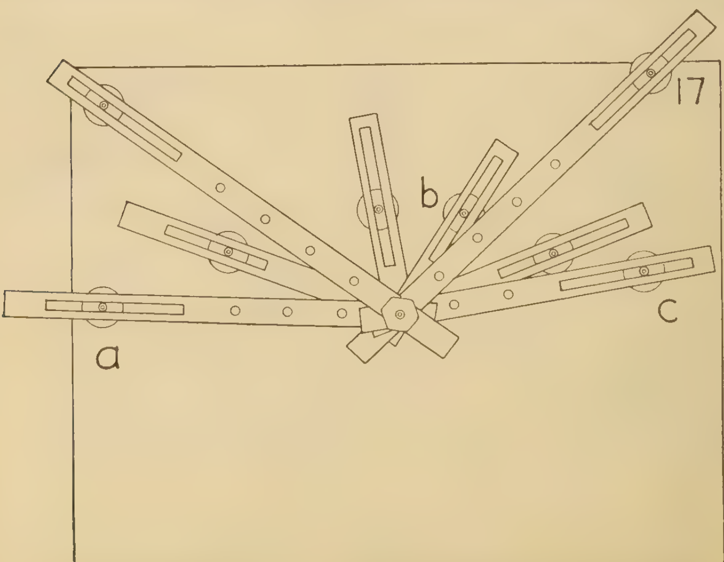

FIG. 4-39. SPIDER TEMPLET FOR PHOTOGRAPH 17.

through the point half way along the line between the principal point of the given photograph and the conjugate image of the principal point of the preceding photograph and another half way along the line toward the conjugate image of the principal

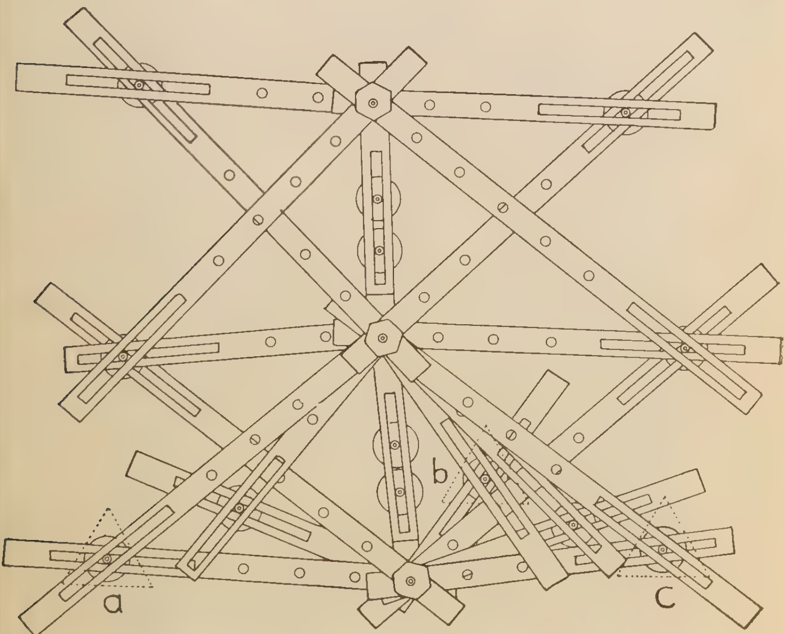

Fig. 4-40. Spider Templets in Place for Photographs 17, 18 and 19.

point of the succeeding photograph. Unthreaded studs are placed over these pins.

Spider templet arms of varying length are set so that the threaded stud is through one of the holes and so that the unthreaded studs are about in the middle of the slots. One arm is set for each control point. After this has been done, a nut is screwed onto the threaded stud which has been set over the principal point and the assembly held in place by tightening securely the nut on the threaded stud. Fig. 4-39 is an illustration of an assembly for Photograph 17.

The laydown of these templets is done in a similar fashion to that of the slotted templet with the exception of the connection of the principal point radials. The line between the principal points is held by inserting two unthreaded studs in the slots noting the radial direction between principal points. Fig. 4-40 shows a laydown for the three photographs discussed in Art. 4-39.

When a laydown is made covering several flight lines, picture control points must be so chosen that they will appear in the overlapping portions of the prints of adjacent flight lines. The slotted templet and spider templet methods are especially well suited for tying together laydowns comprising several flight lines.

The spider method has the advantage that the arms can be used over and over. Disadvantages are the length of time required to assemble the spiders and the possibility that the angles may change due to slippage of the arms.

4-43. Controlled Mosaics. From the control established by ground surveys and as the result of a radial plot, it is possible to lay out a base map of accurately plotted points to which single photographs or portions of photographs may be adjusted to form an accurate mosaic. By comparing distances between control points as they appear on the photograph with the same distances accurately plotted on the base map, the average scale of the print may be determined and also the location of the axis of tilt and the amount of tilt in the photograph. With this information known the photograph may be placed in a rectifying camera and rephotographed to the scale of and in the plane of the base map.

Some rectifying cameras require manual setting of the picture frame using tilt determinations and scale ratios obtained by comparing the radial plot with the photographs. Another method of rectifying the negative is to place it in an automatic rectifier such as that illustrated in Fig. 4-41. From the radial line templet a plot is prepared representing the true positions of several points in the photograph. This plot is then set on the table of the rectifier and the camera is adjusted for enlargement or reduction and tilt until the projected image points correspond with the plotted points. When this correspondence has been

achieved by trial and error, the plot is replaced by a piece of sensitized paper and a print is made from the negative. In Fig. 4-41 the movement of the lens and the table are linked together to provide proper focus and to maintain the proper relation with the negative plane for all angles of tilt.

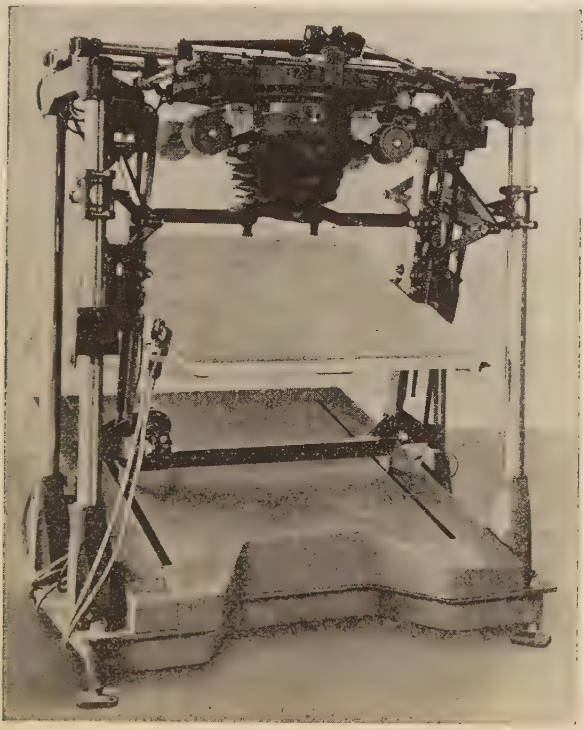

FIG. 4-41. AUTOFOCUS RECTIFIER.
(Courtesy, Bausch & Lomb Optical Co.)

Different portions of the photograph may be rectified separately to eliminate the effect of relief. For example, areas in the photograph representing tops of ridges, sides of valleys and valley floors would be rectified separately.

In blending the mosaics, photographs of nearly the same tone are selected and some tinting or touching up of the negatives is

done to minimize the contrast in color between the many patches comprising the mosaic. The patches are cut to a feather edge thus producing a smooth joint and patch-free appearance. A title, names of features, boundaries, etc., may be lettered on the mosaic before it is rephotographed to final scale. Mosaics are frequently printed on heavy paper or mounted on composition board. They may also be prepared for use as plane table sheets.

4-44. Transferring Details to the Map. There are a number of comparatively simple ways of transferring detail obtained from the photograph onto a line map. One way is to trace the completed radial control plot onto a sheet of tracing cloth or acetate indicating the positions of the center and the wing points of every picture. The photograph is then adjusted under the tracing medium so that its center corresponds with its plotted position and image points are on radials through their true positions. If it is desired to make the map of high accuracy, critical points are added to the transparent sheet by drawing radial lines through image positions, and then intersecting these with radials through the same images from adjacent pictures. In this manner, using the center point and the nine basic points of each picture as a foundation, additional intersections are developed for the control of the features which are desired on the map. In the case of a stream it would be necessary to make frequent intersections at each change of direction. This tedious process is seldom used, however, because if a map is desired to this degree of precision, the use of one of the instruments described in this chapter will prove more economical.

A common method for tracing detail between two control points when the images of these points do not exactly fit their plotted positions on the map is to shift the photograph under the tracing medium so that the detail is plotted on the map in the same relative position to the control points as it appears in the photograph. A tracing table with a transparent top illuminated from below helps materially in bringing out detail so that it can be easily traced.

A degree of precision between that obtained by the rather crude adjustment method described above and that possible with the elaborate stereoscopic plotting machines to be described

in Chapter 5 may be obtained with the *vertical reflecting projector* (Fig. 4-42). An individual print is inserted into the housing at the top of the projector and held in place by a glass pressure plate. The image is reflected by a diagonal mirror in the housing and is projected in an erect position onto a table top having a

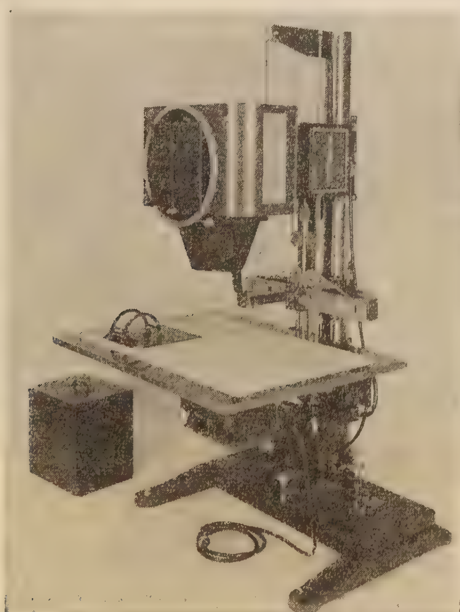

FIG. 4-42. VERTICAL REFLECTING PROJECTOR.
(Courtesy, J. G. Saltzman, Inc.)

tilting easel. By varying the focus, the scale of the projected image may be varied and by tilting the easel, compensation may be made for any tilt in the photograph or for the effect of sloping terrain. The usual procedure is to match the images of three control points with the corresponding map positions and then trace the detail within the triangle formed by these points.

A less expensive instrument is the *vertical sketchmaster*, Fig. 4-43. The detail on the vertical aerial photograph is reflected from a large front-surfaced mirror (which prevents "ghost"

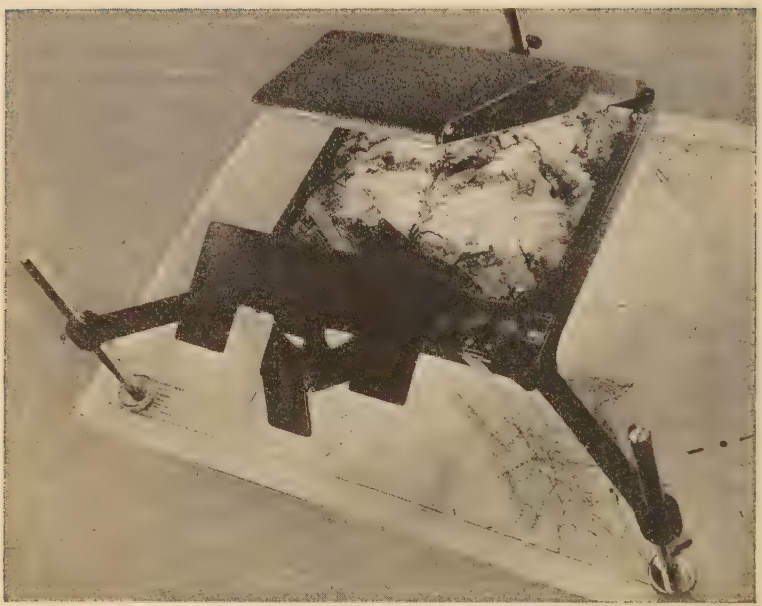

FIG. 4-43. VERTICAL SKETCHMASTER.
(Courtesy, Aero Service Corporation.)

images) to a small semi-transparent mirror. This latter mirror has the properties of both a mirror and a transparent glass. The meniscus lens, Fig. 4-44, serves to focus the virtual image of the photograph on the plane of the map. An observer looking through the eyepiece sees a vertical image of the photograph directly on the map sheet where it may be easily traced. By adjusting the legs of the sketchmaster, compensation may be made for variations in scale and tilt. The procedure in using this instrument is similar to that of using the vertical reflecting projector. Three images of control points on the photograph are placed in coincidence with their corresponding positions on the map sheet and the details within this triangle traced onto the map sheet. Magnetized metal bars are provided for holding the photograph in place on the instrument.

Both of these instruments have the advantage that a clear traceable image is cast onto the map sheet and that compensations

may be made for differences in scale and for tilt. Also an opaque map sheet may be used since no tracing is done through the projection.

4-45. Contours. There are two general methods of obtaining the contours on a map made from photographs. The first is by means of the stereoscopic principle applied to two overlapping photographs (Chap. 5). The second is by using the ordinary ground methods (plane table, stadia, etc.) after the horizontal locations have been completed by means of the aerial photographs. If the former method is used there must be sufficient ground control to enable the stereoscopic pairs to be set up in the stereoscopic plotting machine at proper scale and orientation. In the second method vertical and horizontal ground controls should be similar to that required for plane table surveys.

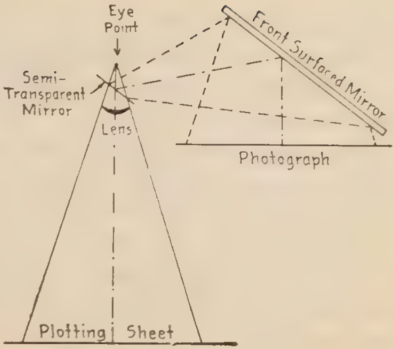

FIG. 4-44. PRINCIPLE OF VERTICAL SKETCHMASTER.

Aerial contact prints, enlargements and precise mosaics are extensively used as plane table sheets, especially in reasonably flat country where the picture or mosaic if properly rectified becomes a true map. A photographic plane table sheet has the great advantage that all of the planimetry is already on the map sheet in correct scale and relationship. The sole task remaining for topographer is to add the contours and complete the planimetry in spots where it is not properly recorded by the photograph (such as under trees) and adding geographic names, political boundaries and similar features. The photograph is frequently prior-mounted, either on composition board or on an aluminum sheet. The contours are drawn directly on the photograph which, when completed, may be left in its photographic form or it may be traced on some transparent medium for reproduction. Another approach is to " delineate " the planimetry by ink lines on the photograph. The photographic image may

then be bleached out, leaving only the ink lines. A photographic copy can then be made either as a negative, a positive, or both.

Sometimes contours are drawn on a mosaic either by stereoplotting techniques or by field methods and published that way. This combines the infinite detail from photo maps with the advantages of three-dimensional representation by contours (Fig. 6-3).

4-46. Oblique Aerial Photography. Oblique aerial photographs may be economically employed in small scale mapping of relatively level areas or areas of low relief. The three commonest types of oblique photography are *low oblique*, *Tri-Metrogon* and *air views*.

4-47. Cameras for Oblique Photography. Oblique photography may be taken by two separate aerial cameras of conventional style, each securely and rigidly attached to a single sturdy mount so that their relative orientation remains fixed throughout the survey. Two separate windows must usually be cut in the floor of the aircraft for taking photographs. Cameras specially designed for oblique photography feature the two cameras mounted in a single case, with their lenses next to each other. A special type of nine-lens camera developed by the U. S. Coast and Geodetic Survey takes one octagonal vertical photograph and eight obliques each of which overlaps one of the edges of the vertical. All are exposed on a single negative. The effect is somewhat equivalent to the use of a wide-angle lens after the obliques have been rectified. The purpose of these oblique cameras is to enlarge the coverage per photograph.

4-48. Low Oblique Photography. Two cameras are used in low oblique photography, each intentionally tilted in a plane perpendicular to the line of flight. The amount of tilt varies with different applications and cameras, but it is always such that the field of view includes the nadir point, so that there is overlap in the coverage of the photos taken by the two cameras. A common configuration uses two cameras, each mounted so that the optical axis is 20° up from the vertical. Each camera covers a trapezoidal-shaped portion of terrain. See Fig. 4-45. Both cameras are triggered simultaneously and the interval between successive exposures is such that there is 60% overlap along the line of flight. Since the scale along the flight line is

the same as it would be in single-lens vertical photography, the amount of film consumed on a single flight line is twice what it would be with vertical photography. The chief economy of the oblique photography, however, lies in the fact that the lateral coverage per two-camera exposure is much greater than with a single camera, permitting fewer flight lines to cover the same area,

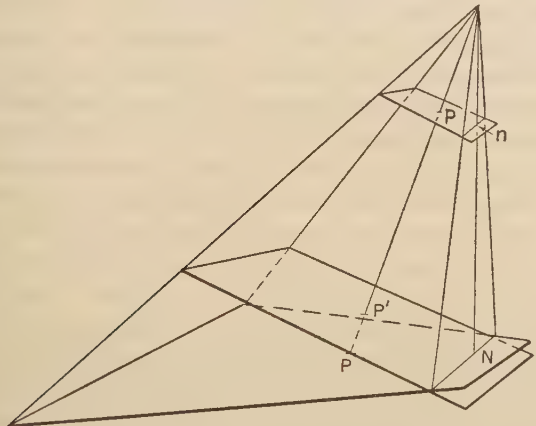

FIG. 4-45. TRAPEZOIDAL AREA COVERED BY 20° OBLIQUE PHOTOGRAPH.

with a substantial saving in flying costs. Side lap between successive flight lines can be the conventional 30% as with vertical photography, or it can be greater, to permit rejection of the extreme edge of each tilted photograph, where the scale is small and detail may be hard to see.

The tilted photograph can be handled like any vertical which has been accidentally tilted, provided that due recognition is given to the correct locations of principal point, isocenter and photo nadir (see Arts. 4-26–29). It can be handled similarly in stereoplotting equipment (Chapter 5) provided the instrument can accommodate such large tilts without exceeding its structural limitations. The Balplex plotter was designed with this type of *convergent photography* in mind.

The scale of a tilted photograph is constant in the direction of flight but varies continuously across the line of flight. For con-

struction of mosaics and interpretation of photographs for geologic purposes it is generally desirable to have a uniform scale over the entire print. This can be achieved in a rectifying printer (Art. 4-44) in which the easel, or print paper holder, negative holder and lens are tilted with respect to each other so that the resulting scale changes on the print are equal and opposite to those of the original negative, resulting in a uniform scale throughout the print. Since the area covered by the original square photograph was trapezoidal in shape, the rectified constant-scale photograph will also have a trapezoidal shape.

4-49. Tri-Metrogon Photography. Oblique photographs which show the horizon are known as high obliques. The Tri-Metrogon technique is employed for mapping from such photography.

This system derives its name from the fact that three K-17 cameras with 3 Metrogon lenses are employed. The cameras have 6 inch focal length and take 9 by 9 inch photographs. One camera is mounted so that its vertical axis points downward and the other two are mounted to take high obliques, one to the left and the other to the right. The optical axes of these cameras are inclined about 60° with respect to the vertical. Since the angle of coverage in the plane of the optical axes is 74°, there will be about a 14° overlap between the vertical and each oblique, and the coverage above the horizon will be about 7°. Fig. 4-46 shows a perspective view of a single exposure of the three cameras.

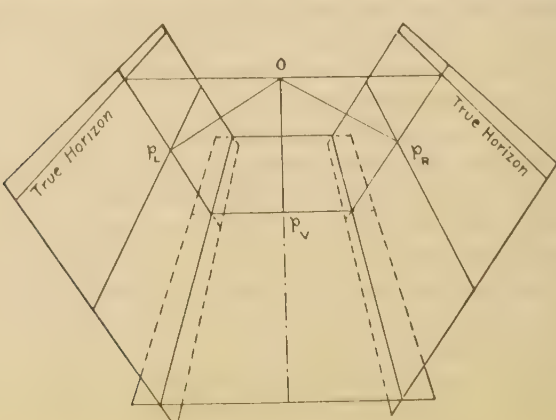

FIG. 4-46.

Most of the Tri-Metrogon installations have not had rigid connections between the cameras; this makes it necessary to establish the correct geometrical relationships between the vertical and the two oblique cameras as they were mounted in the ship when photographs were taken. The tilt determinations of the exposures are measured from the visible horizon.

A radial line plot is prepared by either the slotted or spider templet method. An instrument called a rectoblique plotter, designed by J. G. Lewis and J. L. Buckmaster of the U. S. Geological Survey, graphically obtains horizontal angles from the oblique views for the preparation of the radial line plot.

The detail from the vertical view is obtained by the use of the vertical sketchmaster (Art. 4-44) and the detail from the oblique views from an oblique sketchmaster, Fig. 4-47, an instrument similar in operation to the former but which casts an image from the oblique photograph rectified into the horizontal plane.

FIG. 4-47. OBLIQUE SKETCHMASTER.
(Courtesy, Abrams Instrument Corporation.)

The advantage of the Tri-Metrogon method is that greater coverage may be obtained at each exposure. Flight lines may be spaced farther apart and, therefore, less flying is necessary. Maps plotted from Tri-Metrogon photographs are only of sufficient accuracy for small-scale charts, such as those used in aerial navigation.

During World War II the U. S. Air Force obtained Tri-Metrogon photographs of many previously unphotographed portions of the world. In many areas this continues to be the only existing photographic coverage. Most of it is available from Air Force sources to agencies having a legitimate need for it, and provided consent is obtained from the local government involved.

4-50. The Photoalidade.* The Photoalidade, Fig. 4-48 is an instrument similar in principle to the plane-table alidade by

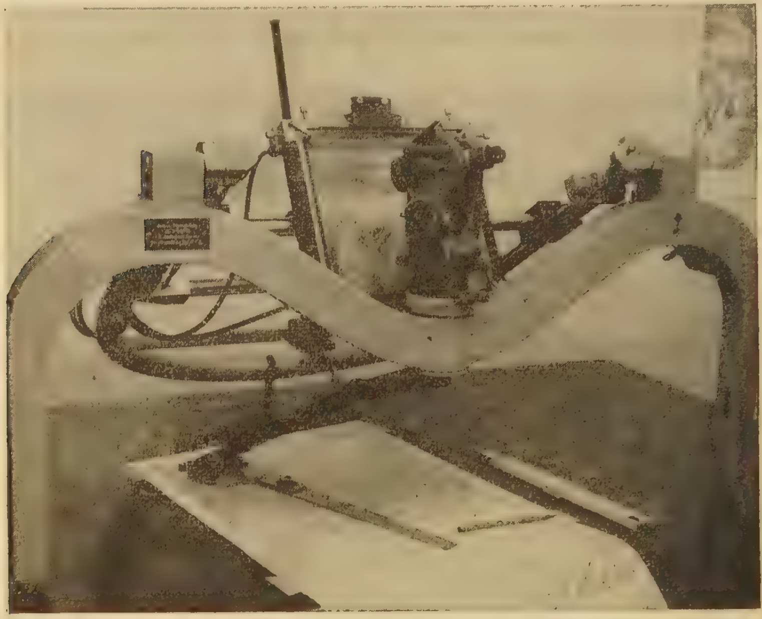

FIG. 4-48. PHOTOALIDADE.
(Courtesy, U. S. Geological Survey.)

* Devised by R. M. Wilson of the U. S. Geological Survey.

means of which the horizontal direction and the depression angle to any point on an oblique photograph may be determined. After a photograph has been placed on the viewing frame at a distance from the telescope equal to the focal length of the camera with the principal point at the center of the frame, the print is adjusted so that the trace of the true horizon has zero depression angle.

By means of a centering microscope at the base of the telescope, the entire instrument is then centered over the point on the map sheet corresponding to the position of the air station. The frame holding the instrument is in position for plotting when the ruler edge passes through the plotted positions on the map of the air station and a known point, while at the same time the image of the known point on the print is centered on the cross-hairs of the sighting telescope.

In operating the instrument, the principles of resection and intersection are applied as in plane-table work.

4-51. Air Views. These are oblique photographs taken at any suitable angle with a hand-held camera. For pictorial purposes, an oblique photograph presents features in a form more readily appreciated by persons unfamiliar with stereoscopic examination of vertical photographs, and they are suitable for visual study of engineering projects and for illustrating progress reports and similar documents.

PROBLEMS

1. Aerial photographs of an area are to be taken to a scale of 1/15,000 with a camera having a focal length of 8.25 inches.

(*a*) At what altitude above the average elevation of the ground should the exposures be made?

(*b*) What difference of elevation on the ground will cause a scale error of 1%?

2. If the size of the area to be photographed is 5 miles in a north-south direction and 4 miles in an east-west direction and the specifications call for a 60% end lap and a 30% side lap, and if the size of the prints is 9 by 9 inches, compute the number of flight lines necessary, and show on a sketch how they would be spaced on a flight map. Flight lines are to run in a north-south direction. For other necessary data, see problem 1. What is the theoretical number of exposures needed along a flight line?

3. A control point has an elevation of 200 ft. above the datum; $H = 10{,}000$ ft. The photograph of the point is 4″.300 from the center of print. By what distance should the photographed position be moved in toward the center to correct it for displacement?

4. In the print of a photograph, the image of a tall chimney occurs. The distance from the principal point to the top of the chimney is 36.15 cm., and the length of the chimney (its displacement) on the print is 3.15 cm. The exposure was made at an altitude of 2310 feet. What is the height of the chimney if its base is at sea level?

5. In a 9×9 print taken with a camera having a focal length of 8.25 inches, there is a tilt of $3° 26'$. Two points, a and b, are located on either side of the principal point and along the principal line. Point a is on the raised side of the print and 4.125 inches from the principal point and point b is on the depressed side of the print and a like distance from the principal point. What are the displacements due to tilt for these points?

6. Referring to Figs. 4-26 and 4-27, the elevation of point A above sea level is 500 ft. and of B it is 280 ft. The ground distance between A and B = 3000 ft. On the picture $r_1 = 3.8$ in., $dp = 4.5$ in. and $pb = 3.2$ in. $f = 8.25$ in. Find elevation of camera station.

7. Using the approximate method of Art. 4-27 and referring to Figs. 4-21 and 4-22, compute the angles of y-tilt for Photographs I and II, respectively, from the following measurements taken from these photographs: $a_1c_1 = 6.50$ in., $a_2c_2 = 6.32$ in., $b_1d_1 = 5.72$ in., $b_2d_2 = 5.80$ in., $P_{1t}p_2 = 3.40$ in. and $p_1p_{2t} = 3.28$ in. $f = 8.25$ in.

CHAPTER 5

STEREOPHOTOGRAMMETRY

5-1. **Stereophotogrammetry** is the science of producing maps from photographs by the application of the principles of stereoscopy. The method involves the study of stereo-pairs of photographs by means of the stereoscope, an instrument which shows the terrain in three dimensions.

Stereophotogrammetry applied to terrestrial photographic surveying requires taking two photographs from the ends of a base, which should be so short that both pictures will overlap and show the same landscape. In aerial stereo-surveying it is customary to take the photographs from an air base of such length that about 60% of the terrain shown in one picture will show again in the adjacent one. Photographs taken in the above manner are referred to as *overlapping* or *stereo-pairs* (Figs. 6-1 and 6-2).

Sterescopic instruments provide a means of plotting maps from photographs with greater precision than can be done by the other methods of plotting. In some instruments the map is constructed by drawing directly on the stereoscopic pictures. Since the scale of the photograph varies with the distance of the terrain from the camera, high areas will appear in the photographs at a different scale than low areas, and each contour line that is drawn will be to a different horizontal scale. Such a topographic map will be a perspective. It must be transformed to an orthographic projection to produce a map of uniform scale. Machines of this type are sometimes called " two-dimensional."

In other instruments a space model of the terrain is formed as it was photographed at the instant of each exposure. By means of optical and mechanical devices the map is traced from this model and at a uniform scale. Such instruments are sometimes called " three-dimensional."

As in the case of natural vision the model gives a much better conception of what the object photographed looks like than can be obtained from viewing the photographs in the ordinary manner. When observing the stereoscopic model it is easier to identify the features, to interpret the detail, to locate the true position of the points, and to estimate relative distances, than by inspection of the individual pictures.

The principles involved in viewing and making measurements on stereo-pairs are dependent on the principles of binocular and stereoscopic vision.

5-2. Binocular Vision and Stereoscopy. A close examination of a stereo-pair of photographs would show that while in general the overlapping portions appear alike they are actually slightly different because they were taken from different viewpoints (from each end of a short base).

In natural or unaided binocular vision the interocular distance is the base; therefore when an object is observed a slightly different view of the object is seen by each eye. The two views are interpreted by the observer as a single view in which there is solidarity; also relief which is not apparant when the observer covers one eye and views the landscape with the other.

The same impression or effect of binocular vision may also be obtained artificially by viewing with a stereoscope two photographs each similar to the view seen by each separate eye in natural vision. The two photographs seem to merge or fuse into a single model of the object which stands out clearly in three dimensions, as they do in natural binocular vision. With some practice it is entirely possible to obtain this effect without instrumental aid by concentrated direct vision. For rapid examination of considerable terrain much time may thus be saved.

In unaided binocular vision there are many contributory factors which enable us to estimate distances; one of these called *stereoscopic parallax* is of particular importance in photogrammetry. As stated above a slightly different view is seen by each eye when observing an object. To prove this it is only necessary to note the apparant change in the position of a nearby object relative to a distant point when first viewed with one eye and

then with the other. This lateral displacement is because the eyes occupy different viewpoints.

In Fig. 5-1 let E and E_1 represent the left and right eyes respectively, the curves representing the retinas. $EE_1 = b$, the interocular distance (Eye Base).

If the two objects, A and C, are observed with the left eye they will be impressed on the retina at c and will appear to be in line. If observed with the right eye they will appear on the retina at a and c'. If C is observed with the left eye with reference to A it will appear on line to A, but if observed with the right eye it will appear considerably to the left of A, that is, C is apparently displaced angularly with respect to A by an amount equal to the angle aE_1c'. This displacement or difference in direction of an object as seen with one eye and then with the other is called stereoscopic parallax. It is obvious that for the object A the difference in direction or the angle of parallax is equal to the angle β and for the object C is equal to the angle β_1. The angle α, called the *differential parallax*, is equal to $\beta_1 - \beta$. It will be noted that angle β is equivalent to the convergence of the eyes when A is observed. The angle of parallax β_1 defines the distance EC while the angle of parallax β defines the distance EA. Hence it is seen that the angle of parallax is associated with distance and may be considered as a measure of distance. A point subtending a greater angle of parallax is nearer to the observer than a point subtending a lesser angle of parallax. The differential parallax ($\alpha = \beta_1 - \beta$) is a measure of the difference in the distances EA and EC, or the distance AC. The above principle, as will be seen later, is employed in

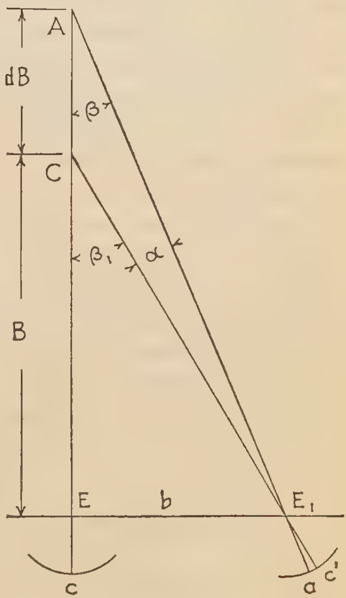

FIG. 5-1.

some stereoscopic instruments for determining the relative distances between points from stereo-pairs of photographs.

Stereoscopic perception enables us to observe solidity and distance only within certain limits. Beyond those limits stereoscopic perception ceases. To illustrate the principle assume in Fig. 5-1 that AC represents the side of a building. An observer comparatively nearby will perceive that the side AC is solid and he will be able to estimate the distance between A and C fairly accurately. As he moves farther and farther away from the building the distance AC or width of building will apparently become less and less. Eventually he will reach a point where he cannot perceive any width or solidity at all.

There is a great difference in the ability of different people to observe parallax. Some can detect a difference of as little as 5 seconds in parallax angle, the average observer can detect 20 to 30 seconds, others can detect one minute only and a few cannot see stereoscopically at all.

If the interocular distance, which also varies with different observers, is taken as $2\frac{1}{2}$ inches, and the parallax difference is taken as 20 seconds, then the distance beyond which no stereoscopic perception with the unaided eye is possible is approximately 2150 ft., as demonstrated below:

In Fig. 5-1,

$$B = \frac{b}{\tan \beta_1} = \frac{2.5}{12 \times .000097} = 2150 \text{ ft. (approx.)}$$

When pictures are to be used for stereoscopic measurements, it is desirable to have a long base line; i.e., one that will give a relatively large angle of intersection between the rays fusing the conjugate images.

5-3. Extending Range of Stereoscopic Vision. The range of natural stereoscopic vision may be increased by increasing the eye-base artificially by interposing a system of mirrors or prisms, thus increasing the angles of parallax approximately in the ratio $\frac{D}{b}$ (Fig. 5-2). Thus the interocular distance is now apparently D, and the object appears to be viewed from E with one eye and E_2 with the other eye.

A second way of increasing the stereoscopic range is by magnifying the field of view by a system of lenses. The parallactic angles are thereby increased in direct proportion to the magnification. In prismatic field binoculars and in range finders a combination of both methods is used.

Assuming that the interocular distance b is $2\frac{1}{2}$ inches and that by means of prisms or mirrors this is increased to $12\frac{1}{2}$ inches and that lenses are introduced to magnify the field twice, then the effective range will be increased $\frac{12.5}{2.5} \times 2 = 10$ times. The corresponding distance in which depth could be observed would under these conditions be $2150 \times 10 = 21500$ feet.

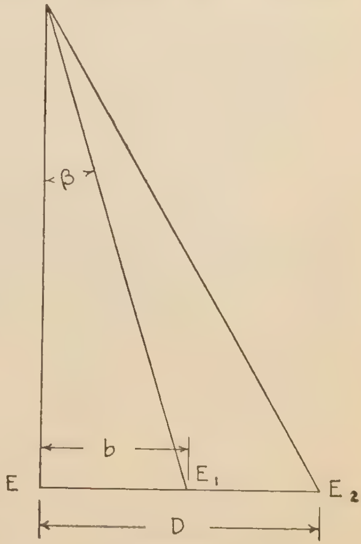

FIG. 5-2.

5-4. Stereoscopic Fusion. When looking into a stereoscope at a pair of aerial pictures, the observer soon perceives what is called stereoscopic depth, that is, he notices the tops of mountains appear to be much nearer to him than their bases, and that even slight differences in elevation can be detected. Similarly in ground photographs, objects in the foreground appear close to the observer. The following devices (dots in Fig. 5-3 and pyramids in Fig. 5-5) are used to illustrate this effect.

In Fig. 5-3, the pairs of dots, aa' and bb', represent corresponding points in two overlapping pictures. The pair, aa', may be assumed to be on the datum plane of an aerial photograph or in the far distance on a ground photograph. By looking with the left eye at the left dot and with the right eye at the right dot, it is not difficult to cause these images to "fuse" into a single image. If one has difficulty with this he may copy these dots on the top edge of a piece of paper and hold it up a foot or

so away and focus the eyes to look at various objects beyond the card. When the eyes are looking at objects the right distance

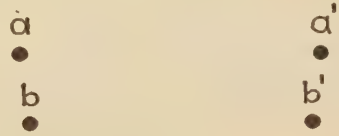

FIG. 5-3. STEREOSCOPIC FUSION.

away suddenly the two dots will appear to coincide. In fusing the dots three images usually appear. Those on the sides (satellites) should be disregarded.

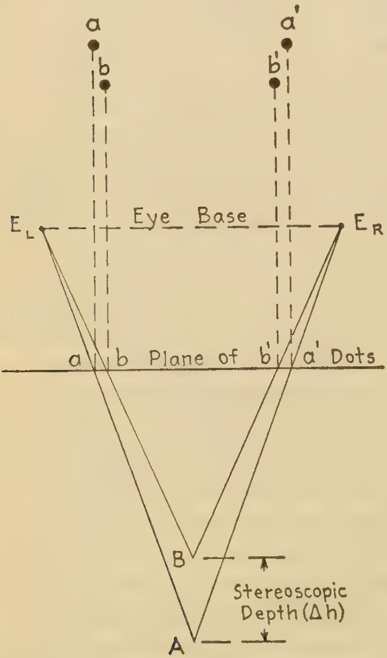

FIG. 5-4. PRINCIPLE OF STEREOSCOPE.

The observer will notice that the fused dot from the pair bb' will appear closer to the eye than the fused dot aa'. This is an illustration of depth perception obtained by varying the stereoscopic parallax. The physiology of this phenomenon is shown geometrically in Fig. 5-4. This diagram is a vertical section where E_L and E_R are the left and right eyes, a and a' one pair of dots and b and b' the other pair. When the dots are fused, aa' appears to be at a point A below the plane of the page and bb' to be at B.

The appearance of stereoscopic depth (Δh) is given by the difference between angles of parallax at A and at B. The difference between these angles is directly related to the difference between the lengths aa' and bb'. This difference, ΔP, is the quantity usually measured when determining elevation

differences from aerial photographs or when obtaining horizontal distances from the camera stations on ground photographs.

FIG. 5-5. STEREOSCOPIC FUSION.

Fig. 5-5 is a stereogram of a square pyramid, that is, two photographs of the same object taken from different viewpoints. When fused in the same manner as the dots, the fused image will

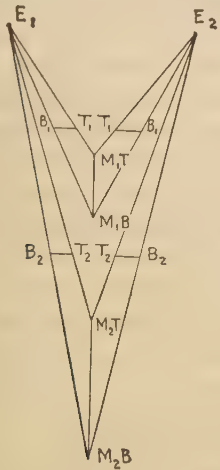

FIG. 5-6.

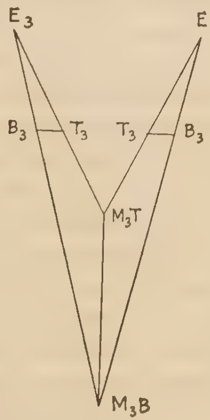

FIG. 5-7.

appear as a solid pyramid, the vertex appearing nearer to the observer than the base. It will be noted in the stereogram that the distance between the vertices is less than the distance between the centers of the bases. Had the reverse been the case, the base of the fused image would appear to be nearer the observer than the vertex. This is similar to what appears in stereo-

scopic pairs of photographs. In aerial photographs mountain peaks appear higher (nearer) than their bases; in terrestrial photographs the objects in the foreground appear nearer than distant objects. For example, if the views of the hill in Figs. 4-15 and 4-16, are observed stereoscopically the hill will appear in three dimensions with the summit S closer to the eye than the base x–y. In viewing a stereogram it is advisable to tilt the diagram slightly to left or right until a position is found which gives the clearest image. This will occur when the stereoscopic axis or imaginary line connecting the two images becomes parallel with an imaginary line connecting the pupils of the eyes.

5-5. Stereoscopic Depth. There is a difference between stereoscopic perception and the apparent height of a stereoscopic image. Stereoscopic perception is associated with the ability to perceive differences in parallax and is attained by viewing the model under high magnification. The apparent depth of the image depends upon the relation between the angles at which the objects were photographed from two camera stations and the angles at which they are viewed stereoscopically. The viewing angles may be varied by changing the separation between the pair of photographs being viewed or by varying the viewing distance.

The relief will appear normal when the eye base is to the picture base (picture separation) as the viewing distance (eye to picture) is to the focal length of the taking camera. The exaggeration of relief may be expressed approximately as follows in which M is the ratio of magnification.

$$\frac{\text{Eye base}}{\text{Picture base}} M = \frac{\text{Viewing distance}}{\text{Focal length}} \, *$$

For example, if the eye base is 2.5 inches, the picture base 3.5 inches, the viewing distance 14 inches and the focal length 6 inches, the exaggeration of depth will be $\frac{2.5}{3.5} M = \frac{14}{6}$; $M = 3+$.

Fig. 5-6 illustrates the effect on depth of increasing viewing

* If a lens stereoscope is used, the right-hand side of the expression should be divided by the magnification of the lens.

distance when identical pictures are viewed at a fixed separation in the same stereoscope. The two points T_1 and B_1 may be considered as representing the top and bottom of a pole, respectively, as they appear in the left and right pictures. The stereoscopic image of the pole at the nearer viewing distance will be at $M_1T - M_1B$. If the viewing distance is increased by moving the

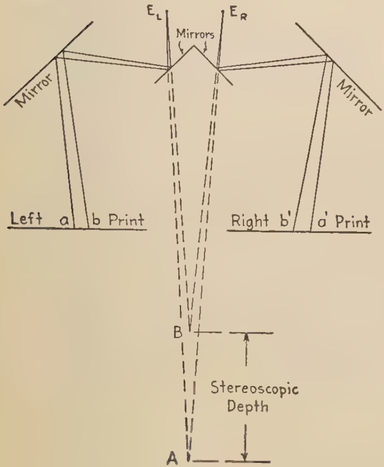

FIG. 5-8. MIRROR STEREOSCOPE.

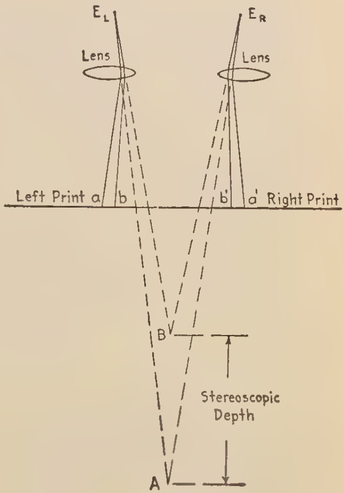

FIG. 5-9. LENS STEREOSCOPE.

pictures farther from the eyes but at the same separation as at B_2T_2, the depth of the image will increase to $M_2T - M_2B$.

Fig. 5-7 illustrates the effect on depth of moving the pictures apart. The images of the pole B_3T_3 are of the same length and at the same viewing distance as in the upper part of Fig. 5-6, but they have been moved farther apart. The eye base E_3E_4 is also the same as E_1E_2 in Fig. 5-6. The depth of the image, however, has been greatly enlarged to $M_3T - M_3B$.

The pictures may be spread apart until the axis of the eyes are actually diverging and the depth of image will continue to increase. The image which the brain sees is not the actual geometric image as constructed on paper; it is an image translated from

the difference in angle of viewing, either converging or diverging.

Increasing stereoscopic perception by magnification is limited by the grain of the emulsion of the photograph. The optimum magnification appears to be about seven times. Below this magnification the eye is being deprived of maximum parallax, and above this magnification the image becomes vague because the brain finds it difficult to fuse an image consisting of a non-homogeneous distribution of grain particles.

The stereoscopic view is usually greatly exaggerated in depth, giving an unnatural appearance to the terrain because the slopes look much steeper than they really are. The appearance of the relief can be flattened by rotating the pictures in the stereoscope through the same angle about the principal point of one picture and the conjugate image of the same point in the other picture. The relief can be reduced as much as desired until it becomes zero when the pictures have been rotated through 90 degrees.

5-6. Stereoscopes. When overlapping aerial or terrestrial photographs are to be viewed stereoscopically, it is usually impractical to do so with unaided eyes. An instrument designed to assist an observer in combining the images from a stereo-pair of pictures into a single view having the effect of solidity or depth is called a *stereoscope*. Stereoscopes are commonly of two types: lens or mirror. Sometimes the two types are combined.

FIG. 5-10. LENS STEREOSCOPE. (Courtesy, Abrams Instrument Corporation.)

Fig. 5-8 illustrates the principle of a mirror type stereoscope and Fig. 5-9 illustrates that of a lens or "direct vision" stereoscope. In aerial photographs 9 by 9 inches in size with a 60% overlap (60% of the area of one photograph included in the other photograph of a stereo-pair), the entire stereoscopic model may be viewed in

the mirror stereoscope, but only a portion of the overlap may be viewed with one setting of the lens type.

For most satisfactory use, the alignment of stereo-pairs of pictures should be done carefully. Corresponding image points should be separated by the amount for which the instrument is designed. The lens stereoscope shown in Fig. 5-10 calls for a

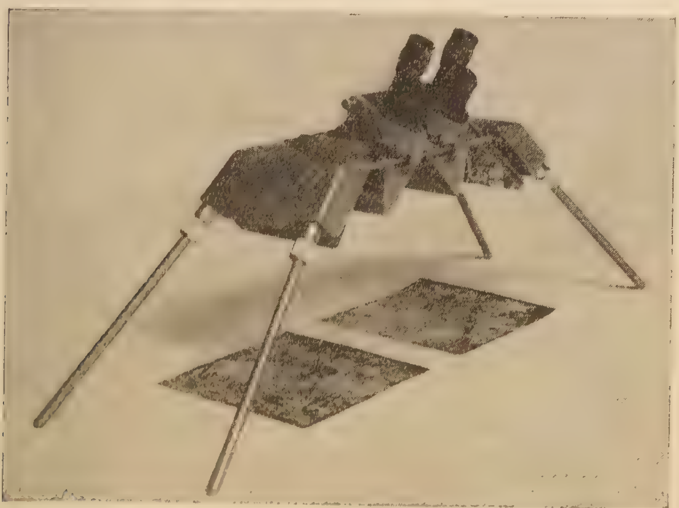

FIG. 5-11. MAGNIFYING MIRROR STEREOSCOPE.
(Courtesy, Société d'Optique et de Mécanique de Haute Précision — SOM.)

separation of $2\frac{1}{2}$ inches and the mirror stereoscope shown in Fig. 5-11 requires a separation of $8\frac{1}{4}$ inches. The alignment of the photographs is best done through use of the perspective centers and their conjugate images. In an aerial photograph taken with the optical axis truly vertical, the perspective center is at the center of the photograph. This point (principal point) is found through use of fiducial marks placed either near the corners or along the sides. In Fig. 5-12 p_1 is the perspective center of the vertical aerial photograph I and p_2 is the perspective center of photograph II. p_2' is the image on photograph I of p_2, and p_1' is the image on photograph II of p_1. p_2' and p_1' are often called conjugate images of the principal points.

A satisfactory method of aligning these photographs for stereoscopic examinations is to take a sheet of drawing paper, say of

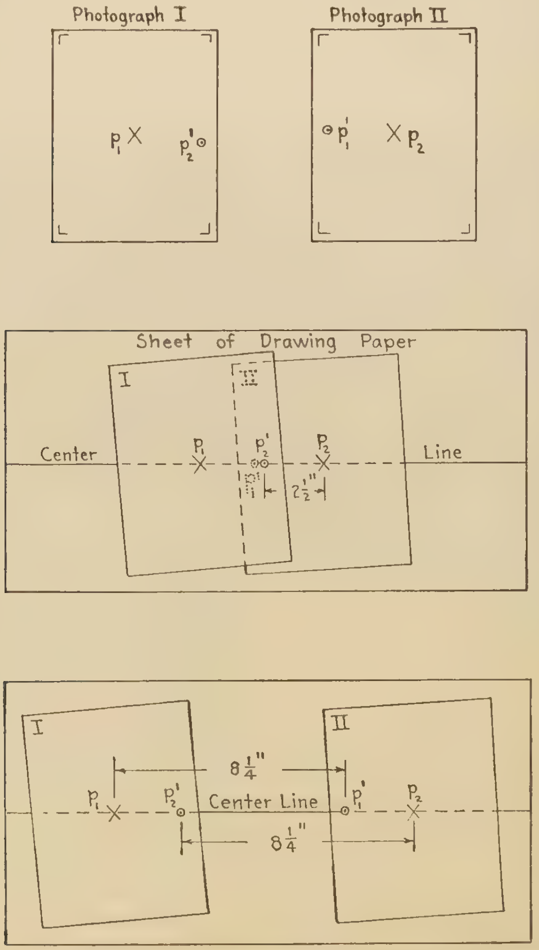

FIG. 5-12. ARRANGEMENT OF PHOTOGRAPHS FOR VIEWING IN STEREOSCOPES.

size 11″ by 22″, and draw a bisecting line parallel to the long dimension. The principle to be followed is to have the perspective centers and their conjugate images fall along this line and

to have corresponding images such as p_1 and p_1' separated by the amount required by the stereoscope. The axis of the stereoscope should be placed parallel to the edge of the sheet of drawing paper. The middle view in Fig. 5-12 shows photographs aligned for examination by means of a lens stereoscope and the lower view Fig. 5-12 shows the same pair adjusted for a mirror stereoscope.

When examining photographs for details, it will be found that the stereoscopic model will render the identification of details easier than if the photographs were examined singly. The value of the relief impression gained cannot be overstated. When transferring points from one photograph to another, as is the case in the preparation of a map by the radial line method, the identification of the conjugate images may be done with greater accuracy and certainty when a stereoscope is used.

5-7. Application of Stereoscopic Principles to Aerial Photogrammetry. In applying this principle to aerial photogrammetry, two overlapping photographs are taken at the ends (*air stations*) of a base-line in the air (or air base). If the specification of 60% end lap is followed, 60% of one photograph will appear in the other and this common portion may be viewed stereoscopically.

Fig. 5-13 shows a stereo-pair of vertical aerial photographs. It is assumed that these photographs have been taken with the camera axis truly vertical and that the air stations, O_1 and O_2, are at the same elevation, H. The distance O_1O_2 is the air base of length B. A point above the datum, such as A, appears on photograph I at a_1, and on photograph II at a_2. The x-axes of the photographs are assumed to lie along the flight line.

The total parallax of point a is given by the *algebraic* difference between the coordinates parallel to the air base. Thus, in Fig. 5-13, the total or " absolute " parallax, P, is

$$P = x_1 - x_2 \tag{5-1}$$

From similar triangles in Fig. 5-13 it will be seen that the air base $(X_1 + X_2 = B)$ is to the picture base $(x_1 + x_2 = P)$ as

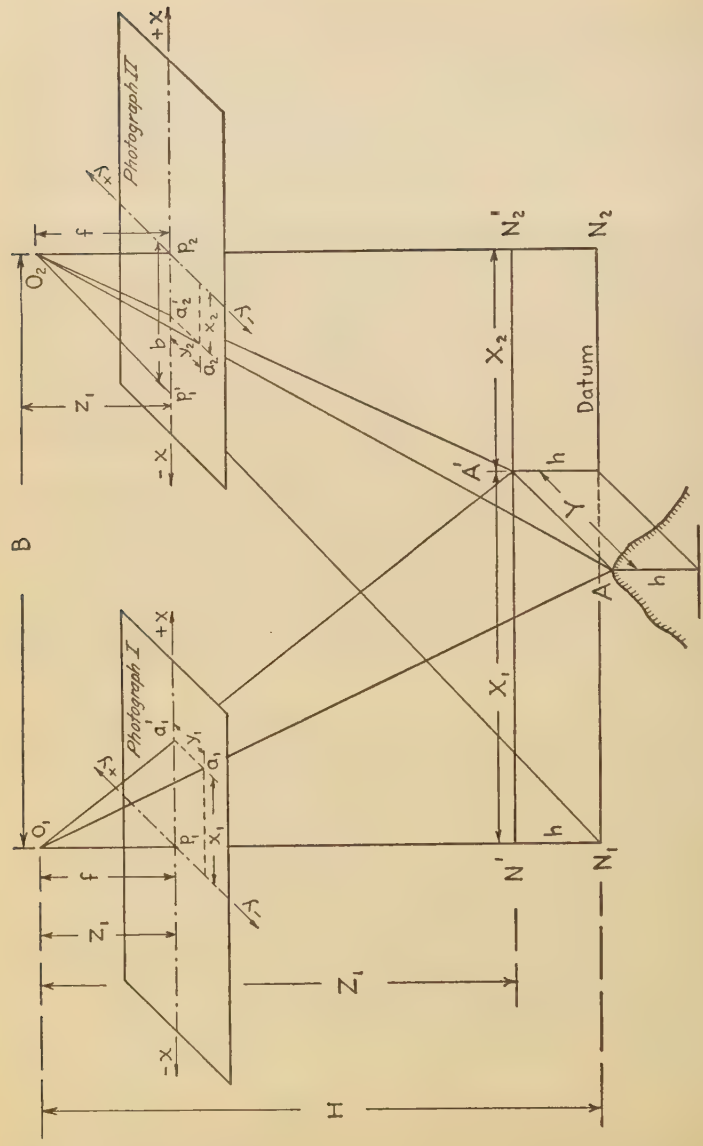

FIG. 5-13. DERIVATION OF PARALLAX RELATIONSHIPS.

the elevation above the ground $(H - h)$ is to the focal length (f). Thus

$$\frac{B}{P} = \frac{H - h}{f} \tag{5-2}$$

and

$$P = \frac{Bf}{H - h} \tag{5-3}$$

Any distance on the ground in the horizontal plane containing point A will, in the picture, be the ground distance multiplied by the scale $\frac{f}{H - h}$.

Thus

$$x_1 = \frac{f}{H - h} X_1 \tag{5-4}$$

$$y_1 = y_2 = \frac{f}{H - h} Y_1 = \frac{f}{H - h} Y_2 \tag{5-5}$$

and

$$z_1 = \frac{f}{H - h} Z_1 = \frac{f(H - h)}{H - h} = f \tag{5-6}$$

Referring to photograph II, Fig. 5-13, the distance $p_1'p_2 = b$ represents the distance on the photograph between the principal point of photograph II and the conjugate item of the principal point of photograph I, when these points are referred to the datum plane. Then

$$\frac{b}{B} = \frac{f}{H}$$

or

$$Bf = bH$$

Hence

$$P = \frac{bH}{H - h} \tag{5-7}$$

This last equation (5-7) is the basic relation for stereoscopic parallax applied to aerial photographs.

If P_a is the total parallax of a point in the datum plane and P_{a1} is the total parallax in the plane of desired elevation, the parallax difference $\Delta P = P_a - P_{a1}$. This parallax difference is

the sum of the two x components of the relief displacement of the two displaced images referred to the plane from which it is desired to measure the elevation. It is a manifestation of change of scale between horizontal planes which lie at different distances away from the camera. In the individual pictures the images of the pole are displaced by the quantity $\frac{hr}{H}$, as explained in Art. 4-25.

Fig. 5-14 represents a normal pair of overlapping aerial photographs, the left one being designated as I and the right one being

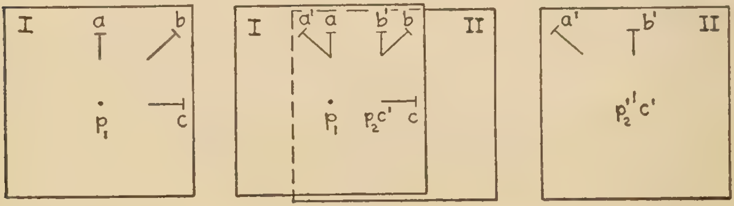

FIG. 5-14. DEMONSTRATION OF PARALLAX IN OVERLAPPING AERIAL PHOTOGRAPHS.

designated as II. Three telegraph poles, A, B and C, have their bases in one horizontal plane and their tops in a higher horizontal plane. The three poles are the same height.

In the center of the illustration, the two pictures are shown as superimposed, with the images of the base of the poles matched. The images of the top of the poles are separated by the equal quantities aa', bb' and cc'. This quantity is the parallax difference ΔP between the top of the poles and the plane of the base of the poles. It may be noted that the parallax direction is always parallel to the picture center line p_1p_2.

An expression for difference in height h in terms of differences in parallax may be developed from Fig. 5-15, which is a view of Fig. 5-13 in the plane of the vertical through the camera stations.

From similar triangles

$$\frac{h}{H} = \frac{A''A_1''}{B_1 + A''A_1''} = \frac{A''A_2''}{B_2 + A''A_2''}$$

Also $\dfrac{f}{H} = \dfrac{b_1 + \Delta P_1}{B_1 + A''A_1''} = \dfrac{\Delta P_1}{A''A_1''} = \dfrac{b_2 + \Delta P_2}{B_2 + A''A_2''} = \dfrac{\Delta P_2}{A''A_2''}$

From which
$$\frac{A''A_1''}{B_1 + A''A_1''} = \frac{\Delta P_1}{b_1 + \Delta P_1}$$

and
$$\frac{A''A_2''}{B_2 + A''A_2''} = \frac{\Delta P_2}{b_2 + \Delta P_2}$$

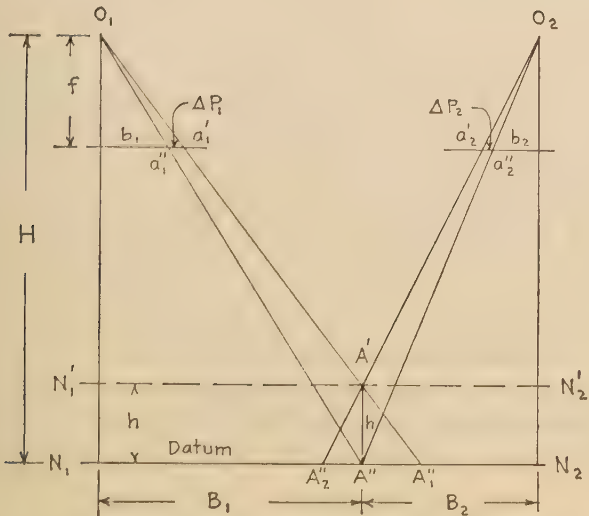

FIG. 5-15. RELATION BETWEEN PARALLAX AND HEIGHT OF OBJECT.

So $\dfrac{h}{H} = \dfrac{\Delta P_1}{b_1 + \Delta P_1} = \dfrac{\Delta P_2}{b_2 + \Delta P_2} = \dfrac{\Delta P_1 + \Delta P_2}{b_1 + b_2 + \Delta P_1 + \Delta P_2}$

The total parallax difference $\Delta P = \Delta P_1 + \Delta P_2$, and the picture base $b = b_1 + b_2$.

Therefore
$$\frac{h}{H} = \frac{\Delta P}{b + \Delta P} \text{ and } h = \frac{H \Delta P}{b + \Delta P} \qquad (5\text{-}8)$$

For measuring parallax and determining the height of an object from the photograph, a common practice is to select the lowest point of known elevation in the picture and to consider a

horizontal plane through this point as datum. If no known elevation is available, one may be assumed. If the only known point of elevation happens to be a high point, the datum may be taken through this point and the elevations of other lower points obtained by subtracting differences of elevation instead of adding them. It is not necessary to select sea level as the datum plane for determining differences in elevations. The height of airplane H must, of course, be the height above the datum used.

In preparation for measuring parallaxes the pictures should be oriented as illustrated in Fig. 5-12. The value b is derived by measuring in each picture the distance between the conjugate images of a point at datum elevation and perpendiculars constructed from the respective flight lines at the principal points, and adding these two amounts; i.e., $b_1 + b_2$ in Fig. 5-15. ΔP is the difference in parallax of the conjugate images of a datum point and the conjugate images of the desired point at elevation h above datum. These distances between the two sets of conjugate image points actually may be measured in total across parts of the two oriented pictures and the intervening space, and ΔP taken as the difference in these two measurements. Only the difference, however, is significant and it can be measured directly by special instruments described later in this chapter.

For example, if $\Delta P = 2.31$ mm. and $b = 90$ mm. and $H =$ 8600 ft.

$$h = \frac{2.31 \times 8600}{90 + 2.31} = 215 \text{ ft. above datum}$$

5-8. Locating Contours from Parallax Measurements. In the above example the parallax difference for each foot of elevation is $\frac{2.31}{215} = .0107$ mm. This is not a constant for all elevations because parallax changes with each elevation. However, the relation may be assumed constant without any significant error over a range of 1/20th of the camera height H.

If the elevation of the datum used for the pair of pictures in the above example is 246 ft. above sea level, the parallax difference reading for the 260 contour with reference to datum at elevation 246 is $(260 - 246)\ .0107 = .149$ mm.

A table of parallax differences for 20-foot contours on this pair of pictures may be prepared as follows:

Contour Elev.		ΔP
260		0.149 mm.
280	$0.149 + 20(.0107) =$	0.363
300	$0.149 + 40(.0107) =$	0.577
	etc.	
460	$0.149 + 200(.0107) =$	2.289

At this elevation a corrected value is computed for the parallax per foot of elevation. The limit of $1/20\ H = 430$ ft. is assumed to apply for 215 ft. above and below the base elevation of 246 ft. Therefore $246 + 215 = 461$ ft. Thus the 460 contour is as far as the value of .0107 mm. per foot should be used. A new value is computed for elevation $246 + 430 = 676$ ft., or for the nearest contour interval of 680 ft., as follows:

$$h = 680 - 246 = 434 = \frac{\Delta P \times 8600}{90 + \Delta P}$$

From which $\Delta P = 4.78$ mm.

The parallax difference per foot for this value of ΔP is $4.78 \div 434 = 0.0110$ mm. per foot.

The parallax table is then continued:

Contour Elev.		ΔP
480	$4.78 - 200(.0110) =$	2.58 mm.
500	$4.78 - 180(.0110) =$	2.80
	etc.	
680	$4.78 - 0(.0110) =$	4.78
700	$4.78 + 20(.0110) =$	5.00
	etc.	
880	$4.78 + 200(.0110) =$	6.98

A corrected value for the parallax difference per foot will be used for contours 900 to 1300, computed at elevation $246 + 2(430) = 1106$; i.e., at the 1100 contour. The foregoing process

may be continued until the reading for highest contour interval required is obtained.

In order to draw a contour, the parallax difference for the desired contour elevation is set on the micrometer screw of the parallax measuring instrument, and the floating mark or dot in the instrument (see Art. 5-13) is brought in contact with the stereoscopic image at one side of the model. The floating dot is then moved over the model following the undulations of the image so that it always appears to lie exactly in the surface of the image. A pencil attached to the mechanism moves with the floating mark and draws in the contour in conic projection. When one contour has been traced, the micrometer is set for the next contour elevation, above or below, and the process is repeated.

Other features, such as roads and streams, are drawn by causing the floating mark or dot to follow along the feature. At the same time the micrometer screw is turned to keep the dot rising or falling so that it appears to stay on the surface of the model. Buildings will usually be drawn at the same parallax setting as for the nearest contour.

Contouring by parallax methods is rather difficult and complicated. Successful results cannot be obtained unless the pictures are free from tilt. The pictures must be carefully rectified or a dense pattern of control points must be available from which a graph may be prepared giving the various corrections required to parallax settings in different parts of the picture. The contours obtained are form lines rather than true contours since each contour is drawn to a slightly different horizontal scale. A further correction is needed to bring all contours to a uniform scale.

Parallax measurements are useful for measuring heights of objects such as trees. The images of the top and bottom of the tree are very close together and for most purposes may be considered to have no relative error due to tilt or lens distortion.

5-9. Parallax Ladder. A simple form of stereoscopic measuring device is the parallax ladder (Fig. 5-16). This consists of a piece of transparent celluloid on which are printed two black lines about 10″ long. These lines will be 2 inches apart at one

end, and 2.5 inches apart at the other. The rungs of the ladder are placed in steps at each point where the separation of the parallax lines increases .02″, and in between these are four light tick lines which represent .005″ each. On each rung the separation of the parallax lines in inches and 100th inches is printed near the right-hand side of the ladder.

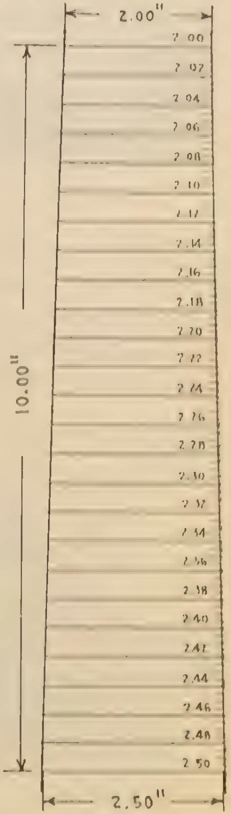

Fig. 5-16. Parallax Ladder.

The ladder is used on a pair of pictures placed under a direct vision stereoscope so that conjugate images are about $2\frac{1}{4}$ inches apart. To measure the parallax of a pair of images, the ladder is moved up or down over the images until a position is reached where the separation of the two sides of the ladder is the same as that of the conjugate images. At this point the fused side lines of the ladder will appear to touch the ground in the stereoscopic model. The parallax is read from the scale at this point.

To measure the height of a tree, for example, the ladder is moved up and down on the stereoscopic model until the fused side lines or parallax line (which appears to be rising steeply) intersects the stereoscopic image of the base of the tree and the parallax read from the scale, such as 2.20 inches. The ladder is then shifted until the parallax line appears to just touch the top of the tree and another reading taken, such as 2.05 inches. The height of the tree, then, is represented by a parallax difference of 0.015 inches. If the picture base is 3.6 inches and the height of the airplane is 15,000 ft., the height of the tree is $\frac{.015}{3.6} \times 15{,}000 = 62.5$ ft.

5-10. Height Finder. An instrument known as the height finder has been devised for measuring parallax differences on

stereo-pairs of aerial photographs. It consists of an attachment (Fig. 5-17) which is placed under a lens stereoscope (similar to Fig. 5-10). The floating mark consists of two dots etched on two

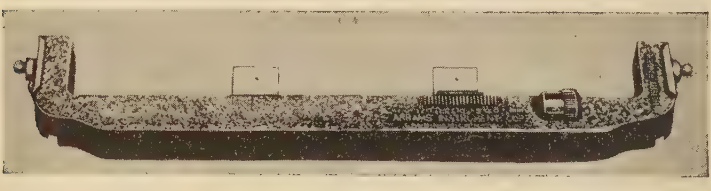

FIG. 5-17. HEIGHT FINDER.
(Courtesy, Abrams Instrument Corp.)

pieces of glass. Parallax distances up to 20 mm. may be measured on the micrometer drum.

5-11. Ryker and KEK Plotters. The Ryker Plotter (Fig. 5-18) consists of a mirror stereoscope viewing paper photographic

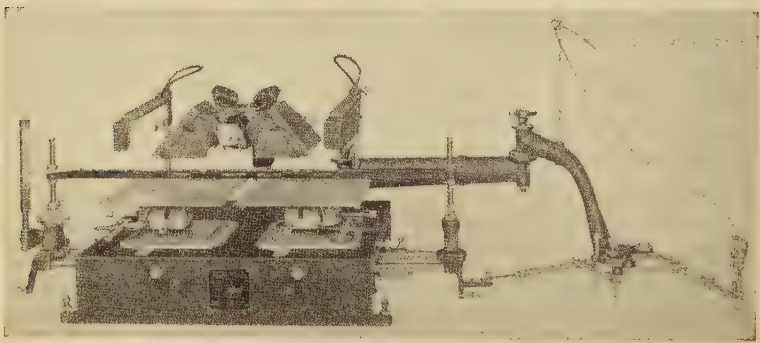

FIG. 5-18. RYKER PLOTTER.
(Courtesy, Harrison C. Ryker, Inc.)

prints supported on two tables, which are adjustable in swing (about vertical axis), x-tilt and y-tilt, and in the "x" and "y" directions in a horizontal plane. The two floating marks are carried on two pieces of glass which move between the pictures and the stereoscope. Change of elevation is effected by raising and lowering the floating marks. Contours and planimetric data

are drawn by causing the floating marks to follow the image in the same manner as with other machines. The floating marks are supported by a bridge-like mechanism which is operated by two hand wheels on its supports. The suspension of this bridge is very delicate mechanically so that the motion is smooth and easy. The movements are translated to a pantograph which permits the scale to be varied from 0.38 to 2.5 times the scale at which the floating marks move in the plotter.

The instrument is most appropriate for mapping from pictures taken with an $8\frac{1}{4}''$ focal length camera. Due to the approximate nature of the tilt correction certain errors are introduced when the viewing distance, from eye to picture, is not identical to the focal length of the taking camera. This variation in distance causes a difference between the viewing angle and the taking angle. The residual error from this cause limits the accuracy of the instrument to drawing contours at intervals of approximately 1/400th of the airplane altitude. The Ryker Plotter is compact and easily portable.

The KEK Plotter is a type of instrument that is similar in principle and construction to the Ryker, except that elevations are measured by raising or lowering the two photo holders rather than by moving the floating marks. The KEK Plotter is used exclusively by the U. S. Forest Service.

With respect to precision and cost these machines lie in between the two-dimensional types, such as the height finder, and the fully correcting types such as the Kelsh or Multiplex.

5-12. Stereotope. The Zeiss Stereotope, Fig. 5-19, is a device for measuring parallax from the prints and plotting contours at scales from three- to five-times magnification by means of a pantograph. Four horizontal and vertical control points are required in each stereoscopic model.

5-13. Anaglyphs. An anaglyph is a kind of picture produced when the overlapping portions of a stereo-pair of photographs are printed *one over the other* in complementary colors. When viewed through spectacles of like complementary colors a model is observed in the third dimension that is most pronounced.

The complementary colors commonly used are blue-green and red. If one print of the superimposed pair is printed in blue-

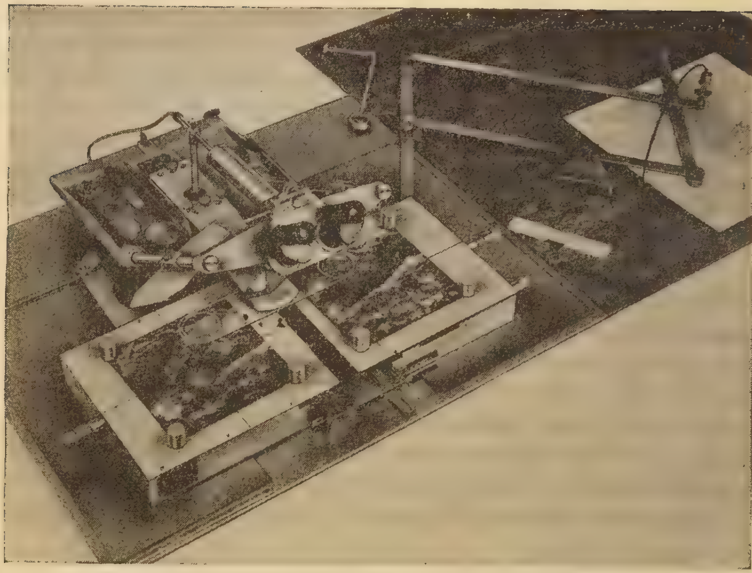

FIG. 5-19. STEREOTOPE.
(Courtesy, Zeiss-Aerotopograph)

green and the other print is red, then in order to see a three-dimensional model the anaglyph is viewed with spectacles (called a " Macyscope ") through a red filter by one eye and through a blue-green filter by the other eye.

The underlying principle of the anaglyph method is based on the reproduction of the conditions of binocular vision. In Art. 5-4 it was stated that the effect of binocular vision may be reproduced artificially by viewing two photographs each similar to the view seen by each eye in natural vision. From the anaglyph we obtain the two necessary views by means of the filters; that is, the filters separate the two superimposed prints so that each eye will see the picture taken from its own viewpoint only. The left eye can see only one of the colored pictures and the right eye can see only the other colored picture through the spectacles.

Fig. 5-20 is an anaglyph of a section of terrain. When viewed through the accompanying spectacles or filters (blue-green to

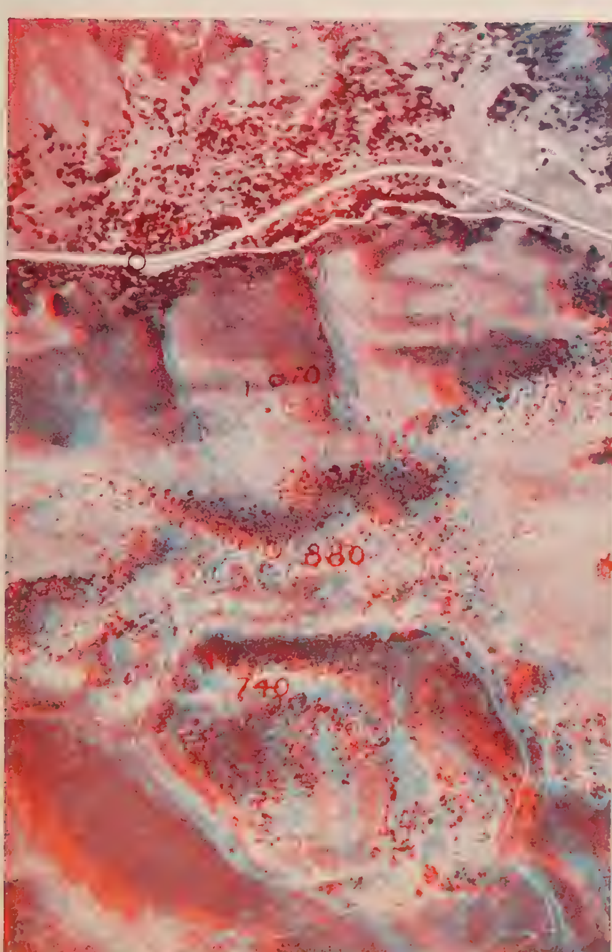

Fig. 5-20 Anaglyph.
(Courtesy, Corps of Engineers, U. S. Army)

the right eye and red to the left eye) a spatial model, in three dimensions, is observed. If the spectacles are reversed or if the anaglyph is viewed when upside down, what is called a pseudoscopic model is obtained: the hills will appear as valleys and vice versa. When viewed from the right or left side no stereoscopic effect will be noticed.

The effect of relief may be accentuated by holding the filters farther away and diminished by holding the filters closer to the anaglyph.

The " Macyscope " for viewing Fig. 5-20 will be found encased in an envelope on the inside of the back cover of this book. To obtain the best results, the anaglyph should first be placed in a strong light. The spectacles should then be placed in front of the eyes with the book about 10″ away; then move the head slowly back until the relief of the hills and valleys is perfectly distinct. If one then moves the head slowly in a direction parallel to the page, the tops of the hills will appear to move and the shape of the hills will change somewhat.

The circle on the road in the view should appear to be directly **on the surface** of the ground; the circle to the left of the elevation marked " 880 " should also appear to be resting **on the ground.** About $\frac{1}{4}''$ to the left of the elevation marked " 740 " a black arrow " V " will be seen. When the view appears to be in **perfect focus** through the spectacles, this arrow should appear to be **floating above** the surface of the ground. This floating point and its significance are later described in Art. 5-15.

5-14. Kelsh Plotter. Almost all three-dimensional stereoscopic machines re-create a stereoscopic model in miniature of the terrain as viewed when the exposures were made. The two projection cameras of the machine are adjusted to the same spatial orientation as existed at the time of two successive exposures of the taking camera in the air, except that all ordinates on the model are reduced in the ratio of the plotting scale to the natural scale. If the plotting scale is 1:10,000, and the air base line was 10,000 feet long, the base line in the plotting machine will be 1 foot. If the altitude of the air base was 15,000 feet, then the altitude in the machine will be 1.5 feet. Adjustments are made in the machine about the x, y and z axes of the projection

cameras to obtain the same angles and orientation which existed in the air camera at the time exposures were made.

The plate holders or projection cameras of the Kelsh Plotter (Fig. 5-21) are functionally identical to the air camera. In place

FIG. 5-21. KELSH PLOTTER.
(Courtesy, The Instrument Corporation.

of the air camera negatives glass diapositives are used which were made by contact printing to exactly the same size as the original negatives. The distance from the diapositive to the interior perspective center of the projector lens is made the same length as that of the taking camera. The lenses, however, are not Metrogons, but the less expensive Hypergons. Since the dis-

tortion characteristics of the Hypergon lens differ from those of the Metrogon lens, an adjusting cam is provided for correcting distortion differences. This cam is actuated by the arms which run from the lens positions down to the plotting table. As the cam is rotated through the angle of the projected rays, it raises or lowers the lens minutely to compensate for the distortion. Distortion is a concentric change of scale, and therefore it can be corrected by slight enlargement or reduction, which is accomplished by the small adjustments to the focal length effected by the cam.

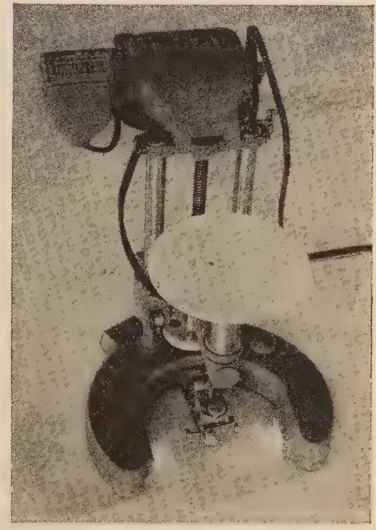

FIG. 5-22. TRACING TABLE.
(Courtesy, Bausch and Lomb Optical Company.)

The Kelsh Plotter projects its conjugate images on to a plotting screen on a tracing table, as more fully explained in Art. 5-15. Fig. 5-22 shows a tracing table and screen suitable for use with all anaglyph-type instruments. The images are projected onto the screen in complementary colors, as illustrated in Fig. 5-20, and the stereoscopic impression is received by wearing a pair of glasses with red and blue lenses.

A dot of light in the center of the screen serves as a floating mark which can be raised or lowered to the desired elevation. A pencil directly beneath the floating mark draws on the map paper over which the stand is moved by the operator as he causes the floating mark to follow the contour or planimetric feature. A pantograph attachment may be added to the Kelsh Plotter which permits a choice of drawing scales.

The complete orientation of the spatial model is achieved in two stages known as *relative* and *absolute orientation:* first, the two projectors are brought into the same relative angular re-

lationships as existed in the aerial camera when the two successive exposures were made; secondly, absolute orientation is obtained by adjusting the machine as a whole so that the scale, position and orientation of the spatial model agrees with the plotted positions of ground control points. Absolute orientation is attempted only after relative orientation has been attained.

Relative orientation is concerned with the complete removal within the overlapping area of the presence of y-parallax.* This is effected by an orderly and systematic procedure of adjustments designed to remove the y-parallax caused by swing, x-tilt, y-tilt, and difference in elevation between exposure stations. The sequence of operations is planned in such a way that after the y-parallax has been removed in one portion of the model, succeeding adjustments will have either little or no effect on the areas already adjusted. After the first sequence of adjustments has been completed, the model is checked for residual y-parallax. If any is found the entire sequence of operations is repeated until a model free from y parallax is obtained.

Absolute orientation is accomplished by first adjusting the scale of the spatial model as a whole by changing the separation of the projectors until distances on the model agree with known distances on the ground plotted to the scale of the map. The model as a whole is then rotated about axes parallel to and perpendicular to the bar supporting the projectors until the elevation readings on the screen of the tracing stand agree with the elevations computed from ground control points. When this last adjustment has been perfected, all elevations will read correctly. The map sheet is then shifted on the drawing table until all plotted control points correspond with their images. The model is then ready to draw.

The Kelsh Plotter has an advantage in the use of contact-scale diapositives, and is commonly used in aerial survey work where control extension has already been accomplished by templets and no bridging (Art. 5-22) is required. The three projector model of the Kelsh Fig. 5-23, however, permits an elementary type of extension control. The stereoscopic model represented

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* Also termed "want of correspondence" or "vertical parallax."

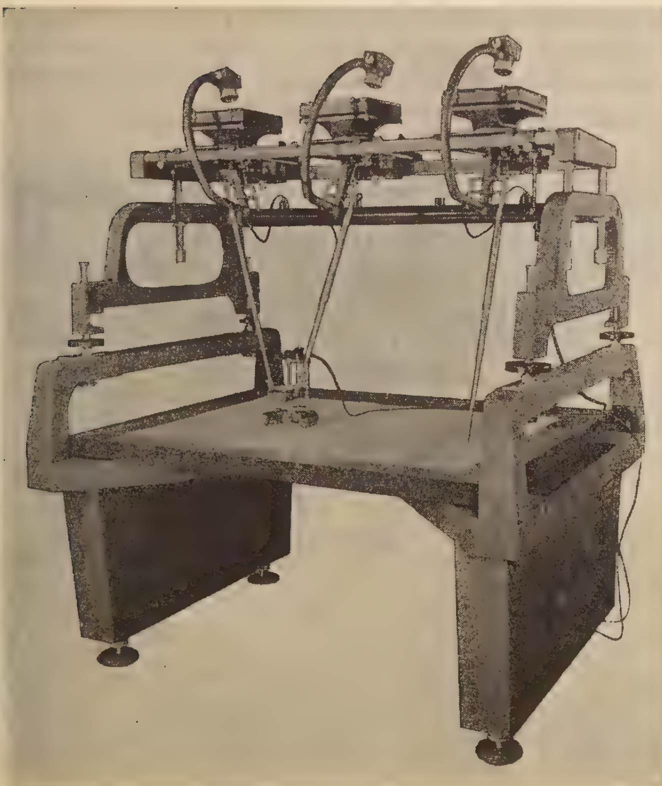

FIG. 5-23. THREE-PROJECTOR KELSH PLOTTER.
(Courtesy, Kelsh Instrument Company, Inc.)

by the first two projectors is leveled and oriented to the control. The projection of a third picture can then be oriented to the middle projector without disturbing the latter.

5-15. The Multiplex. The multiplex aero-projector, for map compilation, is an instrument in which a spatial model or image is produced by direct optical projection from a pair of overlapping photographs. For the construction of the spatial image, use is made of the anaglyphic principle.

The multiplex, Fig. 5-24, consists of two or more projection

cameras, smaller in size, but with similar characteristics to the aerial camera used in the plane.

The projection cameras are mounted on a horizontal bar (Fig. 5-24) in such a manner that each may be:

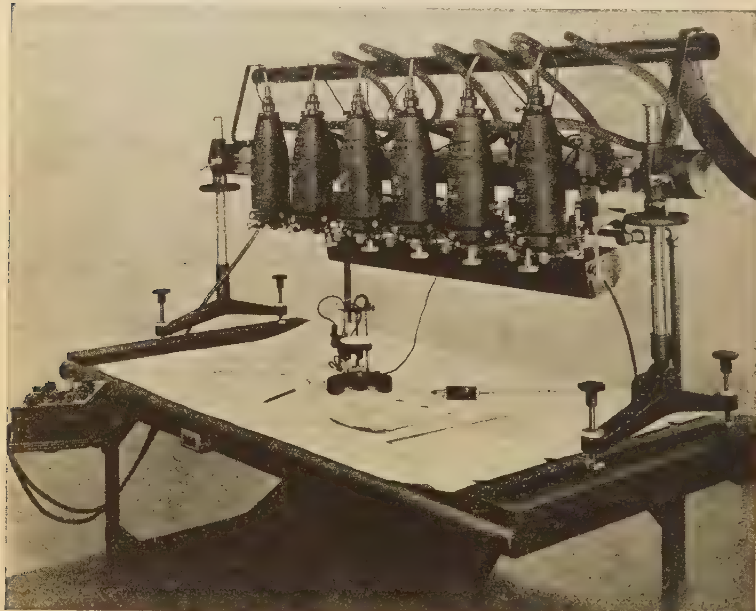

FIG. 5-24. MULTIPLEX PROJECTION CAMERAS.
(Courtesy, U. S. Geological Survey, through Bausch & Lomb Optical Co.)

(1) Moved horizontally along the bar.
(2) Raised or lowered vertically.
(3) Moved horizontally toward or away from the bar.
(4) Swung about its own principal axis.
(5) x-tilted about an axis normal to the base bar.
(6) y-tilted about an axis parallel to the base bar.

By means of these motions, which are controlled by screws, each camera may be adjusted so as to duplicate the position of the aerial camera at the instant the exposure was made. In the

cases of (1), (2) and (3) the amount of movement may be measured by means of graduated scales.

Each camera contains in addition to its projection lens:

(a) A condensing lens system in order to obtain uniform illumination of the image to be projected.
(b) A platform to support the image plate (diapositive) to be projected.
(c) A colored filter.
(d) A source of light for projecting the image on a screen.

The image plates to be projected are called diapositives. They are positives made by photographing each negative, with a specially designed reducing camera, on a glass plate. The image of the aerial negative is reduced to one quarter of the original size; i.e., the original 9×9 inch negative is reduced to $2\frac{1}{4} \times 2\frac{1}{4}$ inches.

The drawing mechanism consists of a small tracing stand supporting a plotting screen, which may be moved in space in any direction. A measuring or floating illuminated dot is provided in the center of the screen. Beneath the measuring mark is a pencil which records on the map any horizontal movement of the stand. Since the distance the screen is moved vertically is a function of the difference or elevation on the earth's surface, the measuring mark may be set to correspond to any desired elevation. By bringing the mark in contact with the surface of the ground (spatial model), and continually keeping it in contact as the stand is moved about, contours are drawn on the map by pencil. Other data shown on the model may be drawn on the map by following it in all three dimensions with the measuring mark.

5-16. Theory of Multiplex. Fig. 5-25 entitled "Schematic Diagram of the Theory of the Multiplex" illustrates the theory of those plotting instruments which completely solve the parallax equation. Plate 1 and Plate 2 are two transparent positives made from two successive negatives of the taking camera; they are mounted in the projection cameras of the multiplex. By cut and try methods these projection cameras may be revolved and

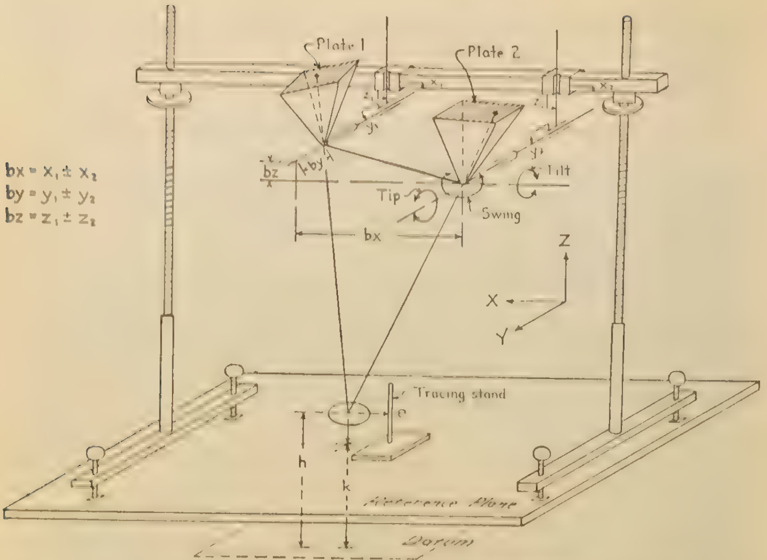

Fig. 5-25. Schematic Diagram of the Theory of the Multiplex.

moved to such positions as to bring them into exactly the same orientation as the aerial camera at the instant the exposures were made. Lines of sight through the plates of the projection cameras to all objects that are common to both plates should intersect at these objects when each camera has the correct setting. The first step is to find the setting of the cameras; the second is to cause the intersections to occur at points so that the scalar relationship between the points is true.

The reference plane or map plane need not coincide with the datum plane, but must be parallel to it. In Fig. 5-25 the reading at c above the reference plane plus a constant distance k will give the proper elevation reading h.

The Multiplex is fundamentally similar in function to the Kelsh Plotter. The chief differences are: (1) the Multiplex projection cameras have one quarter the focal length and use a reduced diapositive one quarter the size of the original 9″ × 9″ negative, (2) the Multiplex projection cameras are smaller than

the Kelsh and may be placed closer together thereby enabling plotting at a smaller scale, (3) the Multiplex can be used in bridging by stringing a number of projection cameras on a single Multiplex bar.

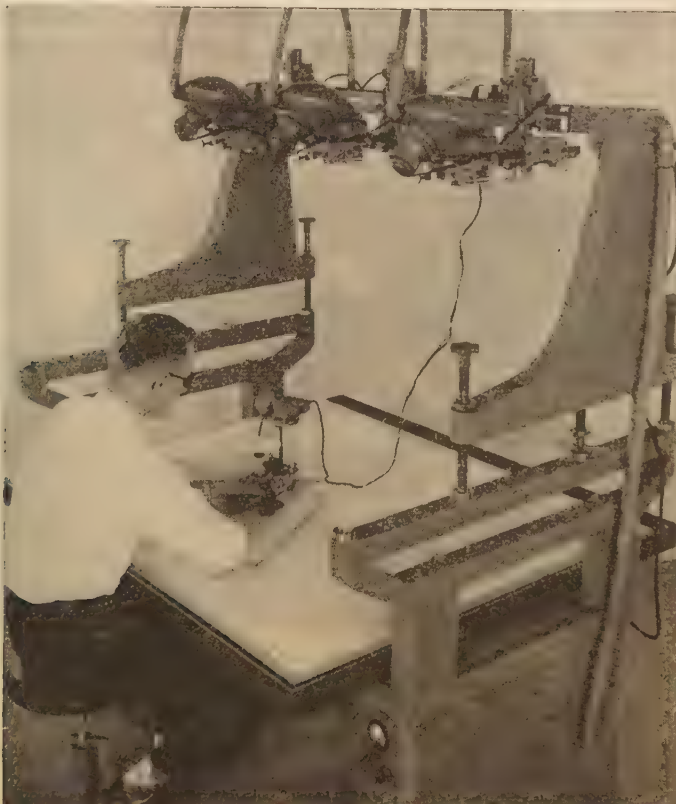

FIG. 5-26. THE BALPLEX PLOTTER.
(Courtesy, Bausch & Lomb Optical Co.)

5-17. Balplex Plotter. The Balplex Plotter, Fig. 5-26, is similar in design to the Multiplex. It projects the image of the entire model on the plotting table rather than only a small portion, scarcely larger than the tracing table, as is the case with the

Kelsh Plotter. This allows a number of projectors to be strung on a bar for aerial triangulation. The Balplex uses reduced size diapositives, 110 mm. square compared to 64 mm. in the Multiplex. The magnification over the original negative is five times. A special feature is that the projectors may be tilted for use with convergent or oblique photography, including 20° convergent photography used extensively by U. S. Geological Survey.

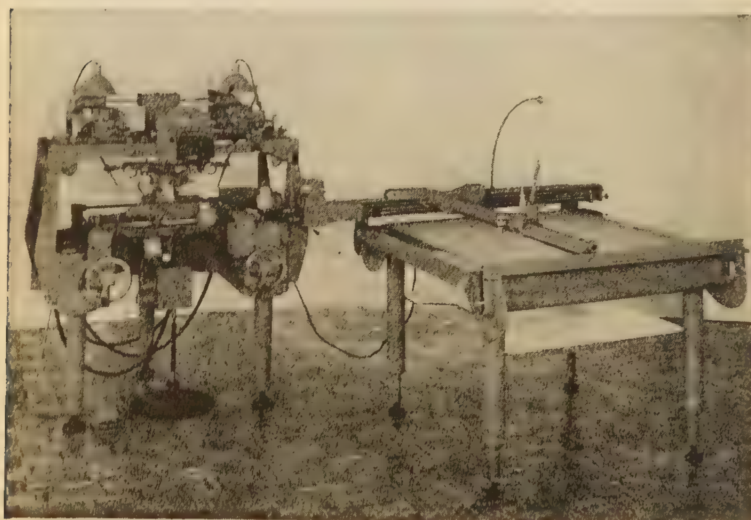

FIG. 5-27. WILD A-9 AUTOGRAPH.
(Courtesy, Wild Heerbrugg Instruments, Inc.)

5-18. Wild Autograph. In this machine (Fig. 5-27) there is no actual optical projection duplicating the angles at which the picture was taken. Instead, there are two mechanical rods representing the two rays of light from a ground point to the two lenses. They pass through a mechanical nodal point, which is adjusted at the exact focal length from the mechanical focal plane. These guide rods are secured at their lower ends to the "base" which is the mechanical representation of the ground. As the base moves in x, y and z it moves the guide rods which are always at the exact angle of the rays of light which formed

the conjugate images. Each guide rod as it moves from ground point to ground point actuates a prism which observes the corresponding point on the diapositive and brings it to the observer's eyes via a floating dot which is incorporated in the optical system.

The Wild plotting instruments include five models, A-5 to A-9. Those with odd numbers, called "Autographs", are universal types adapted to bridging (Arts. 5-21 and 5-22). The A-5 has the basic features of the Autograph line; A-7 and A-9 embody improvements suggested by experience with earlier models. Among other changes, the A-7 uses 9″ × 9″ diapositive plates instead of a reduced size. The A-9 shown in Fig. 5-27 is designed for use with super-wide-angle photography taken with short-focal length cameras, particularly the Wild RC-9a, having a focal length of 3.46 inches. Diapositives are at half the scale of the photographs. The A-6, and its successor the A-8, are less complicated stereo-plotting machines suitable for compiling maps and drawing contours from vertical photography after bridging has been accomplished by the Autograph machines.

5-19. The Stereoplanigraph. In the Stereoplanigraph, Fig. 5-28, two full size glass diapositives are inserted into projectors and their images are projected onto mirrors containing the floating marks. The stereoscopic image and the floating marks are enlarged in the viewing system giving a model magnified from five to ten times. While the Autograph represents the lens and the ray of light mechanically, the Stereoplanigraph exactly duplicates in its lens and plate holder the optics of the taking camera. An image is projected onto the floating marks located on a surface of a mirror which constantly turns the image into the observing system. Between the projection camera lens and the mirror is a telephoto lens which constantly compensates for the change in distance between lens and mirror which occurs as the machine operates. Contours are traced by a pair of hand wheels controlling the x and y motions and the elevation determined by the floating mark is set by the foot wheel.

The x and y motions are transferred to a separate table on which the map is prepared. An assistant to the operator raises and lowers the pencil point when required and helps edit the map.

The Stereoplanigraph is comparable to the Wild Autograph

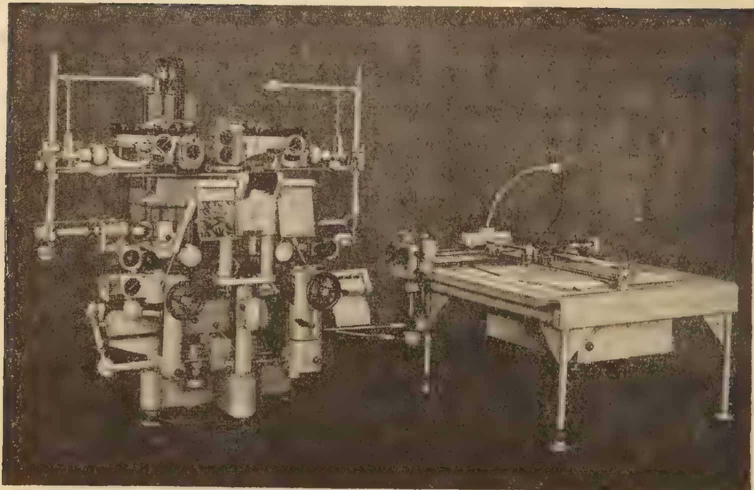

FIG. 5-28. STEREOPLANIGRAPH C8 WITH PRINTING COUNTER AND COORDINATOGRAPH.
(Courtesy, Zeiss-Aerotopograph.)

in its ability to draw from pictures taken at any angle, its wide range of plotting scales, high precision and speed of production. Both machines have a fixed eye position, x and y hand wheels and a z foot wheel and plotting is done on an adjacent table. Both could be operated by one man, but are usually considered more efficient with two operators, one at the machine and one at the plotting table.

5-20. Other Stereoplotting Instruments. Three other stereoplotting instruments similar in general operation to the Wild Autograph (Swiss) and the Zeiss Stereoplanigraph (German) are the Nistri Photostereograph (Italian), the Galileo-Santoni Stereosimplex (Italian) and the SOM Stereotopograph (French). While all the instruments are of comparable quality, the Wild and Zeiss are most used in the United States, the Nistri Photostereograph with digital read-out equipment is illustrated in Chapter 6, Fig. 6-5. Other makes of comparable capacity also provide for the print-out feature.

5-21. Comparison of Stereoscopic Plotting Instruments. The fundamental principles of all stereoscopic plotting instruments

are the same. The Stereoplanigraph can be thought of as a Kelsh Plotter, made much more convenient to operate. The Wild Autograph is similar in capacity, range and use to the Stereoplanigraph. Both of these instruments have a wide range of plotting scales, from substantial reductions to eight-time enlargement. The Kelsh Plotter and also the Multiplex are somewhat limited in scale of enlargement, which in the case of the Kelsh is five times the negative scale, or very close thereto. If the map is desired at some other scale, the map as drawn by the plotting machine must be enlarged or reduced either by a pantograph attachment or by some intermediate step.

The more complex machines have an elaborate optical system whereby the operator keeps his head and eyes always in the same place while, by turning hand wheels, he can cause the image to move before him. The operation is much more comfortable and convenient than leaning constantly over the table and moving from one side to the other, as is necessary when operating the Kelsh and the Multiplex. The hand wheels on the more complex machines permit a given line to be drawn faster than it can be produced by pushing the plotting stand around on the Kelsh and the Multiplex.

Machines such as the Stereoplanigraph and Wild are sometimes called "universal" machines, because they will permit plotting of maps from pictures taken at substantially any angle. Thus a contour map can be drawn readily from oblique pictures or from photographs taken from ground stations. One of the limitations of machines of the Kelsh and Multiplex type is that the lenses only produce a sharp focus in one plane. For this reason the projectors are made substantially pinhole cameras, which depend upon the small aperture of the lens to give sufficient depth of focus above and below the focal plane. This arrangement works out satisfactorily when the ground relief does not exceed perhaps plus or minus 10% to 25% of flying height. Beyond this the image on the screen becomes blurred. In plotting from oblique photographs, however, it is not unusual for the plotting distance from lens to map table, or from lens to representation of the ground, to vary more than the total altitude.

The Stereoplanigraph and Wild instruments handle such great differences most easily. It is not entirely practical to plot from obliques with the Kelsh machine. The Multiplex requires special attachments for plotting from obliques and even so the quality of definition is badly strained. Consequently, this type of plotter is not extensively used for plotting from obliques. The Balplex plotter can be used for this purpose, however.

5-22. **Bridging.** Ideally, ground control for aerial photography should consist of a great many points of known elevation and horizontal position in every model (Art. 4-30). In practical applications, however, the relatively high cost of obtaining ground control makes it desirable to devise expedients to keep the amount of control to a minimum. On large surveys covering many stereoscopic models, this can be achieved by *areotriangulation* or *bridging*.

In principle, bridging can be described as follows: a stereoscopic model on which there exists adequate control is carefully set up on a stereoscopic plotting instrument and adjusted to the ground control. For convenience call these two photographs No. 1 and No. 2. Then, No. 2 is left untouched and No. 3 is set up in the plotter to make a model with No. 2, all necessary adjustments being made on No. 3. Then No. 3 is left untouched while No. 4 is adjusted to No. 3. This cantilevering operation is continued until a model is reached in which there is once again adequate ground control. A comparison is made between the orientation of the model as cantilevered from 1-2 and the orientation the model should have according to the control that falls within it. There will probably be a difference due to systematic errors to which the operator may be prone, errors introduced by the design of the particular instrument, stretch or shrink of the negative film, curvature of the earth, and similar sources of error that are too small to be detected on a single model but which can be cumulative over many models. The closure error is then distributed through the intervening models and the resulting orientation of those models is used for drawing contours.

In practice, bridging is accomplished in various ways depending upon the type of plotting instrument used. The Kelsh plotter, in its common two-projector form, is unsuitable for bridging.

The Multiplex bridges by means of stringing a number of projectors from a very long bar. There can be twenty or thirty of them, or more, and each can be adjusted to the next in a continuous string. Most operators prefer to bridge with the more complex instruments, such as the Stereoplanigraph or the Wild.

Bridging is accomplished in these two projector machines by setting up the first stereoscopic model consisting of pictures 1 and 2 to fit the known control. " Pass points " for bridging are selected at the edge of the model toward the direction in which the bridge is being extended and their positions on the map and elevations determined. Then if the bridge is being extended from left to right, picture No. 1 is replaced by picture No. 3 in the left projector. Picture No. 2 remains anchored in the right projector, and all adjustments are made on picture No. 3. In order to accomplish this without physically moving the left projector to the position of projector No. 3, which would normally be to the right of the projector carrying picture No. 2, an optical " switch " is turned permitting the right eye to view the left projector and the left eye the right projector. The left half of the projected image of picture No. 2 was used in Model 1-2; the right half must be used in Model 2-3. The " base " is reversed, or placed in what is sometimes termed the " pseudo " position. Due to the optical switch, the No. 3 picture is being seen by the right eye and is therefore in effect the right member of the model. By the optical trick of switching the eyes and changing the " base " from normal to pseudo, the effect of stepping the projectors along the bridge is actually accomplished with only a two-projector machine.

On these fixed eye machines the change in " z," which on the Kelsh and Multiplex is accomplished by moving the plotting screen up and down, is done with a foot wheel control which varies the distance between the lens and the floating mark, or, in the case of the Wild, between the mechanical representation of the lens and the mechanical representation of the ground point.

In the universal machines, the exact spatial orientation of each model as the bridging operation progresses is noted by the operator and recorded for use later in setting up the model when

the proper corrections to the settings have been calculated. In the Stereoplanigraph and the latest models of all the European universal instruments, pressing a lever will automatically print out the x, y and z coordinates or encode them on to punched tape (Art. 6-21).

The way in which the " closure error " of the bridging operation is distributed throughout the section bridged varies with the number of models, the distribution of the control, and a number of other factors. Generally, the mathematics of it is complicated, particularly when an attempt is made to adjust simultaneously the bridges of several adjacent strips, so that the results should be consistent in the area of common sidelap between strips. Electronic digital computers are generally used to perform such calculations.

5-23. Bridging with Distributed Control. Several organizations have revised the bridging procedure by allowing the control to be scattered throughout the strip of photographs, rather than requiring a concentration in any single model. The first model is oriented on any assumed provisional datum and scale, and the three instrument coordinates are recorded for each of the set of six pass points, as well as points whose positions are sought and any control points which might fall in the model. Models are added in the customary manner but no attempt is made to adjust any model to fit control. The result of the instrumental operation consists then of a list of the three-dimensional instrument coordinates of points, a few of which (the control points) also have known ground coordinates. The set of coordinates is then transformed by computation into the ground coordinate system to fit the control data. The transformation usually includes terms which compensate for the systematic effects propagated in the instrument and include a least squares adjustment to fit all the control in the area. Although the transformations have been performed by desk calculator and graphic operations, the usual practice is to use a small or medium size electronic computer. To compile a map, each model is then reset, usually in a more economical instrument, to fit the transformed positions of the six pass points.

The procedure has the advantage of reducing field survey

operations by significantly relaxing the specifications as to the number, density and location of the control points. Existing control can frequently be used and new control established at sites which are readily accessible. An increase in accuracy ordinarily results from the systematic manner of adjustment and the fact that models are usually located relatively short distances from the control.

5-24. Analytical Triangulation. From Art. 5-7 it can be seen that the true horizontal coordinates of a point can be deduced from the x and y measurements on the two photos of a stereoscopic model and so can the parallax and therefore the elevation of the point. In principle, therefore, it would appear that careful measurements of coordinates of the " pass points " on a series of photographic plates would provide all the information needed to reconstitute a mathematical equivalent of the stereoscopic models; electronic computers could handle the necessary computations. In practice, however, the necessary measurements must be made to an extremely high degree of precision, requiring precision comparators as shown in Fig. 5-29.

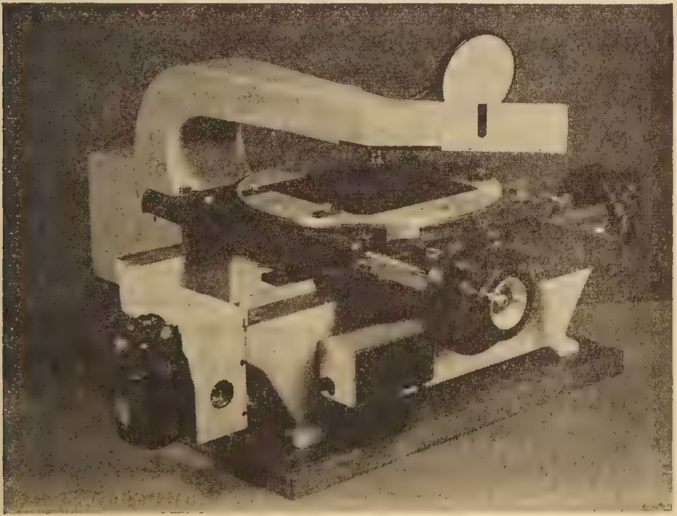

FIG. 5-29. PRECISION SCREW COMPARATOR.
(Courtesy, David W. Mann Co.)

5-25. Application of Stereophotogrammetric Apparatus. Satisfactory topographic maps may be compiled by all of the previously described stereophotogrammetrical methods. Instrumental errors in the horizontal position of a plotted point are less than the usual errors of plotting at these small scales.

The accuracy of contour location is affected by the angular field of the aerial camera, type of machine used, skill of operator, quality of photography and precision of the camera.

The table on p. 325 gives a summary of the important features of different types of stereoscopic plotting instruments. The "C-factor," also known as the "altitude-contour ratio," is the ratio between the flight altitude and the contour interval of which a particular stereoscopic plotter is capable at that altitude. The specified contour interval multiplied by the C-factor of a given stereoscopic plotter determines the flight altitude from which the photography must be taken to yield contours of the desired accuracy.

5-26. Automatic Plotting Devices. These are being developed to automate the process of drawing contours from stereoscopic models, and thus eliminate the dependence on the visual acuity of the operator and his judgment of the precise point at which the floating mark is "on the ground." A promising system is the "Auscor" (Automatic Scanning Correlator) also known as the *Stereomat* (Fig. 5-30). The stereoscopic model is set up in a two-projector plotter such as the Kelsh, and a spot of light from both projectors scans a small area around each point. The spot senses boundary crossings (from light to dark areas) and causes changes to be made in the level of the tracing table until the crossings coincide at both scanners. This, in effect, eliminates x-parallax and places the tracing table at the correct elevation. A combination of servo-motors linked to the parallax detector can move the tracing table in a horizontal direction, always along a line where parallax has been eliminated, thus tracing out a contour. Development work is continuing on automatic plotters, but they can already be used for problems, such as drawing cross-sections. When taking cross-sections, the horizontal motion of the tracing

CHARACTERISTICS OF STEREOSCOPIC PLOTTING INSTRUMENTS * COMMONLY USED IN THE UNITED STATES

Instrument	Type	Optics (Magnification)	Depth Range	Tilt Correction	Drawing Method
Ryker	3-dimensional	Fixed Stereoscope (1.0)	0.25H	empirical	move mark
KEK	3-dimensional	Fixed Stereoscope (1.0)	0.25H	empirical	move mark
Multiplex	3-dimensional	Anaglyph (2.4)	0.25H	true	tracing stand
Kelsh	3-dimensional	Anaglyph (5)	0.20H	true	tracing stand
Stereoplanigraph	3-dimensional	Fixed eye position (4–10)	0.80H	true	hand wheels
Wild A7	3-dimensional	Fixed eye position (8–9.5)	0.80H	true	hand wheels
Balplex	3-dimensional	Anaglyph (5.0)	0.50H	true	tracing stand

Instrument	Scale Range	Angular Range	"C" Factor[1]	Bridge	Approx. Weight in Case	Approx. Space Occupied
Ryker	0.38–2.5	vertical	400	empirical	160 lb.	on table
KEK	0.75–2.25	vertical	400	no	200 lb.	on table
Multiplex	2.4[3]	vertical[4]	700 800	yes	1000 lb.	5′ × 5′
Kelsh	5.0[3]	vertical[4]	800 1200	Yes, but inconvenient	1000 lb.	5′ × 5′
Stereoplanigraph	0.25–30.0[2]	universal	1000 1500	yes	5000 lb.	10′ × 10′
Wild A7	0.33–20.00[2]	universal	1000 1500	yes	5000 lb.	10′ × 10′
Balplex	4.5–5.5	universal	800 1500	yes	1000 lb.	5′ × 5′

* Table prepared by Leon T. Eliel.

[1] Average opinion of several users. [2] Practical enlarging limit 8.0. [3] Can attach pantograph. [4] Some claims are made for mapping from obliques.

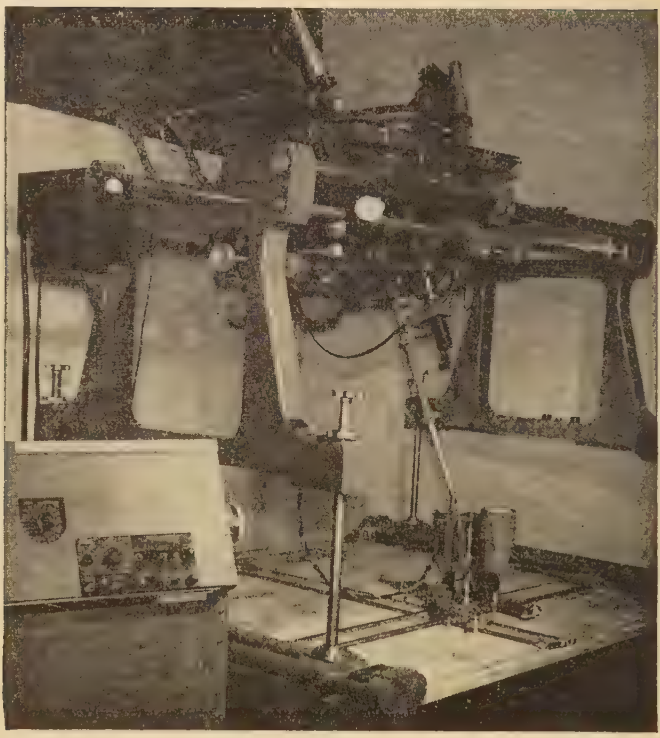

FIG. 5-30. STEREOMAT.
(Courtesy, Benson-Lehner Corp.)

table is a straight line, and the floating mark is made to move in a vertical plane instead of in a horizontal one as in drawing contours. The cross-sectioning operation itself, whether with or without automatic contouring, can be performed on either Kelsh-type or other universal instruments and can be automated for punched-card printout (Art. 6-5).

PROBLEMS

1. From a radial line laydown, a solution for the correct location of a point on the projection to the scale of 1/10,000 was obtained, Fig. 5-31. The elevation of the datum plane is mean sea level and the focal length of the camera is 12 inches.

Both pictures were free from tilt and both exposures were made at an altitude of 10,000 feet. The distance between the principal points of prints I and II measures 2.702 inches. The distance between the image points a_1 and a_2 is 0.409 inches and is found to be parallel to p_1p_2. From the above data, what is the elevation of point A above the datum plane? Point A on the ground corresponds to point a on the print.

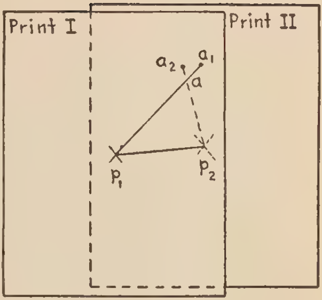

FIG. 5-31.

2. From a stereo-pair, a parallax difference of 1.29 mm. is measured between the image of the top of a tall chimney and an image near the base. The average distance between the principal points and their conjugate images (b) measures 72.5 mm. and the altitude of exposure is 15,000 ft. Using the above data, what is the height of the chimney?

3. From a stereo-pair of photographs, the following data were observed:

$b = 75.7$ mm. $H = 13{,}800$ ft.

Point	Elevation	Observed Micrometer Reading
a	212 ft.	10.85 mm.
b	483	12.42
c	519	12.68
d	634	13.28

(*a*) What is the average micrometer reading for mean sea level?

(*b*) What are the micrometer settings for tracing the 400- and 500-ft. contours?

4. The location of the control points on the left-hand print of the stereo-pair for which data were given in problem 3 is as shown in Fig. 5-32. When tilt, variations

FIG. 5-32.

in scale, distortion in paper are present, the micrometer readings for mean sea level vary from one point to another. Often a sketch similar to a contour map is prepared giving lines of equal-micrometer readings for zero elevation. Using the results of problem 3 and tracing Fig. 5-32, prepare a chart showing lines of equal-micrometer readings for each 0.01 mm.

5. In a stereo-pair of photographs, the datum elevation is taken as 350 ft. above sea level, the picture base $b = 86.4$ mm., and the height of the camera H above picture datum is 9800 ft. The parallax difference ΔP for a point A above picture datum is measured from the prints and found to be 1.08 mm.

(*a*) Find the height of A above sea level.

(*b*) Prepare a table of parallax differences for finding each 50 ft. contour between 400 and 1000 ft. elevation on these photographs using method of Art. 5-8.

CHAPTER 6

APPLICATIONS OF AERIAL SURVEYS

6-1. Choice of Ground or Aerial Survey. Very often, an organization in need of a map will be faced with a choice of aerial survey or conventional ground survey methods. To a large extent the choice is one of economics. The specialized equipment required for aerial survey work, such as aircraft, cameras, darkroom facilities and stereoscopic plotters is costly and requires a large number of skilled technicians which, in turn, incur a certain amount of administrative overhead. The cost of organizing and commencing an aerial survey, therefore, is large compared with that required for ground surveying techniques. The cost of sending a crew aloft to photograph ten acres is virtually the same as to photograph twenty acres. The savings in aerial survey work, therefore, are great only when the size of the survey justifies the use of aerial methods. A county-sized survey obviously calls for aerial techniques, whereas a survey of a three-acre tract should be done by ground survey methods.

There will be many intermediate-sized areas, however, where the cost differences will be small enough to be outweighed by other factors, and such factors should be evaluated in selecting the most suitable technique.

Conditions favoring ground surveys over photogrammetric methods in topographic or planimetric mapping are:

1. Where the area to be mapped is partially or entirely obscured by vegetation of non-uniform height, so that the ground cannot be seen from the air. Aerial surveys of large areas, where cost considerations would normally make aerial work economical, must often be scheduled for fall or spring, when there are no leaves on the deciduous trees, but before or after the ground is covered with snow. Such delays may not always be allowable, and ground surveys may be required to obtain needed information within time limit imposed.

2. Where the climate is such that clear, cloud-free days, suitable for aerial photography, are rare. An aerial survey that might otherwise be economical can become costly if an aircraft and crew must stand by for long periods awaiting suitable weather.

3. Where the scale or precision required is beyond the capability of routine aerial surveys. Maps at scales larger than 1 in. = 40 ft., or contour intervals smaller than 1 foot, present technical difficulties in aerial surveys which generally make ground surveys more economical. Since the areas covered by such large scale maps are usually relatively small, ground survey methods are often preferred. Exceptions will be found, however, such as in taking a stock-pile inventory (Art. 6-20).

Aerial surveys, however, have certain characteristics which often make them superior to ground surveys for the following reasons:

1. The aerial photography itself provides infinite detail which can be of value to the user of the map, in addition to the map itself.

2. Areas which are inaccessible because of rough terrain, or to which access on the ground is denied can be photographed from the air and mapped. Legal precedents have established that a property owner cannot prevent aircraft from flying overhead and taking pictures, provided there is no invasion of privacy and military security is not involved.

3. The time required to obtain field data is usually much less by aerial survey than by conventional ground survey methods.

4. The aerial photographs, even after the required map has been made and their intended function served, can be kept on file and used again for other purposes at a later date.

5. Topographic maps made from ground surveys contain a considerable element of subjectivity and are subject to the varying skills of individual topographers, as much of the contouring must be sketched by the field surveyor from a comparatively few points of reference. In aerial surveying every point of the terrain is available to the plotting machine operator for careful quantitative analysis, eliminating the subjective element. In general, a map made by aerial survey techniques will have a greater uniformity of accuracy than one made on the ground.

6-2. Aerial Survey Products. The principal products of an aerial survey fall into four major categories, which often overlap each other and are closely related. A purchaser will often take several of the products simultaneously, but in general terms they can be classified as (1) *aerial photographs*, (2) *mosaics*, (3) *planimetric maps* and (4) *topographic maps*.

6-3. Aerial Photographs. While aerial photographs are necessary for the making of all aerial survey products, they are also an end in themselves. When observed in pairs through a stereoscope (Art. 5-6) they give the same three-dimensional effect as seen through the Macyscope (Fig. 5-20).

Even without the aid of the stereoscopic effect, a skilled interpretor can tell a great deal from black-and-white aerial photographs. For example, the stereo-pair of photographs, Figs. 6-1 and 6-2, show an infinite amount of detail; street striping, automobile parking, building lines, etc. Long shadows depict high buildings which also appear to tip away from the vertical due to relief effect (Art. 4-24). This effect is particularly noticeable for the tall white building in the lower left corner of Fig. 6-2 and also in the lower center of Fig. 6-1. Due to differences in perspective of the two views, the right side of this building appears to lean to the left in Fig. 6-2 and to the right in Fig. 6-1. In both views the building appears to lean away from the center of the picture. The overlap of the two photographs is about 60 percent, as can be seen by the shift in street locations between exposures.

6-4. Interpretation of Aerial Photographs.* The value of photo interpretation for military intelligence has been recognized and used for over a century, but only comparatively recently have the importance and variety of commercial applications been recognized. Deriving the maximum use from aerial photographs calls for a high degree of skill and experience, but a few elementary principles of interpretation can give the unskilled user some assistance.

Man-made features, such as roads, railroads, canals and buildings, usually stand out quite clearly in aerial photographs. Roads

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* See "Manual of Photographic Interpretation" published by American Society of Photogrammetry, 1960, for complete coverage of this subject.

FIG. 6-1. LEFT VIEW OF STEREO-PAIR OF PHOTOGRAPHS.
(Courtesy, Fairchild Aerial Surveys, Inc.)

of concrete or gravel appear as white bands. Bituminous roads show darker in color. The importance of a road is not always indicated by its prominence in the photograph. An unsurfaced local road may be more striking than a dark-surfaced paved road, particularly if the latter is in a wooded area. On large-scale photographs, oil stains on concrete pavements, and to a lesser extent on bituminous types, clearly define the paths of heavy traffic flow. In areas laid out by Public Lands system roads will appear in a rectangular pattern usually following section lines; most of these roads are secondary highways. The

FIG. 6-2. RIGHT VIEW OF STEREO-PAIR OF PHOTOGRAPHS.
(Courtesy, Fairchild Aerial Surveys, Inc.)

more important highways are usually laid out along direct routes cutting diagonally across section lines.

Railroads appear as regular scars across the landscape, consisting of long tangents connected by flat curves. The track will appear as a dark band, and at some scales the rails can be distinguished. Since railroad gradients are quite flat there will be frequent cuts and fills. Railroad bridges will usually be narrower than highway bridges and will be found over or under highways as well as over streams. Sidings and station buildings will appear along the track and trains or standing freight cars may appear in some photographs.

Streams and other bodies of water usually appear black but they may also appear white if the photograph was taken in such a position that the rays of the sun were reflected from the water into the lens. A body of water may appear black in one part of the picture and white in another. If the water is muddy it has a tendency to photograph lighter than if it is clear. Water also appears light in color when a bright sky is reflected. Deep water usually appears darker than shallow water. In clear, still water features on the bottom, such as boulders, shoals, logs, etc., may show in the photograph. Streams are usually well defined by black twisting lines and by the presence of dense vegetation. Drainage ditches are detected by their straightness and regularity. Canals are also regular in shape. In hilly country a canal or irrigation ditch will follow along the contour of the terrain. A stereoscope is of great assistance in tracing drainage patterns and drainage areas.

Shadows play an important part in photograph interpretation. In order to obtain the most natural effect from a photograph it should be viewed so that the shadows are falling toward the observer; otherwise the relief may appear in reverse, the valleys looking like ridges and the ridges like valleys. Vegetation looks dark on photographs partly because its green color reflects less light than other colors and partly because the upper branches of the tree cast a shadow on those below. Evergreens appear somewhat darker than deciduous trees. In winter the evergreens stand out and the other trees almost disappear. For preparing contour maps photographs taken in the winter or early spring have the advantage that the ground surface is most exposed.

Tall grasses or crops cast longer shadows than short ones and therefore appear darker on the photographs. Different types of crops and those cut and uncut can be distinguished by differences in color. These color differences delineate the boundaries of fields and also serve to identify property boundaries. Fence lines may be detected at field boundaries or at the edges of cleared areas. The height of trees is indicated by the length of shadow, and the presence of low bushes is evidenced by the absence of shadows.

Some idea of the shape and types of bridges, water tanks,

towers and buildings is revealed by their shadows. A transmission line location, for example, will appear on the photographs as a cleared swath with an occasional pair of small squares indicating tower foundations. The line itself will be practically invisible except for the shadows cast by the towers.

Shore lines with beaches appear prominently in photographs, but shore lines in marshy areas where both the water and marsh vegetation print in dark color may be difficult to trace. Gravel pits and strip mines appear as irregular white patches of extreme prominence, like great sores eating into the landscape.

An observer skilled in geology and land forms can tell much of the geology of the region which he sees in the photograph. Also he can predict the kind of soils that exist in different areas and the presence of sand and gravel materials useful in construction. Soil maps have been prepared of large areas of the country using aerial photographs supplemented by a relatively small amount of field checking.

6-5. Color Aerial Photography. Color photography has been developed which is useful for such purposes as crop inventory and identification of soil and rock forms. Color photography, however, is more costly than black-and-white because (1) it requires better weather and illumination conditions, (2) it permits less latitude in exposure and development, thus requiring more skilled personnel and more possibility or reflights, (3) the film itself is more costly and deteriorates more rapidly before exposure, making special handling necessary, and (4) the best way to view color photography is by transparencies, requiring special illumination for viewing and making field use awkward; duplicates are more expensive than black-and-white prints. Skilled interpretors can generally derive the information which they desire from the gradations of shades of grey on black and white photographs. Problems do arise, however, where color photography is worth the extra cost, such as in geological surveys where colors help to identify minerals in rock or soil. Color photography is also helpful in making crop inventories and diagnosing plant diseases.

Photography taken on film which is sensitive to infra-red light has occasional special use. Water appears as dense black on such film, and hence swamp or flooded areas can be distinguished from

dry soil. Also, certain types of trees can be recognized from their infra-red activity, assisting in identification of species in a mixed stand of timber. Finally, infra-red photography can penetrate haze somewhat better than that taken with panchromatic film.

6-6. Mosaics. A mosaic is a continuous representation of the ground obtained by piecing together individual photographs into a composite picture. Various types of mosaics can be prepared depending on the desired degree of accuracy; the cost, however, increases as the standard of accuracy is raised.

A photo index or uncontrolled mosaic is the simplest type of mosaic, assembled by laying out contact prints from the aerial negatives on a flat surface and photographing the entire assembly. When the corner of the photo containing the print number is left uncovered, this is known as a photo index (see Fig. 4-28). When the number of photographs covering an area is large, an index is very useful in showing the relation of prints to each other and to make it easy to identify the print or prints covering a point of interest.

When the corners of the photos containing the print numbers are not shown this is known as an uncontrolled mosaic. In every group of aerial photographs, variations in true scale can be expected to some degree due to variations in terrain elevation and in flight height of adjacent flight lines. These, together with tilt and relief displacements (Art. 4-28) will result in a certain number of mismatches at the edges of pictures and an inability to measure accurate distances on the mosaic. Nevertheless, uncontrolled mosaics are useful in displaying features that cannot be encompassed by single pictures such as large-scale geological trends and general distribution and shape of forested areas, lake shorelines, populated areas, drainage patterns, road and railroad networks and the like.

6-7. Photo Map or Controlled Mosaic. Controlled mosaics are made by correcting the scale of individual photographs, or even parts of them separately, and removing the effects of tilt. Edges of individual prints are artfully concealed by cutting irregularly along photographic features that would reproduce as lines anyway, and the whole is blended to impart a uniform tone to the resultant mosaic (Fig. 4-29). The final controlled mosaic,

sometimes called a photo map, is accurate to scale and measurements of distance and angle can be made from it, subject only to the stretch and shrink likely to be encountered in photographic print paper. The photo map has uses in making timber inventories, in drainage studies, land use studies, as a base for geologic field work, and in many aspects of urban and county administration, such as planning, engineering and traffic studies.

6-8. Planimetric Map. A map showing natural or cultural features but no information about topography is called a planimetric map or line map. If a planimetric map is to be prepared concurrently with a controlled mosaic, the map is most easily made by tracing the required features directly from the mosaic, in the appropriate symbolic notation. If not, the features can be drawn by placing the contact print from the photography in one of several projection devices and transferring detail to the radial control plot (Art. 4-39). With either technique, information required for the planimetric map which is not obtainable from the photographs would have to be derived from other source material. Depending on the uses to be made of the map, this might include such items as political boundaries, place names and street names, identification of types of fence or of prominent public buildings, and similar information.

Planimetric maps can serve as transportation maps or for drainage studies. They can be used as base maps upon which information from specialized surveys can be plotted, such as land ownership, population distribution, soil and vegetation types, wildlife studies, utility distribution systems, delivery route systems, etc. The familiar service station road maps, city street maps, and outline maps of the states and U. S. are all examples of planimetric maps.

6-9. Topographic Map. A map showing contour lines as well as planimetric detail is called a topographic map (Art. 4-37). Alone or in combination with other types of aerial surveys products, topographic maps have a great variety of use. They are indispensable in virtually all applications of civil engineering, and have important applications in geological studies, urban planning, hydraulics and military operations.

Every map is a special-purpose map. It is, or should be, made

with one or a number of special purposes in mind and the map specifications written accordingly. If successfully executed, it should be completely satisfactory for those purposes but may be inadequate for other uses for which it was not intended. Maps produced by the Topographic Division of the U. S. Geological Survey come closest to an all-purpose map. (See insert on back cover of this book.) These are mostly in sheets $7\frac{1}{2}$ minutes of latitude by $7\frac{1}{2}$ minutes of longitude, with contour intervals of the various sheets differing according to the terrain. The most common scale is 1:24,000, which is 2,000 feet to the inch. While the sheets are suitable for a variety of uses, they should not be employed where measurements must be made from the map with such precision that stretch or shrink of the paper will affect the accuracy, nor should elevations be interpolated to a precision greater than is warranted by the compilation accuracy of the map. As an illustration: where the contour interval is 10 feet, no elevation should be interpolated from the contours to a precision greater than 5 feet, except in the immediate vicinity of a plotted bench mark. Similarly, no distance which is approximately twenty inches long on the map can be reliably measured to within .01 inch, particularly if there has been a fold in the paper in the intervening portion. Furthermore, necessary distortions due to the symbolism must be recognized. For example, a levee or a stream valley along which a one-lane dirt road runs will be shown wider than its true dimension because the standard symbol for the road is wider than the true road. The engineer using this map must be aware of these conventions.

Certain specialized topographic maps have been developed with the aid of photogrammetry, such as controlled mosaics with contours, superimposed as shown in Fig. 6-3. These combine the visual and numerical indication of elevations obtainable from contours with the intimate terrain detail and physical features available from the photography. Combination maps are also available with mosaic in upper plan of plan and planimetric or topographic detail in lower half.

Another specialized use of contour maps obtained by photogrammetry is that used for making an inventory of a stock-pile described in Art. 6-20.

FIG. 6-3. CONTOURED CONTROLLED MOSAIC.
(Courtesy, Fairchild Aerial Surveys, Inc.)

6-10. Specifications for an Aerial Survey. No two aerial surveys are ever exactly alike. The specifications for the desired end product and the means used to attain them will vary with the purpose of the survey, the characteristics of the terrain, the equipment available for performing the work and the time allowed for doing it. Nevertheless certain generalities can be made regarding various types of aerial surveys.

Specifications are generally written in great detail when the client is a frequent user of aerial surveys and thoroughly familiar with them, such as a state highway department or a consulting engineer. Infrequent users will generally consult with the aerial survey firm and give the company greater latitude, after outlining their requirements.

In detailed specifications, which may be written by the client or by the engineering staff of the aerial survey company for internal guidance, the optical and mechanical properties of the camera to

be used are set forth. The scale of the pictures and the overlap are also established as well as the details of construction and reproduction of the index map.

Mosaic specifications usually stipulate the accuracy desired in five ways: (1) the accuracy of the radial control including the distribution of ground control; (2) the extent to which relief displacement and camera tilt are to be corrected; (3) the scale of the copy negatives; (4) the scale and dimensions of the photo maps, type of grid projection, marginal data and title; (5) the material on which the photo map is to be reproduced, such as double weight paper or mounted on cloth or mounted on composition board.

Specifications for topographic maps prescribe the scale and contour interval. Tolerances are established for vertical and horizontal accuracy. A very complete list is generally given of the planimetric features which must be shown, and it is not unusual to include a complete set of conventional signs and a style sheet.

The specifications usually state the minimum size of building which must be drawn in exact detail and specify such things as how to designate solid business blocks of contiguous adjacent buildings.

In the case of a city map at large scale the specifications must indicate whether or not curb lines and property lines are to be shown, and in the case of an unimproved street the width to which it is to be plotted should be specified. Oftentimes it is desirable to show dedicated but unconstructed subdivisions on a map and the method of handling such a situation must be established.

The specifications will generally differentiate between "permanent control" and "picture control." It is often required that the former be monumented and established to a different order of accuracy than the latter.

Specifications will also cover such matters as the number of colors to be provided for in reproduction and whether "color separation prints" shall be compiled.

The most universally accepted specification for vertical accuracy is as follows: "90% of all contours shall be within ½

contour interval of their correct position. The remaining 10% shall not be in error by more than one contour interval."

The most generally used specification for horizontal accuracy is as follows: "90% of all identifiable horizontal features shall be within .025 inches, their correct position as determined from the nearest grid lines and the remaining 10% shall not be more than .05 inches in error."

Frequently specifications contain a provision for relaxing the accuracy when the ground is obscured by vegetation. Sometimes the above tolerances are doubled. Again, the map maker is required to show contour lines of uncertain accuracy as dashed lines or to designate the obscured areas with the timber outline symbol. In very heavy vegetation such as Douglas fir in the Pacific Northwest, a specification frequently encountered requires the contours to be accurate to one-half the height of the timber plus the one-half contour interval.

6-11. Scale of Photography. The scale of the aerial photography to be taken should be decided with the utmost care, as this will be the biggest single factor in determining the cost of the aerial survey work. In general the scale should be as small as possible consistent with the following requirements:

1. If the photography is to be used for drawing contours, the "C-factor" for the stereoplotting instrument which will be used will determine the smallest scale (Art. 5-25).

2. If the photographs are to be studied for detail they could be enlarged, but then the use of a stereoscope would be quite awkward. For non-stereoscopic examination portions of photographs can be enlarged at least three diameters with no loss of sharpness, and as much as ten diameters if the user is willing to lose some photo quality. But the resolution of small objects is of course not improved by enlargement, and the original scale of the photography will ultimately depend on the size of the smallest objects sought.

3. A small scale of photography may call for an aircraft with so high an operating ceiling (Art. 6-14) as to make it necessary to use special aircraft; the cost of using such aircraft may outweigh the savings to be accrued from the small scale of photography.

4. When using color photography, a lower flight altitude will give truer colors than one which must penetrate many thousands of feet of haze.

5. Some types of clouds, as high stratus, cast thin enough shadows to permit aerial photographs to be taken of the ground, provided that the flight altitude is low enough so that the cloud is not itself visible in the photograph. In some areas where the climate is such that good photographic weather is scarce, this may have an important effect on costs.

6. Shorter focal length cameras will yield smaller scales for the same flying heights but the wide angle required to produce 9-in. photographs with a short focal length camera usually creates intolerable distortions. Most stereoplotting equipment is, therefore, designed for a six-inch focal length camera.

For some purposes a small scale is desirable without regard to cost. A photogeologist, for example, examining geological trends stereoscopically appreciates having photographs that cover a large enough area so that a large portion of such a trend may be followed on a single print or pair of prints.

6-12. The Aerial Survey Project—Flow of Work. The following paragraphs (Arts. 6-12 to 6-17) describe the sequence of operations in conducting an aerial survey project, once the specifications, type of camera and desired scale of the photography have been decided.

The *scale* of the photography will determine the mean terrain clearance at which the survey is to be flown (Arts. 4-14 to 4-16). This must be added to the mean terrain elevation to obtain the required flight altitude. An aircraft must then be selected which has the necessary operating ceiling, or else the scale of the photography must be changed. For example, an aircraft with an operating ceiling of 21,000 feet can obtain 1:40,000 photography with a six-inch focal length camera in Louisiana but not in Wyoming. Range of the aircraft is also a factor, particularly in remote areas where the survey area is a considerable distance from the operating base.

Flight lines are laid out on the best available maps of the survey area at a scale convenient for handling by the pilot or navigator (Art. 4-12). The cameraman is given instructions concerning the

type of camera and film specified and the required amount of overlap. Depending on the circumstances of local weather or season, an indicated maximum percentage of snow cover, tree leaves or clouds may be specified. The flight crew are also told the hours during which the sun is high enough for taking pictures in the survey latitude and the dates for satisfactory photography, and told of any special limitations that may be imposed by the particular survey, such as the need to take the photographs when the tide is low, or on Sundays to avoid factory smoke that would impair picture quality.

After the photography is taken the crew stands by until the film is developed and checked to see that all requirements mentioned above have been met, as well as others concerning permissible amounts of overlap, crab and tilt. Gaps in coverage due to poor navigation or to incorrect source maps on which the flight line were laid out must be reflown. Flying a straight line exactly as laid out on a map calls for a high degree of skill but the rewards are great as a well-flown survey provides the necessary coverage with the fewest possible photographs, and the cost of compilation increases directly with the number of photographs.

6-13. Control. Before the flying gets under way the compilation group makes a thorough search of source material for existing ground control. Use of existing control will of course save the cost of establishing new points, but usually such points cannot be precisely identified on an aerial photograph from the description alone. In that case a survey party may go out in advance of the flying, recover the points and mark them with strips of white cloth weighted down with stones, to make the points visible in the photographs. For repeated use, some cities paint white circles or crosses around control points embedded in asphalt pavements. These procedures speed up the compilation time considerably once the flying is performed, providing the existing control is sufficient for the needs of the survey. But if additional control must be established anyway it is generally more economical to have the survey crew go out after the flying is complete and identify the existing control point by occupying it in the field and putting a tiny pinhole at the proper location on a stereo-

scopic pair of photos taken along for the purpose. On the same trip the crew establishes the new control.

If additional control points are needed the prints of the aerial photography are studied in the laboratory. The optimum density of control per picture or per flight strip (Art. 4-30) may have to be modified in accordance with problems of accessibility (the photographs themselves are very useful for studying this), distribution of existing control and special requirements laid down by the client. Many highway departments call for more ground control and more monumenting than would be needed for the aerial survey alone, knowing that this will be useful to them in later stages of the construction work.

When all the field work has been performed and computed (see Arts. 4-30, 36) supplementary control points are added in the laboratory to provide the required density of control per stereoscopic model. This may be done by templets (see Arts. 4-38 to 4-42) or by bridging (see Arts. 5-22, 23). If a controlled mosaic is to be laid, the templet control is used, and the laying of the mosaic is the next step (see Art. 4-43).

6-14. Stereoplotting. Diapositives are made from the original negatives for use in plotting instruments; these are positive prints, on glass, sometimes at the same scale as the negative and sometimes smaller, depending on the particular plotting instrument used. Electronic dodging * is often used at this stage to decrease extreme contrasts between light and shadow on the original photography.

The atlas sheet layout for the survey is determined, generally in accordance with detailed instructions from the client. Often, in highway work, a map sheet will be many feet long, though not more than 42 inches wide, which is the width of most standard reproduction machines. A coordinatograph, which is a machine for drawing lines and plotting points in an x-y coordinate system, is used for drawing the boundaries or neat lines of the atlas sheets and for plotting the original and supplementary control points. It is necessary, therefore, that all points shall be given in some

* A process whereby the light intensity used to rephotograph strip of film is automatically varied so as to equalize the tone of the photograph.

plane coordinate system, generally utilizing the map projection system in use in the particular state (Art. 1-62). The scale of the plot is one which is compatible with the plotting instrument being used (see table in Art. 5-25) but should never be smaller than the scale at which the final product is to be delivered. In general, the larger the ratio of compilation scale to delivery scale, the easier will it be to perform drafting and other types of operation, as errors will be reduced in size during the final photographic reproduction. The material used is generally one with high non-shrink characteristics, such as glass cloth or drafting film (Art. 9-17). With the control and neat line plotted and drawn on it, this is known as the manuscript sheet.

The plotting machine operator places the two diapositives for a stereoscopic model in his instrument, places the manuscript sheet on the plotting table, and, armed with a pair of paper prints of the model on which the control is pinpricked and described, proceeds to draw the contours and required planimetry. Edge matches with adjacent atlas sheets are made where necessary.

6-15. Editing. The manuscript sheet then goes to an editor. His functions include the drawing of political boundaries and the assignment of place names, all obtained from other source material, the placement of proper symbolism for swamps, orchard, woodland, culture and other features, the selection of proper type size and face for internal and external lettering, and general review and check of the manuscript to be sure that no contour has been omitted and no blunder made in drawing. The editor supervises the work of the draftsman.

Sometimes the stereoplotting machine operator does his own drafting. This can be most conveniently accomplished when the instrument is a Kelsh or any of the similar simpler instruments. On the universal instruments, which operate with a two-man team and where the equipment has a high hourly amortization rate, it is generally not economical to do this.

6-16. Drafting. Drafting of a map for putting it in its final form can represent a substantial percentage of the cost of the entire product and sometimes a purchaser will ask that this step be omitted and that the manuscript sheet be delivered to him

instead. Actually this sheet has all the essential engineering information, even though it may not all be clearly labelled or look like a finished map. Government agencies at all levels, large corporations, and purchasers who make frequent use of the map and by many different persons, tend to require that the map be finished in conventional form.

Most drafting consists of tracing all the required features in ink, from the manuscript sheets. Frequent users of large numbers of maps will specify exactly the type of symbol to be used for each feature; thickness of inked lines; dimensions to be used with the symbols, such as separation of the two lines that indicate a road, or length and separation of the dashes constituting a political boundary; size and face of type to be used; and content of marginal information to appear on each sheet.

The tracing is done in ink on some translucent material such as tracing cloth or a stable plastic. Lettering may be set in type on transparent, gummed tape somewhat like ordinary adhesive mending tape, which is carefully placed on the proper locations on the map, or the lettering may be inked directly with the aid of lettering guides.

If the map is to be delivered and used at the scale at which it is drafted, reproductions are made in duplicating machines for actual field use, while the original, with its " stick-up " in the form of pasted-on lettering, swamp, woodland and other symbols, is saved for use as a master copy. If a smaller scale is desired the map must be photographed and the negative then reproduced by photographic processes at the desired scale. In the event that there is a possibility that the original drafted map will be reduced in scale at any time after it is produced, care should be taken to change or omit a numerical scale, and to show the scale graphically.

Drafting for maps which are to be reproduced in more than one color is done on separate sheets, one for each color. The sheets are then photographed, and printed by lithographic processes.

A new and effective technique of drafting single-color or multi-color maps is known as scribing. A uniform coating of a translucent emulsion is applied to the base material. Lines are "drawn " by scratching off the emulsion with a specially designed stylus. With the proper size stylus proper width of line is auto-

matically achieved and none of the nuisances of keeping drafting pens properly inked need be experienced. Errors are easily rectified by coating the mistake with liquid emulsion applied with a brush. Since the end product is a " negative " consisting of clear light lines on a dark background, the type is set in negative form too and pasted on spots cleared of emulsion. This negative form also can save one step in photographic reproduction but the biggest saving comes from the fact that the scribing technique requires less drafting skill than the conventional kind.

The final edit reviews the drafted sheet for compliance with all pertinent specifications, and the chief engineer adds his signature.

6-17. Checking. Before accepting and paying for a topographic map a purchaser will want to inspect it and check it. He can assign his own surveyors to do it or hire an independent firm. Aside from office inspection for conformity to specifications in such matters as sheet size and symbolism, a field check is made to determine accuracy of the topography and planimetry. Sample profiles are run both on the ground and by scaling from the map. Elevations taken from the map must conform to specifications, i.e., 90% of all points shall be within half a contour interval of the correct elevation. Some large users of topographic maps have elaborate statistical techniques for checking, and sometimes use stereoplotting instruments for setting up models to perform spot checks on stereoscopic work.

6-18. Applications. Photogrammetric surveying is employed for producing maps and mosaics for all kinds of comprehensive projects, such as airport, highway, railway, waterway, pipe line, and transmission line locations; for irrigation, water supply, and flood control; for harbor and river improvements, city zoning and planning, traffic studies, parks, grade separations, and assessors' tax plans; for soil conservation studies, soil identification and mapping, and timber estimates; for military surveys and explorations. In a project in which it is necessary to secure right-of-way over private land, such as a transmission line, this method offers the advantage that the entire plan can be made before the matter is generally known, when such rights are much more easily acquired. In harbor improvements it is an advantage to have photographs showing vessels in drydock, or entering or

leaving the harbor; actual conditions are shown better in this way than by plans alone.

For timber studies the method saves traversing much country in which traveling is often difficult; by examining the pictures under the stereoscope an approximate estimate of the different kinds of timber can be made. Soil maps may be constructed from land forms and cultural features appearing in the prints. In making a reconnaissance for an airport or for a dam or reservoir site, the prints may be examined to determine the most desirable sites and to select the sites at which a detailed topographic map is required. Pipe lines or transmission lines may sometimes be located from a study of pairs of prints in a stereoscope. Since the cost of prints and a precise mosaic amount to only a fraction of that of a complete map, the possibility of their use alone should be carefully considered for some projects.

6-19. Use of Aerial Surveys by a Municipality. Some of the uses which may be made of aerial survey by a municipality of say 50,000 population are described below.

The Mayor's Office and City Council can use a mosaic of the city for wall decoration in City Hall; also the mosaic or enlargements of portions of it can be used as exhibits at public hearings. A photographic presentation is usually more readily grasped by the general public than even the best of maps.

The City Planning Department uses individual photographs or controlled mosaics for house counts, studies of means of communication, zoning studies, plans for parks and recreational areas, population density studies, locations of airports, and other land uses. In common with other departments, such as Traffic and Water Supply, its area of interest cannot logically be confined solely to the legal boundaries of the city, and it often requires surveys of surrounding areas as well. Topographic maps help the Planning Department take the maximum advantage of topography in planning future development of the city. New communities being laid out by a large developer or the operator of a newly-discovered mine generally are planned from aerial surveys of the area.

The Police or Fire Departments, at the precinct or fire station level, should have an intimate knowledge of streets, alleys,

fences and structures in their jurisdiction. Enlarged photographs can help immeasurably in this regard.

The Traffic Department can study aerial photographs for information on traffic channelization, striping, problems of complex intersections, and other planning functions. The regular municipal aerial survey pictures can be used, but traffic departments sometimes supplement them with photographs taken at peak traffic hours, or after sporting events that draw great crowds, or with photographs of downtown outdoor parking lots at specific hours of the day. Photographs are particularly valuable for taking parking inventories, as can be seen from an inspection of Figs. 6-1 and 2.

The Department of Education and municipally owned utility and transit companies need mosaics for studying population densities and estimating future trends that affect their interests.

The City Department of Public Works is the municipal agency with the greatest need for aerial surveys, particularly topographic maps. All the civil engineering works of the department require topographic maps in their planning and execution stages. While some of the larger scale maps are most efficiently made by ground survey techniques, many can better be done from aerial surveys. The city engineer can use the aerial survey topographic map as a base on which to draw the special-purpose detail required by the problem at hand or by some other department. Without aerial surveys, he would have to make the entire base map of the city with his surveyors.

The city engineer generally furnishes the ground control required for the aerial survey, or works with the aerial survey agency in obtaining it.

Private citizens and commercial enterprises in the city benefit from an aerial survey. Municipalities often publish the topographic maps and sell them publicly for a nominal charge, and many communities will make arrangements with the aerial survey firm in which the company retains custody of photographic negatives and sells prints or enlargements to the public at a reasonable reproduction price. Real estate promotors, retail food chains, private utility companies and many other enterprises have uses for these photographs and mosaics.

6-20. Specialized Use of Photogrammetric Methods. Certain specialized applications of photogrammetry have been developed such as for stock pile inventory. Large outdoor stock piles of coal or raw materials can be photographed from the air and their volumes determined by stereoscopic analysis. The procedure has these advantages: (1) The volume can be determined as of a given day and hour (photographic weather permitting) whereas additions and withdrawals may take place during the progress of a ground survey; (2) ground survey techniques for determining measurements of the pile may be impossible because of the difficulty of climbing on it, and volumes must therefore be computed indirectly by counting truckloads or carloads added or removed.

In practice, a pair of photographs constituting a stereoscopic model is taken of the stock pile at an altitude low enough to guarantee the necessary precision at the desired contour interval but high enough to encompass surrounding control points and, if possible, include the entire pile in a single pair of photos. Control consists of several points of known position on the ground surface surrounding the pile. Contours are then drawn at intervals of say one foot, and the volume of the pile computed by measuring areas within each contour and summing the volumes between these areas, assuming uniformity of shape from one contour level to the next. The location and shape of the bottom of the pile must be furnished by the owner or surmised from the surrounding terrain or obtained from aerial photographs taken before the stock pile was laid down. While the user is usually interested in weight of stock pile the aerial surveyor can only provide volume, and the user must make his own weight calculations, based on assumptions of uniformity of density, compaction, etc.

Volumes of material excavated from a quarry or open-pit mine can be calculated from aerial surveys in a similar manner.

6-21. Use of Photogrammetry in Highway Engineering. State Highway Departments and Federal agencies, such as the Bureau of Public Roads, Forest Service and Corps of Engineers, make extensive use of aerial surveys in planning, designing, building, maintaining and operating highways. The requirements and practices of the 50 states and other agencies vary. In an effort to obtain some degree of standardization, the U. S. Bureau of Public

Roads has issued a guide to photogrammetry specifications which engineers may follow or adapt to their particular needs in contracting for photogrammetric products. The applications described herein represent a composite of practice rather than that of any one state or agency.

In comprehensive highway planning, aerial surveys are used to obtain detailed land use data, and to compile base maps of areas under study upon which the planning elements may be displayed. The land use data are not only useful for planning highway systems and selecting route locations, but are also used with motor vehicle origin and destination survey data in predicting future traffic volumes.

In the early stages of route selection, small-scale photography covering large areas per print is used, together with available topographic maps, to delineate feasible routes which warrant further study. A band of photography usually covering about a mile in width is then flown along one or more feasible routes, usually to the scale of 1 inch = 1000 ft. for the photographs and compiled in strip-map form to a scale of 1 inch = 200 ft. with 5-foot contours. In rough mountainous terrain the compilation scale might be 1 inch = 400 ft. with 10-foot contours. From these maps, tentative lines and grades are determined with sufficient accuracy to make preliminary quantity and cost estimates for comparing alternate lines.

Fig. 6-4 shows a portion of a highway location study using a controlled mosaic as a base map. The center lines of the two roadways of a divided highway are shown by white lines together with limits of area to be graded and of right of way taking. The profiles of the center lines are also shown for each roadway.

Once the final route has been chosen, large-scale photogrammetric maps are obtained at a scale of 1 inch = 40 ft., or 1 inch = 50 ft., with 2-foot contours. The line is then transferred to these base sheets, refinements and adjustments are made and the alignment tied in to the local coordinate system, and tangents and curves computed mathematically. Design details are added and contract plans drawn. Graded surfaces for complex urban intersections and at freeway interchanges are commonly defined by one-foot contours. Often the only ground surveying needed

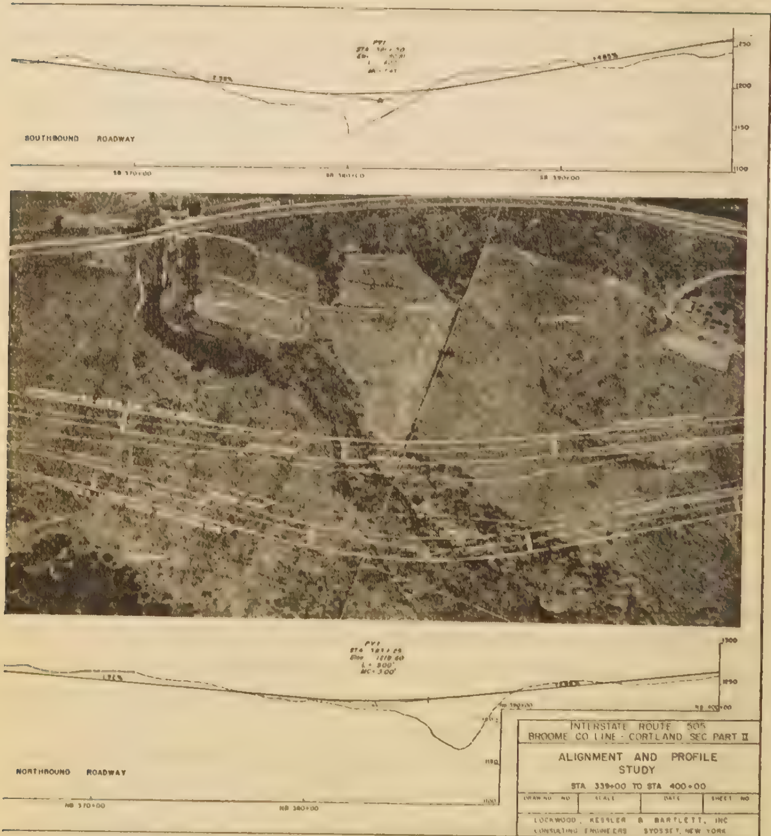

FIG. 6-4. HIGHWAY LOCATION STUDY ON MOSAIC.
(Courtesy, Lockwood, Kessler & Bartlett, Inc.)

for a highway project is for establishing control, filling in details, and staking out work for construction.

Cross sections for computing earthwork quantities can be obtained by scaling map distances from center line and interpolating elevations from contour lines. Preferably, cross sections may be obtained directly by scanning them from the stereoscopic model in a plotting machine by moving the tracing table in a straight line along the line of the cross section being plotted.

The moving dot of light is kept constantly on the ground surface by the operator as it moves along. At uniform distance intervals, or wherever the operator sees a change in the slope of the terrain, readings are taken of the terrain elevation and of the distance from the tentative center line (furnished by the Highway Department). Usually these readings are taken automatically and recorded simultaneously by an automatic typewriter and on punched cards or tape (Fig. 6-5). Thus, a representation of the

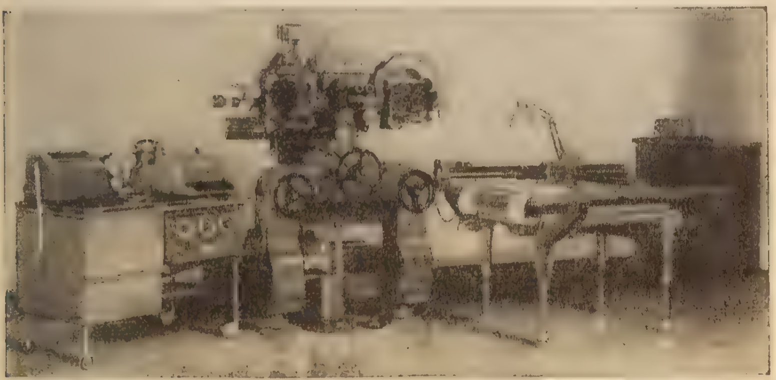

Fig. 6-5. Nistri Photostereograph with Digital Read Out.
(Courtesy, O.M.I. Corporation of America)

terrain is available in numerical form to the highway engineer who can use it in a digital computer for calculating earthwork quantities for optimum alignment and profile, for cost estimates and for pay quantities.

Periodic estimates of earthwork quantities may be obtained by repeating an aerial survey of the location and computing volumes between surfaces obtained from successive flights.

The photography is also useful for locating borrow pits, sources of gravel for subbase, identifying soil types along the location and delineating culvert drainage areas.

Fig. 6-6 shows a controlled mosaic which has been overlaid with a sheet of transparent drafting film on which the locations of the various soil types have been delineated. The soil types are represented by symbols.

FIG. 6-6. USE OF MOSAIC FOR SOIL IDENTIFICATION.
(Courtesy, Aero Service Corporation.)

Mosaics made from the photography are valuable in planning highway locations and for display at public hearings or for public relations releases. Enlargements of individual prints are very helpful to the right-of-way department in determining and explaining limits of land takings, sometimes copies of individual photographs are given to property owners as an effective good will gesture. Oblique views are also effective for this purpose.

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CHAPTER 7

HYDROGRAPHIC SURVEYING

7-1. Definition. Hydrographic surveying is the term applied to the processes used in surveying any body of water. In the case of oceans or lakes this may include the determination of shore lines, soundings, characteristics of the bottom, location of buoys, etc.; the survey of a river may also include the determination of the velocity and characteristics of the flow. In its broad sense the term may be applied to the survey of drainage areas and proposed reservoirs for the storage of water.

7-2. Shore Line and Stream Surveys. The shore line of a body of water may be surveyed by running a transit and tape traverse at a convenient distance from the water and locating the shore by offsets measured with a tape from the traverse line at all points where there is a noticeable change in the direction of the shore line.

Surveys of such irregular lines, however, are often made by first establishing instrument points by traverse or by triangulation and filling in the details by the Stadia or the Plane-Table Method (Chapter VII, Vol. I). If a traverse is used for the control it may be run out by means of the transit and tape, or it may be made a stadia traverse and run at the same time that the side shots are taken to locate the details.

In surveying a river whose width is too great for accurate stadia measurements it is necessary to run traverse lines on both banks and to locate each shore line by side shots from its traverse line. As a check on the survey it is well to connect the traverse lines on each side of the river by occasionally measuring angles to successive transit points in the traverse on the opposite bank. But where the river is not over half a mile wide the opposite bank can be located, within 5 to 10 feet under favorable conditions, by stadia distances from the transit line, or by occasional intersections from adjacent transit points.

It is frequently advisable to make no attempt to run a traverse which will follow very closely every turn of the shore or river; small auxiliary traverses composed of short courses may occasionally be run around the arm of a lake or the bend of a river while the main traverse cuts directly across it. The stadia method has great advantages over the transit and tape for locating a shore line, because the distances to inaccessible points across the water or over the brush along the shore can be readily measured by this method. If the survey covers a long stretch of shore line, and especially if great accuracy is required, it should be based on a system of triangulation executed with whatever accuracy the work itself demands (see Chapter I).

7-3. Shore Lines of Harbors, Lakes and Rivers. The traverse system is generally used for a long and narrow body of water, particularly when no obstructions such as buildings or trees would hamper the work. Otherwise it may be more desirable to establish a triangulation system for the control of the survey. The field work for the triangulation is first accomplished, and plotted on a plane table sheet. Details of the shore line are then determined by stadia, using the triangulation points for control.

In planning a system of triangulation where stations occur on opposite banks of a body of water it must be possible to see between successive stations on one shore or the other in order to complete the system. For example, in Fig. 7-1 it is evident that unless a sight can be taken from 5 to 7 or from 6 to 8 the computation or plotting of the system cannot extend farther than line 5 — 6. It is obvious in Fig. 7-2 also that the system stops on line 4 — 5 unless a sight can be taken from 4 to 6 or from 3 to 6. A stronger network is, of course, obtained if four successive points form quadrilaterals in which all angles can be measured.

It is not usually economical to locate shore lines of lakes by traversing around them, owing to the difficulties presented by natural obstructions, such as woods. It may be necessary, however, to supplement the triangulation and plane-table work by side traverses, in order to secure the necessary details in bays or irregular inlets.

Although the stadia and plane table methods are commonly used in surveys of limited extent, shore lines may be obtained

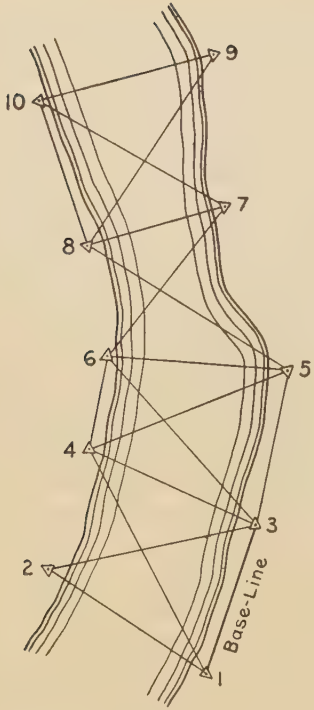


FIG. 7-1.

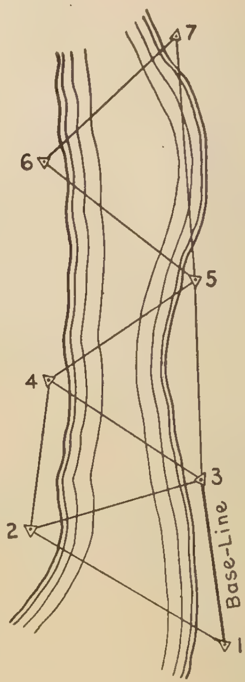


FIG. 7-2.

more readily and in more detail from an aerial survey (Chapters 4–6). When this is done, the ground traverse or triangulation is only needed to establish ground control points necessary for producing a map by the photogrammetric process. Where established points already exist, these can be used for control.

7-4. **River and Lake Surveys in Winter.** Some surveyors prefer to make river and lake surveys in the winter season, where ice can be utilized for both triangulation and traverses, and no boat is required. It is doubtful whether this is more economical than summer work, owing to the unfavorable weather conditions, prevalence of snow on banks, difficulty of securing points, etc. Soundings may be located more accurately through the ice, but at far more expense.

7-5. Ocean Shore Lines. The shore line of the ocean or any other body of water which is affected by tidal action is usually taken as the line of average high tide. When this line is to be located with precision (as when the shore line defines a boundary) it is impossible to judge with sufficient accuracy the position of the high water line from the deposited refuse or weather marks on the rocks. It becomes necessary in such cases therefore to determine by tidal observations (see Vol. I, Art. 262, p. 309) the elevation of mean high water and to locate points on the shore at that elevation. A line connecting these points is the shore line at mean high water.

These points may be determined just as any single contour line is run out, by a leveling party working with, or just in advance of, a transit party, the latter locating the points which have been marked by the leveling party. Such work can also be readily carried on by one stadia party, and this is the method most commonly employed. In using this method the transitman levels the telescope, and the rodman moves up or down the bank until the proper rod-reading is obtained; the rod is then located by an azimuth and stadia distance, both of which are recorded in the note-book.

In order to avoid locating unnecessary points the rodman chooses only those points where the shore line changes its direction. The levels should be checked occasionally on bench marks previously established; large errors in level, however, may be detected by noticing the position of each located point with respect to the water surface or to the line of deposited refuse.

For maps on a scale of $\frac{1}{1000}$ or smaller it is usually possible to run in the shore line with sufficient accuracy by stadia, judging the position of the high water line from the appearance of the drift along the shore; where the slope of the shore is very flat, however, elevations may be required. It is always necessary to bear in mind the scale of the map to be produced, in order that useless refinements of measurements and observations may be avoided and a saving in both time and cost effected without sacrificing accuracy.

A specific shore line elevation may be traced in a stereoplotter

from a stereomodel of a pair of air photographs in the same manner as a contour is traced on the ground.

7-6. Contour Surveys of River Banks. Surveys made for the purpose of investigating water resources frequently include the topography of the banks of a river, the fall of the water surface between successive points, etc., in addition to the location of the shore lines. In such cases the general methods of procedure are like those previously described for locating shore lines (Art. 7-3), but in addition a line of levels must be carried along as a basis for the determination of contours. Levels are taken on the water surface at the head and foot of rapids and wherever it is necessary to determine changes in slope of the water surface. The places where these level readings are taken should be located with reference to the transit line. The bank contours may be sketched in, using the transit line and the levels as a basis, or if greater accuracy is desired they may be located by stadia or by plane table. In a river survey it is often sufficient to determine contours on the bank for an interval of 5 or 10 feet, while water surface contours should be shown for differences of, say, one foot.

7-7. Drainage Areas and Storage Basins. Topographic surveys are required to determine drainage areas and capacities of storage basins before filling with water. Such surveys may be made by transit and tape traverses supplemented by stadia or plane table surveys as described in Vol. I, Chapter XI. Use may also be made of contour maps prepared from aerial surveys as described in Chapters 4 and 5.

After the contours have been plotted the drainage areas can be readily sketched (see Vol. I, Art. 329, p. 395). Also the shore line of the proposed reservoir can be sketched on the topographic map as soon as the elevation of the spillway of the dam has been determined. Drainage areas may also be traced from aerial photographs viewed in a stereoscope which shows the ground in three dimensions (Chapter 5).

Topographic maps of the U. S. Geological Survey, or of some state surveys, are useful for studying drainage areas. A ground survey is necessary, however, for construction purposes and for accurately determining the capacity of a reservoir.

After deciding upon the elevation of the spillway which determines the height of water surface in the reservoir, the survey of the shore line then becomes merely the problem of determining the position of that single contour elevation on the ground.

The capacity of the reservoir can be calculated from data taken in various forms. For instance, a topographical survey of the entire area of the reservoir may be made and the volume calculated by the End Area Method by using the areas of the successive contour planes as bases of vertical prismoids, as is described in Vol. I, Art. 407, p. 486. Or the reservoir may be cross-sectioned into horizontal squares or rectangles as described in Vol. I, Art. 253, p. 302, and the volume calculated by the Borrow Pit Method, as shown in Vol. I, Art. 411, p. 491. Still another method is to take vertical sections across the reservoir site and compute the volume by the End Area Method, using each vertical cross-section as a base of a horizontal prismoid. The areas of these vertical sections (or of the horizontal contour planes in the first case) may be readily determined after being plotted to scale, by the use of the planimeter which is described in Appendix A of Volume I; or this area may be determined approximately by placing over the drawing a piece of tracing linen which has been divided into small squares and counting the number of whole squares and estimating the fractional parts of squares contained in the area.

7-8. The Sextant — General Description. In addition to the instruments employed in land surveying, the *Sextant* (Fig. 7-3) is frequently used in hydrographic surveying. It is an instrument which, unlike the transit, is adapted to measuring angles in any plane. The frame of the instrument is in the form of a sector whose arc is 60 degrees, or the sixth part of a circle, from which its name is derived. It is constructed, however, in such a way that angles as large as 120 or 140 degrees can be measured. Owing to the fact that it can be used by an observer who is on a moving object, such as a boat, it is especially valuable for hydrographic work. It is employed not only for taking angles from a boat in locating soundings but is also in common use for making astronomical observations which are necessary in determining the latitude, longitude, and time at sea.

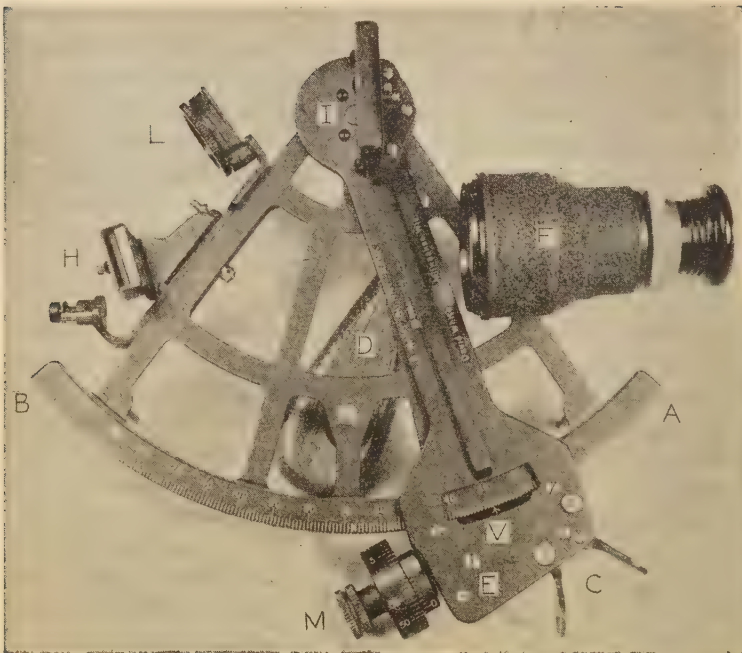


FIG. 7-3. THE SEXTANT.

The frame *ABI* (Fig. 7-3) is usually of brass, on the under side of which is attached the wooden handle *D*. The *index arm IE* is pivoted at *I*, the center of the arc *AB*, and this arm can be swung around *I* as a center so that the vernier *V* which is fastened to this arm can pass from *A* to *B* on the arc (or limb) and can be set at any position on the arc by means of the squeeze clamp *C* and micrometer screw *M*. At *I* is a plane glass mirror called the *index glass;* it is attached rigidly to the index arm and perpendicular to the plane of the sextant, its reflecting surface being over the pivot about which the index arm revolves. Rigidly attached and perpendicular to the frame of the sextant is the plane *horizon glass H*, the upper half of which is transparent, while the lower half is a mirror; this glass is set so that its plane is parallel to the index glass when the vernier is set at 0°. The telescope is at *F*.

A colored glass at L is hinged so that it can be swung on a pivot into the path of the rays of light to protect the eye of the observer in making observations on the sun. There are three short metal legs on the under side of the frame on which the sextant may rest when not in use.

The limb AB is graduated into spaces which are really half-degrees, but on account of the construction of the instrument **each of these is marked as a whole degree** so that the scale has an extent of twice the actual angle swung (120° to 140°). The arc is graduated in degrees and the micrometer drum in minutes so that the angle can be read directly in minutes. In Fig. 7-3 an auxiliary vernier drum permits readings to 0.1 minute.

The arm IE in the usual sextant is from 5 to 8 inches in length. A pocket sextant having an arm about 2 inches long is very convenient for reconnoissance surveys and for filling in the details of more accurate surveys.

Some sextants have removable telescopes so that either low or high power types may be inserted. Others have telescopes with large objective lenses for night observations. For some work, such as for rough location of soundings, the sextant may be used without the telescope.

A pentagonal prism attachment, available with some instruments, will extend the range of angle measurement by 90°, permitting single angles to be measured up to 215°.

7-9. Principle of the Sextant. In Fig. 7-4 the index glass is at I, the horizon glass at H, and the eye at O. AB represents the arc. Suppose the angle to be measured is between two flagpoles, C and D. A ray of light coming from D passes through the upper (transparent) part of the horizon glass H to the eye at O. A ray of light from C strikes the silvered index glass I and is reflected along IH; the lower part of the horizon glass (silvered portion) then reflects the ray along HO, so that both objects D and C can be seen at the same instant, the point D appearing through the transparent part of H to be in line with, or practically to coincide with, the point C as seen in the silvered portion of H. If the arm VI is moved the line IH will move until it is off the horizon glass; it is evident therefore that there is but one position of the arm VI where IH will intersect the horizon glass at a point in the line OD.

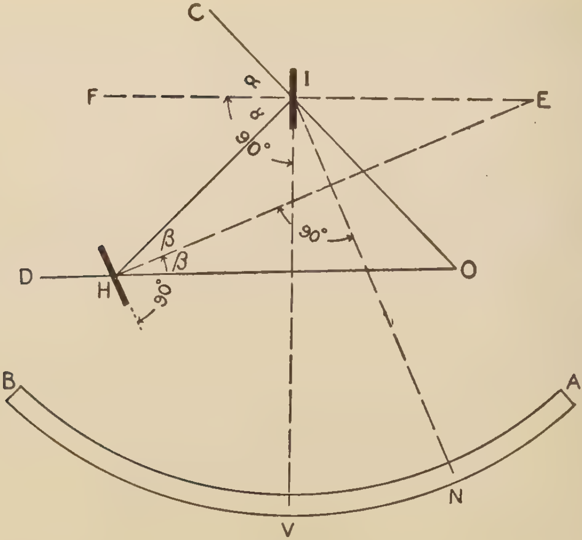


FIG. 7-4. PRINCIPLE OF THE SEXTANT.

Bringing the image of C (by moving the arm) so that it appears on the horizon glass in coincidence with the point D constitutes an observation with the sextant. When this is done the angle COD is read; it is represented by the arc NV as shown below.

The theory of the sextant is based upon the simple physical law that when a ray of light is reflected from a plane mirror the angles of incidence and reflection are equal. In Fig. 7-4 the two angles marked α are equal, and also the angles marked β.

In the triangle OIH

$$\text{angle } O = CIH - IHO = 2\,\alpha - 2\,\beta = 2\,(\alpha - \beta)$$

In the triangle EIH

$$\text{angle } E = FIH - IHE = \alpha - \beta$$

$$\therefore \text{ Angle } O = 2 \text{ angle } E$$

If IN represents the position of the index arm when N is at 0°, then, since IN is parallel to the horizon glass, the angle E

equals VIN. But VIN is the angle passed over by the vernier from the $0°$ mark, and as the half-degree spaces on the circle AB are marked as whole degrees the angle O is read directly from the arc AB.

7-10. Adjustments of the Sextant. To Make the Index Glass Perpendicular to the Plane of the Sextant. Set the arm at a reading near the middle of the arc, then look into the index glass and observe if the image of the arc as seen in the index glass appears to form a continuous arc with that portion of the limb itself which is seen directly. This will be the case if the index glass is perpendicular to the plane of the sextant. If the arc appears to be bend at the edge of the mirror the index glass is not perpendicular to the plane of the sextant; if the image appears above the arc the mirror is leaning forward; if below the arc the mirror leans backward. It should be adjusted by loosening the screws on the back side at the base of the index glass, sliding a thin piece of paper between the frame of the index glass and the arm, and then tightening the screws. This process should be repeated until the adjustment is perfected.

7-11. To Make the Horizon Glass Perpendicular to the Plane of the Sextant. With the sextant held as for taking an altitude set the vernier so that the horizon and its reflected image coincide. Then tilt the sextant right or left. The horizon should still be a continuous line in the horizon glass. If it is not, the adjusting screw should be moved so as to tilt the horizon glass forward or backward until the horizon is continuous. The test and adjustment may also be made by sighting at a bright star. If the tangent screw is moved and the reflected image of the star passes above or below the star as seen direct the adjusting screw should be moved until the two images coincide.

7-12. To Make the Horizon Glass Parallel to the Index Glass When the Vernier Reads $0°$. After the two previous adjustments have been made set the vernier exactly on $0°$, and while looking through the telescope and the transparent portion of the horizon glass at some well-defined line or point such as the sea horizon or a star, observe if the doubly reflected image of this same point in the lower portion of the horizon glass coincides with the direct image. If it does not, the horizon glass must be adjusted by

means of adjusting screws at its base so that it will stand this test. This correction may also be made by holding the sextant horizontally and sighting a flagpole or high chimney.

7-13. Index Correction. In some instruments there is no provision for making the preceding adjustment of the horizon glass, in which case the two images of the same point are brought into exact coincidence and the vernier is then read, giving what is called the *index correction*, which is to be applied to every angle that is read. This cannot be accurately obtained unless the index and horizon glasses have both been made perpendicular to the plane of the sextant. Even though the horizon glass can be adjusted for index error this error is frequently so small that it is not worth while actually to make the adjustment, but the error is determined and applied as a correction whenever the required precision of the results demands it.

If it is necessary to obtain the index correction accurately it may be determined by means of an observation on the sun. The sun and its reflected image are brought so that they are just tangent to each other, one appearing above the other, and the vernier is read. The index arm is then moved so that one image moves past the other and becomes tangent to it on the lower side, and the vernier is again read. One of these readings will fall on the arc while the other will fall beyond the 0° point. Half the difference of the two vernier readings is the **index correction,** which is **plus** if the reading beyond 0° is the greater. It will be found that the contacts can be judged much more accurately if different colored shades are used for the two images, and that the contacts can be made more exact if one of the telescopes is used.

Care must be taken to observe whether this index correction is **plus** or **minus** and to apply it correctly to the measured angles.

7-14. To Make the Line of Sight of the Telescope Parallel to the Plane of the Arc. The telescope usually contains two horizontal and two vertical hairs forming a small square; the images of the objects between which an angle is to be measured are brought into the center of this square.

Set the sextant on a table, sight through the telescope, and mark a point 20 or 30 feet away which shall appear in the center of the square formed by the cross-hairs. Then take two small

pieces of wood, which are practically equal in height to the height of the center of the telescope above the plane of the sextant, and place one of these pieces of wood on top of the arc near the zero end and the other piece near the other end of the arc. Sight across the tops of these wooden sights toward the mark already made when looking through the telescope. The line of sight across the tops of the pieces of wood should coincide nearly with the mark. A difference of half an inch at a distance of 20 feet will make an error of only about a second in the angle measured with the sextant, so that great precision in this adjustment is unnecessary. When the error in this adjustment is large enough to require correction it is done by means of the screws in the collar holding the telescope tube. This adjustment is entirely independent of the others.

7-15. Use of Sextant. To measure the horizontal angle between two objects, hold the sextant in the right hand in a horizontal plane. (For vertical angles, hold in the same hand in the vertical plane.) Sight the left object through the telescope and the transparent half of the horizon glass (*H* in Fig. 7-3). Then turn the index arm with the left hand until the right object appears in the silvered portion of the horizon glass. Clamp the arm when both objects appear to coincide. Make final precision adjustments with the tangent or micrometer screw, for true coincidence of the objects. Coincidence may be tested by twisting the sextant itself back and forth across the plane in which the sextant is being held. The reflected image should move back and forth across the direct image. Read the vernier and apply the index correction (Art. 7-13).

Occasionally the angle to be measured is greater than the range of the sextant arc. In this case the angle is measured in two parts, from one of the objects to some intermediate point **in the same plane,** and then from this intermediate point to the second object.

In making astronomical observations it is frequently necessary to measure with a sextant the vertical angle between the sea horizon and the sun, or some star. When this is done a correction must be applied for the dip of the True Horizon below the Apparent Horizon, as explained in Art. 2-26. In making solar observations the sun's image is dropped down by moving the index arm

until the sun's lower (or upper) limb is tangent to the horizon, and the vernier is read. This angle is then corrected for refraction (Table VII), dip, and the semi-diameter of the sun (Art. 2-27).

7-16. Precautions in the Use of the Sextant. It should be borne in mind that the angle measured with a sextant is **not** the horizontal angle between the two points, as is the case when measured with a transit, but is **an angle lying in the plane defined by the two objects and the eye of the observer.**

Furthermore the **vertex of this angle** (*O*, Fig. 7-4) **is not a fixed point.** It is evident from the figure, since the positions of *I* and *H* do not change, that as the angle diminishes in size the point *O* must move farther away from the instrument, and that for very small angles it may be at a considerable distance back of the observer. For angles taken in astronomical observations where the objects sighted are at a great distance the assumption that *O* is at the same place as the observer does not introduce any appreciable error. Where the instrument is used to locate details such as soundings it is not good practice to measure between points that will give small angles, not only because the vertex of the angle is not at the observer, but also because small angles give poor intersections. The sextant is therefore not used in work where the position of the vertex of the angle must be known with any great degree of accuracy. With a sextant of the ordinary size the vertex of a 20° angle will be about 5 inches back of the index glass; with a 5° angle it is about 2 feet; and with a 1° angle it is about 10 feet. These approximate values will illustrate the importance of keeping this point constantly in mind in all work with this instrument.

7-17. Subaqueous Surveys. The determination of the topography of the bottom of a lake, harbor, or other body of water is one of the common problems in hydrographic surveying. In connection with such surveys the character of the material composing the bottom is often desired. Surveys of this kind are made for a variety of purposes, such as to prepare charts for navigation, to design navigational facilities, to determine where material shall be dredged and where such dredged material may be dumped, or to measure the quantity removed. They are also

made for the purpose of discovering what changes are taking place in the bed of a river, canal, or harbor due to dynamic agencies, such as silting of a reservoir, or to acquire data for projecting wharves, sea-walls, breakwaters, levees, dikes, etc.

This work is usually done by first establishing certain points on shore (by triangulation or traverse) to which the hydrographic survey may be referred, and then measuring, usually from a boat, the depth of the water at various points and determining the positions of these points. The measurements of depth are called *soundings*. Since the subaqueous surface is not visible it is evident that for a given degree of accuracy a great many more points must be located to obtain the shape of the surface than would be necessary in an ordinary topographical survey of equal area.

The points on shore to which the hydrographic survey is referred should be so chosen that they will be in clear view from the water surface. Such prominent objects as church spires, windmills, lighthouses, flag-poles, and the like are serviceable, provided they are near enough to the shore to come within the boundaries of the sheet when plotted on the map. The shore line and some of the adjacent topography is usually desired; this is obtained as explained in Arts. 7-3 and 7-6.

There are two general methods for making subaqueous surveys; conventional methods using surveying instruments and sounding with lead-line or rod, and electronic methods employing a sound-reflecting device (echo depth finder) to obtain water depths and electronic position fixing instruments to locate soundings. Surveys of limited extent and for specific purposes are usually made by conventional methods, although the echo depth finder is increasing in use for taking soundings in combination with ordinary surveying instruments for obtaining locations. Surveys of large extent, such as those of coastal areas and off-shore ocean areas made by the U. S. Coast and Geodetic Survey, employ electronic methods exclusively. The conventional methods readily available to the surveyor are first described in detail, then followed by a general description of the electronic methods.

7-18. Instruments Used. Besides the ordinary surveyor's transit, tape, and lining-poles a sounding-pole or lead-line and a boat with its necessary equipment will be required. In many

sounding operations the sextant, timing clock, signals, buoys, and gages for recording the height of water surface are used.

7-19. The Boat. Sometimes the surveyor has no choice in the type of boat, having to use whatever he can hire in the vicinity. When a choice is possible, consideration of the location of the soundings must be made. For soundings which are off-shore for the most part, a round bottom boat is most desirable because it is less affected by tides and currents and therefore more readily controlled. In locations where soundings extend into very shallow water and frequent landings have to be made, the flat bottomed boat is more serviceable. In any event the boat should be large enough to hold three or four men and wide enough for stability, while work is in progress. As soundings are taken from the bow it is apparent that the boat must be fairly heavy and serviceable.

A power boat or out-board motor boat is better in many respects than a rowboat. It can be kept exactly on a range with little difficulty even if the wind is blowing and the current strong. Its speed is more nearly uniform, and it is much stiffer and steadier. A power boat can be used to good advantage where the water is free from grass and the ranges are long. The speed can be regulated by a drag consisting of a $4'' \times 8''$ timber 8 to 10 feet long (or other size to suit the power of the boat) with a rope attached to each end. One rope is fastened to a cleat on the boat and when running free the other rope is loosened so that the drag tows freely through the water. When it is desired to sound, the loose rope is pulled in until the drag takes a position crosswise to the direction of the boat. By varying the angle that the timber makes with the direction of the boat the speed can be regulated to suit the conditions. An iron bucket tied to a rope and allowed to trail the boat, with its open end opposing the direction of the boat, is also a good drag. If a power boat is used a skiff will also be needed for landing along shore unless there are wharves in the vicinity.

7-20. Sounding-Pole and Lead-Line. *Sounding-poles* are made similar to an ordinary self-reading leveling rod. They are usually from 12 to 20 feet long, $1\frac{1}{2}$ to 2 inches square in section, and with an iron or lead shoe of sufficient area to prevent the pole from

sinking into the mud or sand. The shoe is sometimes provided with a cup-shaped cavity in the bottom which enables samples of material to be collected. This cavity should be smeared with some such material as tallow or soap so that the soil will adhere to the metal shoe. The pole is graduated to feet and tenths, and it will be found convenient in reading the sounding to have these graduations on two opposite faces. If, however, the scale is painted on but one side of the rod the leadsman can, by using proper care, handle the rod so that all the wear will come on the back side. Poles to be used for long periods of time should have the graduations notched.

7-21. The Lead-Line. The lead-line consists of a chain or rope with a lead weight at one end. Cotton or hemp lines may be used but produce questionable results, due to their tendency to shrink or stretch, according to their degree of wetness. The U. S. Coast and Geodetic Survey uses an abraided cotton line with a phosphor-bronze stranded wire core. It does not stretch appreciably. However where high accuracy is required, this type of line should be checked frequently. Most surveyors prefer a chain. Copper sash chain, although more expensive than iron, is more serviceable. The chain should be light for ease of handling.

Usually the line is marked in feet, and the tenths are estimated by the leadsman. Graduation marks should be readily identifiable at every five-foot interval. One system of marking found to be practicable consists of the use of square leather markers, about one inch square, on the 5- and 10-foot intervals, with smaller pointed strips on the intermediate feet. The 5-foot markers have a jagged outside edge, and the 10-foot markers have a smooth edge. Holes are punched into these large markers to denote the number of feet; thus one hole with a jagged edge for 15 feet, 2 holes with a smooth edge for 20 feet, etc. With this system the liability of a 5- or 10-foot error is lessened. All lead-lines should be checked at regular intervals. In marking it is best to avoid metal tags or wire fasteners, as they are likely to injure the leadsman's hands. Narrow strips of cloth of various colors are sometimes used for tags, a simple system for distinguishing marks being readily improvised.

The work of taking soundings may be made easier and faster

by paying the line out over a grooved hand wheel mounted in the boat. The rim of this wheel is graduated in feet and tenths for taking readings.

The weights used with a lead-line vary from 3 to 20 pounds, depending upon the depth of water and strength of current. Where there is not much current a 6- to 10-pound weight will suffice for depths up to about 40 feet.

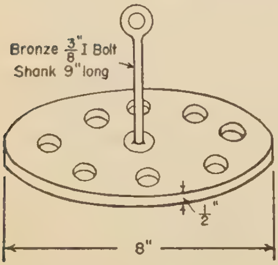


FIG. 7-5. SOUNDING WEIGHT.

The U. S. Corps of Engineers uses a perforated circular, flatbottomed plate weight of 8-inch diameter and weighing 12 pounds (Fig. 7-5). For taking samples of the bottom, a sampler working on the clam-shell bucket principle is used. It opens when lightly dropped on the bottom, and digs in and closes when tension is applied to the line in bringing it to the surface.

7-22. Signals and Buoys. Shore signals, if used to mark the ranges on which soundings are to be taken, may be either common $\frac{7}{8}'' \times 2''$ scantling or poles with iron shoes driven into the ground. On the poles are fastened flags which are visible for a long distance. The color of the flags used will depend upon the background and the distance they must be seen. A white flag is clearly visible against any dark background; a red flag is clear against anything except a medium dark background; and a black flag is the best when the background is very light. The visibility of a flag may be affected by many different conditions, the manner in which the sunlight strikes it being the most important. A shore signal may sometimes consist of a $2'' \times 4''$ or $4'' \times 4''$ mast properly braced at the bottom, with a flag made of cloth nailed to two short horizontal pieces of lath. The ordinary tripod signal employed in triangulation work may also be used (see Art. 1-15). Where there are many signals, such as those used to mark range lines, they are sometimes distinguished by Roman numerals made of short strips of wood which are fastened crosswise to the mast. For short ranges where a flag of any color will be visible a system consisting of a number of colors is often used

to identify the signals where several different ranges are in use at one time.

The visibility of signals can be greatly increased by the use of bright-colored, fluorescent paint or cloth. A flaming orange color is very effective.

Conditions may exist where one of the signals on each range will have to be in the water, and in such cases if the water is deep it may be necessary to use floats, properly moored and provided with a mast to which the flag is attached. When floats are used allowance will have to be made for changes in position due to the action of currents, wind, and tide. Where there is no tidal action a buoy may be anchored by three guys so that it will remain in one position. Sometimes it is found to be so difficult to set shore ranges that it is advisable to put all the ranges in the water. Ordinary scantling $\frac{7}{8}'' \times 2''$ in section may be used for this purpose. These are cut 3 feet longer than the depth of the water; a weight is attached to one end of the stick and a flag to the other, and then it is placed in the proper position in the water, the weight resting on the bottom. Unless the water is quiet it may be necessary to steady these sticks by guys.

7-23. Staff Tide Gage. The staff tide gage usually consists of a board painted white, with a scale of feet and tenths painted on it in black and secured in an upright position in the water. The elevation of the zero point of the gage should be connected by a line of levels with some bench mark on shore. Care should be taken in setting the gage to have it extend low enough for extreme low tides and high enough for readings at very high tides.

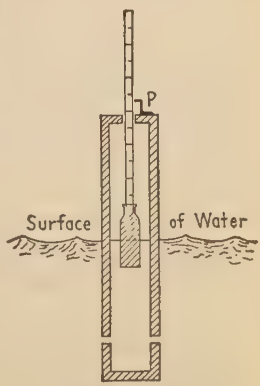


FIG. 7-6. TIDE GAGE FOR ROUGH WATER.

Gage readings are taken at regular intervals of time during the work so that a sounding whose time has been taken may be referred to mean low water or to some other desirable datum plane. (See Fig. 7-10, for form of notes.)

Where tides must be observed on an open coast subject to the action of

waves it is necessary to use a gage that is unaffected by this action. A simple form (Fig. 7-6) consists of a long wooden box in which an empty bottle, having a graduated wooden rod fastened tightly in its mouth, floats up and down with the tide. The box has a wooden bottom and top and has several holes bored low down in its sides to admit the water. The graduated rod moves freely up and down through a hole in the top of the box as the bottle rises or falls, and the scale is read at the pointer *P*. The graduations on the rod should increase from top to bottom. The elevation of *P* must of course be referred to permanent shore points, and the box must be fastened to some stationary object such as a wharf or a pile.

Another form of tide gage for use in rough water is illustrated in Vol. I, Art. 262a.

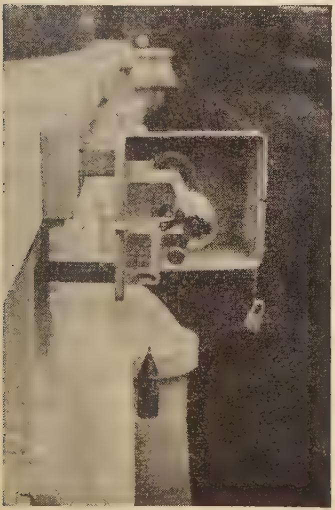
7-24. River and Lake Gages. In taking soundings or in measuring the discharge of rivers it is necessary to erect gages at convenient places, e.g., on bridge piers or abutments, and to have them read at regular intervals. When the gage is fastened to a bridge pier it is usually found that the eddying of the water at the pier makes it difficult to obtain an accurate reading of the gage. This effect can be largely done away with by attaching, lengthwise to the staff, wing pieces making an angle of about 30 degrees with the face of the staff and flaring toward the back of the staff. These wing pieces prevent the eddies and make the water surface in front of the gage smooth.

On a sloping bank an inclined staff may be installed, the graduations on it being so marked that the **vertical** distance between them is the same as on a vertical staff.

7-25. Wire-Weight Gage. The wire-weight gage (Fig. 7-7) is used on bridges and overhanging structures. It consists of a reel on which a stainless steel wire cable about 100 feet long is wound. To obtain the elevation of the water surface the weight, which is attached to one end of the wire, is lowered until it touches the water surface. The gage reading in feet is observed on a counter; the tenths and hundredths are read on a graduated disc.

As these river gages are located at intervals along the shore their zero points are not at the same elevation. They are connected, however, by leveling, so that the water level at the var-

ious gages may be referred to the same datum as the map of the surrounding country. The slope of the river may then be determined from simultaneous gage readings at different points along the river. On account of the slope of a river it is impracticable to refer soundings taken in the river to a datum plane. It is customary, therefore, to refer them to the surface of the water when the river is at a certain stage — high water, mean



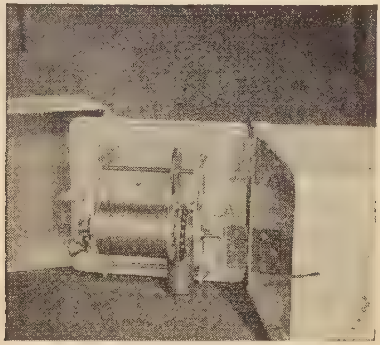


FIG. 7-7. WIRE-WEIGHT GAGE. IN POSITION ON BRIDGE (LEFT). ASSEMBLY (RIGHT).

water, or low water stage, the soundings being the depth of the river bottom below the water surface at this particular stage. As the soundings may have to be taken when the river is not at the assumed stage it will be necessary before taking them to first determine from a series of observations (taken daily or oftener at each gaging station) the various gage readings corresponding to the assumed stage of the river; for example, at the upper gage a reading of 2.3 may be found to represent the mean water stage, while 1.7 is the reading for mean water at the next lower gage, 2.1 at the next staff, and so on.

For lake surveys the datum to which the soundings are referred is often the lowest recorded stage of the water. In this case the frequency with which the gage should be read will depend upon

the various conditions which influence the level of the water surface.

7-26. Automatic Tide and River Gages.* In many branches of engineering it is of inestimable value to have an accurate and continuous record of the fluctuations of water levels. For making such records an automatic (recording) gage is used. Instruments for this purpose are available which are convenient to handle and which require only occasional attention of an observer.

For **tidal observations** there are two general types. In one type the record of the height of the tide is graphically represented by a curve automatically traced by the machine on cross-section paper, the abscissæ representing the time interval and the ordinates the height of the water surface; in some makes of this same general type of machine the representation is by polar coordinates, the time being recorded by the angle and the height of the tide by the radius vector. In the other general type the heights of the water surface are printed as arabic figures at equal intervals of time, such as every 15 minutes; the gage, however, can be set so as to record at any desired time interval.

In all automatic tide gages a dependable clock movement is essential. The rise and fall of the tide is usually transmitted to the machine through a float that is free to rise and fall with the change in water surface in a vertical box or pipe to which the water has access through a sufficiently small opening so that the water surface effect of abrupt wind waves will not be registered. The motion of the float is communicated to the recording device by a wire or perforated tape.

In the first type of gage mentioned above the paper is usually moved by the clockwork and the pencil is operated by the float; in some gages, however, the paper is moved by the float and the pencil by the clockwork.

Two different sizes of automatic tide gage are used by the U. S. Coast and Geodetic Survey, a non-portable and a portable type. The heavier one is used at the principal tide stations. Its record paper is a roll 15 inches wide and 66 feet long, which is sufficient for a month of tidal record when the time scale is one inch per

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* See Art. 8-11 for description of pressure type recording gage.

hour. The gage can be adjusted to different height scales to accord with the observed range of tide. The paper for this gage is not ruled; as the record is being traced by the tide, a datum line is also being automatically drawn by the machine, and from this datum line the heights may afterwards be scaled off. There is also a time-marking device by which the beginning of each hour is indicated on the record.

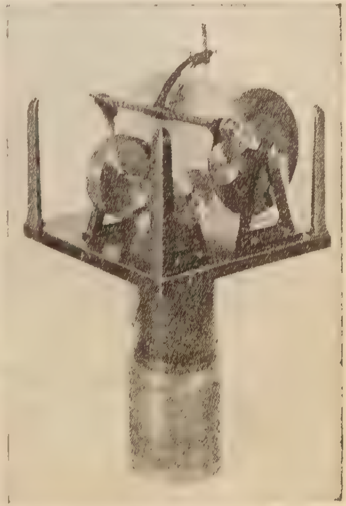

FIG. 7-8. PORTABLE AUTOMATIC TIDE GAGE, U. S. COAST AND GEODETIC SURVEY.

The gage shown in Fig. 7-8 is the portable type automatic tide gage of the Coast and Geodetic Survey, which was designed primarily as a light-weight and easily installed gage for use by hydrographic parties in obtaining a short series of tidal observations for reduction of soundings or for comparative purposes. The paper for this gage is 7 inches wide and 20 inches long, ruled both vertically and horizontally in accordance with the time and height scale to which the gage is to operate. The paper is waxed and the record scribed on it by the stylus. An 8-day clock rotates the drum once in 48 hours, and the paper is usually changed every 4 or 5 days. Because of the advancement of the times of high and low waters on successive days, the paper may be left on the gage for several revolutions of the drum and still leave a distinct record for each day. A detailed description of tide gages is included in Special Publication No. 196, Manual of Tide Observations, U. S. Coast and Geodetic Survey.

Automatic River Gages are used extensively by the Water Resources Branch of the U. S. Geological Survey. They are manufactured by instrument makers under specifications of the Survey. Standard automatic water-stage recorders of the Stevens

type are made by Leupold & Stevens Instruments, Inc., Portland, Oregon. (See Fig. 7-9.)

The essential requirements of the U. S. Geological Survey automatic gages are as follows. They must operate continuously

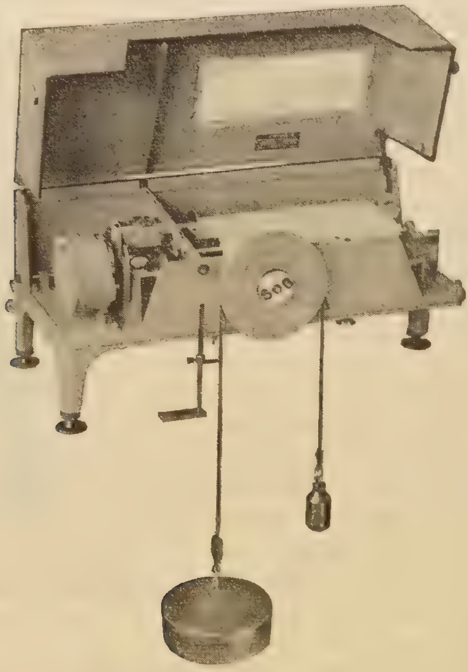

FIG. 7-9. STEVENS WATER LEVEL RECORDER.
(Courtesy, Leupold & Stevens Instruments, Inc.)

for 60 days without requiring attention and the height element and the time element acting conjointly shall trace on a record sheet a continuous graph of the water stage and the time.

The time element has two or more parallel rolls; one is the recording roll, the other is the supply roll. Both are actuated by a weight controlled by a clock in such a manner that the paper will travel at a uniform rate and in a taut condition from the supply roll to the recording roll. The usual time scale is 2.4 inches

per day, but the recorder is so designed that the time scale may readily be changed to 1.2, 4.8, 7.2 or 9.6 inches per day, thus making it adaptable to water surfaces that change slowly or rapidly. The clock is actuated by a weight supported by non-corrosive metal cable; it can be wound without removing either of the rolls and so as not to cause any perceptible movement of the record paper.

The height element consists of a float, float cable or cables with counterweight, a drive float wheel, and some means of reducing the motion by a constant ratio and driving the recording stylus. The latter moves back and forth across the paper in a path parallel to the axis of the roll. The water surface height is usually recorded crosswise on the paper at a scale such that a 1 foot change in water surface is represented on the record paper by either 1 inch (ratio 1:12) or 2 inches (ratio 1:6), both of which ratios are readily interchangeable in the field. Additional gears are supplied with the recorder which can be inserted to provide ratios of 1:24, 5:12 and 10:12, as desired.

The stylus is either a gravity-type pencil or a syphon-type pen. It will record changes of 0.01 ft. in water stage for 1:12 ratio. The stylus carriage is so designed that adjustment for accurate reversals can be made and also so that the recorder cover cannot be replaced unless the stylus is in the recording position.

The paper, which is usually in 25-yard rolls (unspliced), is graduated in units of 0.1 inch in both directions. Height is recorded on the drum crosswise of the record sheet and time is measured lengthwise of this sheet. Every 10th height line is made heavier than the others, and every 24th time line is made heavier to designate midnight; the 12th time lines in between are heavy dashed lines to designate noon.

It is customary for the Water Resources Branch inspector to visit these recording gages at intervals of from one to three months to wind the clock and renew the record sheet. On streams carrying heavy sediment loads, also on those subject to frequent sudden rises, inspection is needed at more frequent intervals in order to see that mud or sand is not clogging the pipe connecting with the river. The well also is subject to sedimentation, and if not flushed out after each flood the float may become stranded

on sand deposits during low stages. This condition is discovered when the recording stylus is found to be moving in a horizontal straight line.

The duties of the inspector at each visit consist of: (a) checking the stage on the graph against the reading of the staff gage, (b) checking the time indicated on the graph and writing in pencil on the graph the correction to be applied and (c) making sure that no sediment deposits in the well or in the pipe connecting with the river are interfering with the registration.

When received at the office the data from the record sheets are tabulated by daily or twice daily gage readings. Before this is done, a red ink line is drawn on the record sheet averaging the jagged tooth-like graphical record and corrections are applied for stages and time as noted by the field inspector.

The method of installing the gage and connecting it with the river is shown in Fig. 8-11. The pipe which runs to the river should be located so that it will be well below the ice at the lowest stage of the river. The shelter * should be weather and dust proof and large enough so that the observer can conveniently work around the recording instrument, which should be rigidly supported on a rigid shelf or table. A trap door in the floor is provided for access to the float well which should be large enough to accommodate a man; 4 ft. square is recommended. A vertical pipe descending well below the orifice of the pipe connecting with the river is provided for flushing out sediment.

Where ice conditions are severe it is advisable to use kerosene in the float pipe, in which case correction of water level, due to the fact that kerosene is of different specific gravity than water, is necessary. A staff gage should also be installed nearby for use in setting and checking the recorder of the automatic gage and for comparing the stage within the well with that of the river outside. Permanent bench-marks or reference marks should be established nearby for use in maintaining the datum of both the automatic and staff gages.

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* See U. S. Geological Survey Water Supply Paper 888, "Stream-Gaging Procedure — A Manual Describing Methods and Practices of the Geological Survey."

7-27. Organization of Sounding Party. The organization of the sounding party will depend upon the method used in locating the soundings. In any event there will be in the boat the *chief of party*, the *recorder*, the *leadsman*, the *oarsman*, or *engineman*, and possibly a *signalman*. This comprises the entire complement of the boat party provided the soundings are located by angles taken from the shore, in which event the transit man and assistants (if needed) will remain on land. If, however, the angles are to be taken from the boat, the *instrumentman* who will use the sextants will also be in the boat. Where work is to be done in tidal water a *gage reader* will also be needed on shore.

7-28. Sounding Procedure. Prior to commencing sounding operations all watches should be synchronized. This is necessary as soundings are to be made at fixed time intervals, and each individual member of the party must be in a position to accomplish his part of the work at the specific interval. The chief of party usually acts as signalman. He begins the work by raising a flag to warn the instrumentmen on shore that the work is about to start. At an even minute he drops the flag and calls out, "Sound." The leadsman immediately takes a sounding, and the instrumentmen on shore locate the sounding at the same instant. Also at that same instant the boat is being propelled along a range, previously determined, and at a uniform speed. The leadsman then takes soundings at full 5- or 10-second intervals. As the next minute approaches, the signalman raises his flag, usually 5 to 10 seconds ahead of the minute, and drops it on the minute. The sounding and location are then made as before. This procedure is followed for the desired length of the sounding range. The recorder, in the mean time, records each sounding and the corresponding time. Basically this method is used for all soundings, although refinements may be made in the procedure when necessary to obtain greater accuracy. Signal flags are made in more than one color. The signalman varies the color on each location and the recorder notes the color of the flag along with the sounding and time. The instrumentmen also note the color of the flag as a further check on the locations and soundings. In tidal waters the tideman is noting the tide level and time of

variation. This provides data for reducing the soundings to whatever datum plane is used in the locality.

7-29. Tidal Observations. The tide gage or staff is usually read at 15-minute intervals, unless the rise and fall is more than a foot per hour. The gage height at any intermediate time can readily be obtained by interpolation. Where it can be done it is best to employ a man to read the tide gage and to record the readings at 5-minute intervals. A convenient form for these notes is shown in Fig. 7-10. The direction and force of the wind should

POINT MERIDETH HARBOR. — TIDE GAGE READINGS.

David Ray, Observer, Sept. 11, 1957

Time	Gage Reading	Correct Tide	Remarks
8.00	2.8	2.2	Fresh N. W. wind.
.15	2.7	2.1	
.30	2.6	2.0	
.45	2.4	1.8	
9.00	2.1	1.5	
.15	1.8	1.2	
.30	1.5	0.9	
.45	1.2	0.6	Gage set on N. E. Pile of Thompson's Wharf.
10.00	1.1	0.5	Bottom of fender cap of wharf reads 12.85 on
.15	0.9	0.3	gage. Referred to B.M. No. 1 on plan of 1933,
.30	0.7	0.1	correction for gage — 0.6.
.45	0.6	0.0	
11.00	0.4	−0.2	
.15	0.3	−0.3	
.30	0.3	−0.3	
.45	0.3	−0.3	

FIG. 7-10. TIDE GAGE READER'S NOTES.

also be recorded, especially when the tide gage is not located close to the place where the soundings are being taken. It is well to locate the tide gage as near to the sounding operations as convenient, and it is imperative that the gage should be located in the same tidal basin as the soundings. In the " Remarks " in the above notes it will be seen that the gage reading of the bottom of the fender cap is recorded. By referring to such a record the position of the gage can be readily checked at any time should there be any suspicion that it had been knocked out of place and

not replaced at exactly the point where it was originally fastened.

Under some conditions the tide gage can be read by one of the boat's crew by means of a field-glass or by the shore assistant or transitmen if the gage is near enough to any of them.

7-30. Methods of Locating Soundings. Common methods of locating **lead-line** soundings are as follows:

(1) The boat is rowed on a range line and the positions of the soundings are " cut in " by a transit angle taken on shore or by an angle taken with a sextant from the boat.

(2) The boat may or may not be rowed on any definite range and its position is located by angles taken simultaneously by two transits on shore or by angles taken simultaneously to shore points from the boat by means of two sextants.

(3) The positions of the soundings are located by the stadia method.

(4) The positions of the soundings are defined by the intersection of fixed ranges.

(5) A wire or line is stretched across a stream from shore to shore and soundings are taken at different points along this wire and located by measured distances from one end of it.

7-31. Locating a Sounding by a Range and an Angle from Shore. In still water where there is no difficulty in keeping the boat in any desired position soundings may be conveniently located by keeping the boat on a range line marked by two objects on shore, such as range poles, and then " cutting in " the position of the leadsman by means of a transit angle taken from shore at the instant the sounding is made (see Fig. 7-11). Where the ranges are marked by poles one of these may be set in the water or a float may be anchored and used to define one end of the line if it is not convenient to place both poles on land.

Usually the ranges are parallel in a straight reach of a stream, or fanned as shown in Fig. 7-11 in rounding a bend. Shore signals are set at regular intervals, 25, 50 or 100 feet, usually off the traverse line. It is not necessary to locate each shore signal as long as the azimuth of the range and its intersection with the traverse line are known. When ranges are fanned, it is of course necessary to know both the location of the fan point and the

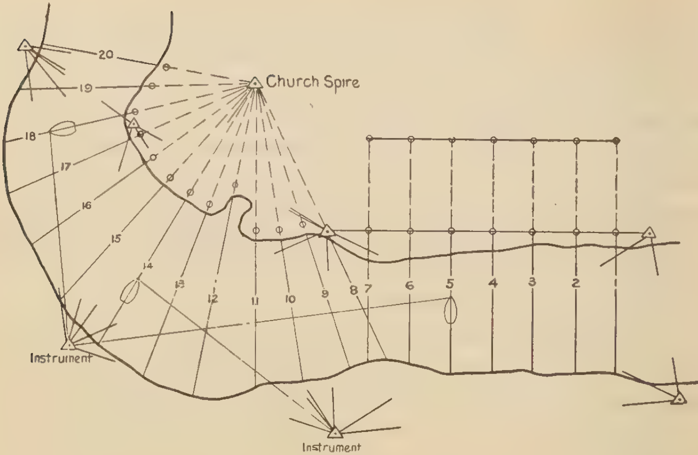

FIG. 7-11. LOCATING SOUNDINGS BY RANGE LINE AND ANGLE FROM SHORE.

shore signals. However sometimes a fan point is located, the transit set on it and then by regular angles the shore signals are fixed. A back signal is then erected on the fan point. In any case the range lines must be so chosen that they will afford a base long enough to be accurately projected across the water and so as to cover the entire area as closely as is desired. When the fan-shaped ranges are used it is advisable on account of their divergence at the further shore to take a series of soundings on other ranges running crosswise. This is particularly desirable where the radial lines are very far apart. In fact it is always a good plan to run at least a few lines crosswise to the first series, as it helps to verify the work and also to fill in the gaps.

7-32. Fieldwork and Notes. In this method of locating soundings the transit point should be so chosen that the intersection of the lines of sight will cut the range lines as nearly at right angles as possible. The vernier of the plate is set at 0°, or if preferred at the proper magnetic or true azimuth, and the telescope sighted at some signal or object on shore whose location is known. The telescope is then turned in the direction of the boat, ready for measuring angles. The transitman keeps the vertical cross-hair of his instrument on the sounding-line, as

nearly as he can, and follows it as the boat moves along the range line. At a signal from the boat the transitman clamps the instrument (sometimes leaves it loose) as nearly as possible on the angle to the sounding line, and reads and records both the angle and the time, as shown in the notes in Fig. 7-12. If colored

POINT MERIDETH. — SOUNDING LOCATIONS.

K. and E. $\overline{/|\backslash}$, at S.B. "A." L. Hayes, Sept. 11, 1957.

Time	Flag	Angles	No.	
0° on S.B. "B"				
h m s				
8–15–20 Begin	B	32° 15′	1	
....	W	34 00	2	
....	W	35 45	3	
....	B	37 05	4	
....	W	38 45	5	
....	W	40 20	6	
....	W	?	7	Yacht in way.
....	R	43 30	8	
....	W	44 40	9	
....	W	46 05	10	
....	W	47 25	11	
....	W	48 35	12	
8–27–10 End	R & W	49 40	13	
8–35–00 Begin	W	50 50	14	
....	W	53 00	15	
....	B	54 05	16	

FIG. 7-12. TRANSITMAN'S NOTES.

flags are used by the signalman in the boat, the color of the flag shown is also recorded. At the end of the day the transitman's notes and the notes of the soundings are compared, and the observations are numbered to correspond. Occasionally the transitman will fail to measure an angle when a certain sounding is taken. This should be discovered when the notes are numbered

and compared, since both parties have recorded the times of their observations.

On account of the rapidity with which the angles must be read the transitman will usually leave the upper plate of the instrument unclamped and simply set by hand. A good transitman can easily take two angles per minute and record them, together with the time; with an assistant to record he can take five or six angles per minute if necessary. The minutes in the angle are sometimes read by estimation instead of by the vernier, as it is usually sufficiently accurate for this kind of work if the angles are correct to the nearest 5 minutes.

In the notes shown in Fig. 7-12 the soundings were not taken at any regular time interval, as is done by some surveyors. In this method it is not usually practicable to record the time of each angle, the signals being given too rapidly, but it is sufficient to take the time at the beginning and at the end of each short range and at a few intermediate points on long ranges, leaving to the recorder in the boat the duty of noting the time when each signal was given. Where the angles are taken on a particular time interval, as illustrated in Fig. 7-14, the time may be recorded for each angle. After the soundings on two or three range lines have been taken the transitman should check his work by sighting again on the signal and seeing that the vernier reads the azimuth originally set. The recorder and the transitman should check up their notes each half-day if possible, but in any case at the end of each day.

It will be observed in the following notes (Fig. 7-13) that the recorder in the boat has recorded the depth of each sounding and also the times of those soundings which are located. The observed times are used to reduce the soundings to the datum, and also to identify the angles taken by the transitman. The letters in the second column indicate which soundings were located by transit angles and the color of flags used; and it will be seen that these correspond to the transitman's notes in Fig. 7-12. The reduced gage readings, which are shown in the column headed "Tide," are copied from the gage reader's note-book; the tide for any time may be obtained by interpolation between the observations taken. The first three columns are the only ones

POINT MERIDETH HARBOR. — SOUNDINGS.

Sept. 11, 1957.

H. G. Wells, in charge. D. Ray, gage.
John Smith, recorder. J. Sidley, leadsman.
L. Hayes, at ⊼ S. B. "A."

Time	Flag	Sounding	Tide	Reduced Sounding	No.	Remarks
Line No. 1. Running South.						Fresh N. W. wind.
h m s 8–15–20 Begin	B	3.0	2.1	0.9	1	125′ ± from shore.
....	......	3.5	....	1.4	......	
....	......	3.7	....	1.6	......	
....	......	3.7	....	1.6	......	
....	W	4.3	....	2.2	2	
....	......	5.7	....	3.6	......	
....	......	5.7	....	3.6	......	
....	......	7.9	....	5.8	......	
8–17–10	W	7.5	....	5.4	3	
....	......	8.6	2.1	6.5	......	
Etc.						
8–26	W	34.3	2.0	32.3	12	
....	......	34.5	....	32.3	......	
....	......	34.5	....	32.3	......	
8–27–10 End	R & W	33.5	....	31.3	13	Near W. end of breakwater.
Line No. 2, Running North.						
8–35–00 Begin	W	34.7	2.0	32.7	14	Near W. end of breakwater.

FIG. 7-13. RECORDER'S NOTES.

that are filled in during the fieldwork. The sixth column, headed "Number," is filled in when the transitman's notes are compared with those of the recorder at the end of the day.

In plotting notes like these where the soundings are quite close together the points which were "cut in" are located on the plan and the intermediate readings are interpolated between them; the soundings are assumed to be equally spaced between the ones which were located.

It will sometimes be found more convenient to take the located soundings exactly one or two minutes apart and to take the intermediate soundings at equal intervals of time, as shown in Fig. 7-14.

POINT MERIDETH HARBOR. — SOUNDING LOCATIONS.

K and E. ⊼ at S.B. "A." Frank Jones, Sept. 11, 1957.

Time	Flag	Angle	Number	Remarks
0° on Lantern Staff on Breakwater. L. Hayes ⊼ occupying S.B. "B."				
		134° 17′		to S.B. "B"
8.53 Begin	W	7° 56′	1	Line No. 7, South.
.54	R	10 14	2	
.55	W	12 47	3	
.56	W	15 15	4	
.57	W	17 46	5	
.58	R	19 48	6	
.59	W	21 41	7	
9.00	R & W	23 28	8	R & W check watch, 9.00 A.M.
.01	W	24 55	9	
.02	R	26 19	10	
.03	W	27 33	11	
Etc.				
.10 End.	W	34 35	18	
9.20 Begin	W	36 13	19	Line No. 8, North.
.21	W	35 01	20	
.22	R	34 03	21	
.23	W	33 51	22	
.24	W	33 00	23	
.25	?	?	24	Schooner in way.
.26	R	31 31	25	

FIG. 7-14. TRANSITMAN'S NOTES.

7-33. Locating a Sounding by a Range and an Angle from the Boat. The method of locating a sounding by a range and an angle from the boat is like the one just described, with the exception that the angle instead of being taken from the shore is measured with a sextant from the boat. It is not as common a method as the previous one because it increases the office work, and the only advantage so far as the fieldwork is concerned is that the instrumentman is in the boat with the chief of party, who can therefore direct the work to better advantage. But since it will probably be necesssry to use a transit in laying out the ranges

the soundings may as well be located with the same instrument and the office work thus simplified.

7-34. Locating a Sounding by Two Angles from Shore. Where it is impossible to keep the boat always accurately on range or where it is not convenient to establish ranges, the position of soundings may be located by two angles taken simultaneously from shore. In this method two transits are used, each being set up at previously determined triangulation points or points on the shore traverse, or else at selected instrument points which can later be tied to the triangulation system. These points should be so chosen that the transit lines locating the soundings will cross **as nearly at right angles as is practicable.** The instruments are both run as described in Art. 7-32, the angles being taken **simultaneously** on signal from the boat. When the boat is a considerable distance away it is customary to take the angle to the leadsman or to the portion of the boat in which he is standing rather than to set exactly on the sounding-pole. Evidently for a given degree of precision this approximation is as applicable for short as for long distances.

This method affords a quick way of obtaining the necessary data for such a survey. The U. S. Corps of Engineers have frequently employed this method, together with the use of approximate ranges. The soundings are taken on ranges determined by pacing off nearly equal distances along the shore and determining by eye the parallel courses. The soundings are taken at equal intervals of time and the recorder's notes are kept practically as shown in Fig. 7-13, except that the time of each sounding, say one every 15 seconds, is recorded in the first column. Each of the transitmen keeps a set of notes similar to those shown in Fig. 7-14.

7-35. Locating a Sounding by Two Angles from a Boat. A common method of locating soundings when ranges are not used is by taking two angles simultaneously from the boat to three signals, or any previously determined points, *A*, *B*, and *C* on shore (see Fig. 7-15). This is an application of the **Three-point Problem** which is frequently used in plane-table work. (See Art. 1-57.)

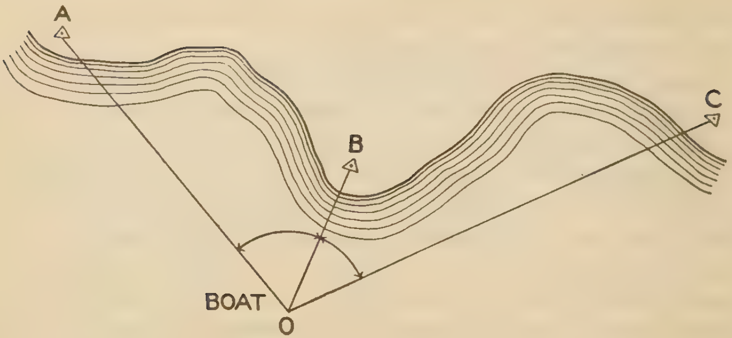

FIG. 7-15. LOCATING SOUNDINGS BY TWO SEXTANT ANGLES FROM BOAT.

Since it is necessary to employ an instrument which does not require a steady support like the transit the angles are usually measured with two sextants. Two angles are sufficient to locate the position of the boat except when the boat happens to be on the circle passing through the three signals; in this case the position is indeterminate. The accuracy of the location will vary according to the relative position of the signals sighted, and will in general be governed by the angle at which the three position circles intersect each other. If the three signals are in line or if the middle signal is nearer the boat than the other two signals and provided no angle is less than 30°, then the location will be strong. If the signals are all very distant and the angle between two of them is small, the location will be weak. If, however, two signals are in range, or nearly in range, so that the observed angle is small, the location will be strong, provided the points on range are not actually near together and provided the second angle is not too small, say not less than 30° to 40°.

Small angles between signals should ordinarily be avoided because they give weak positions in most cases and also are likely to be inconvenient to plot.

Better results will generally be obtained by the use of near points rather than distant ones, because any errors in measuring the angles or in plotting them will have less effect on the plotted position of the sounding.

When the middle signal is very close, however, and the other

two are distant, the angle between the outside signals should be measured, if possible, in order to avoid the error of parallax on the near signal due to the fact that the two angles are not taken at the same instant. This may also be avoided by taking the two angles from the same spot.

The sextant method has the advantage that the entire party is in the boat and can be more easily handled than where there are one or two transit parties on shore. The party can be quickly moved from one place to another.

On the hydrographic work of the Coast and Geodetic Survey the party usually consists of the chief, who directs the work, measures the left (sextant) angle and also plots; the right angle man; the recorder; the leadsman; and the boat crew. On a very small job the left angle man would also record. Soundings are taken on approximate ranges. Sextant angles are not taken for every sounding, but frequently enough so that the remaining soundings may be spaced in between the located points in proportion to the elapsed time without danger of error. The soundings are plotted as the work progresses in order to be certain that all necessary data are taken before leaving the locality. Sometimes the plotting is done in the boat (that is, on the " boat sheet," on heavy mounted paper or film base) on a plotting board; sometimes it is done directly on the original hydrographic sheets in the field office. These latter are completed in pencil before being sent to the main office.

7-36. Locating Soundings by Stadia. As the stadia method is admirably adapted to measuring distances to inaccessible points, and as its results are within the limit of accuracy required for most sounding work, it would appear at first thought to be most useful for locating soundings. This would be quite true if it were not for the fact that both the instrument and the rod must be at rest if an accurate observation is to be made.

The transit may be set up on shore and the stadia rod carried in the boat. The rod if properly painted and of sufficient length may serve the double purpose of a sounding-rod and a stadia rod. At the instant the sounding is taken the transitman observes the interval on the rod, and then records it. The corresponding azimuth angle can be read while the boat is proceeding along its

course. The transit should be set up at a point which will be as nearly at the shore level as possible, so that there will be no necessity for noting the vertical angles. It will simplify his work if the transitman sets up his instrument at the end of a range on which the boat is being rowed, for in this case he will have only the rod interval and the time to observe. It will sometimes facilitate matters if a regular sounding-pole is used for the soundings and a separate stadia rod is used for the transit sights. If the boat is rather heavy and the water calm it may be possible to read the stadia rod when held vertical with its base resting on the bottom of the boat.

It is evident that this method is applicable only in shallow waters. The " range and angle " method is fully as accurate as the stadia method, but where only a few isolated soundings are to be taken the latter method is to be preferred because it requires less fieldwork.

7-37. Locating Soundings by the Intersection of Fixed Ranges. In cases where it is necessary to take soundings at various periods of time at some given point, fixed ranges are located on shore so that their lines will intersect at angles as near 90 degrees as is practicable. The boat will then proceed to the several intersections of these ranges and soundings may be taken as often as desired. This method is especially employed, for example, in making observations at different times to determine whether the bottom of a channel in a given place is filling or scouring, or to determine how much material has been removed by dredging.

7-38. Locating Soundings from a Wire Stretched across a Stream. When taking soundings across a narrow river or canal a wire or rope is sometimes stretched taut from one shore to the other with tags fastened to it at equal intervals, and the soundings taken at these points. This procedure is sometimes called the " tag-line " method. It may also be used in connection with dredging, the soundings being taken before and after the dredging is done (Art. 7-46). The method is accurate but usually more expensive than other methods of locating soundings.

7-39. Reducing the Soundings to Datum. In tidal waters the datum is determined by a series of observations, as explained in Vol. I, Art. 262, p. 309. It is the general practice to use Mean

Low Water as the datum, and to reduce all soundings by subtracting (algebraically) from each sounding the corresponding adjusted gage reading. Besides this reduction, account should be taken of the effect of wind and current, which may be sufficient to make it necessary to apply a correction for errors due to these causes. Furthermore the correction on account of erroneous length of lead-line should not be neglected unless it is small enough to cause no appreciable error.

On the west coast, Mean Lower Low Water is used as the datum for soundings in the Pacific Ocean. In these waters there are two tides of unequal range each lunar day. The mean of the lower tide is therefore specified.

Soundings in rivers or lakes are reduced to the datum to which the gage has been set, usually Mean Sea Level (Art. 7-24). Mean sea-level is a datum plane so placed that the area between this plane and the curve of high waters is exactly equal to the area between this same datum plane and the curve of low waters. This plane is the datum used by the U. S. Coast and Geodetic Survey, Geological Survey, Corps of Engineers and other governmental organizations when establishing control levels. For a single lunar month, the two data may differ by as much as one foot. If the observations extend over just one lunar month the result will be fairly good, whereas in less than one month a close result cannot be obtained; to determine this accurately will require observations extending over a year or more.

Level surveys for waterfront structures, beach and harbor protection works are usually based on Mean Sea Level obtained from bench marks established by government agencies.

7-40. Plotting the Soundings. Where the soundings have been located by ranges and angles taken from points on shore, the ranges and transit stations are plotted on the work sheet, and angles laid out from transit stations by means of a protractor. The soundings are located at intersections transit sights and range lines.

Several methods have been devised for rapidly plotting the positions of soundings which have been located by the various field methods; most of these require that the plan shall be on some transparent material. Where for example the soundings are

located by angles from two ends of a base-line, this base-line is plotted on the plan which is made on tracing cloth. Paper protractors with the radial lines drawn on them and properly marked may be made and kept in the office for general use. One of these protractors should be drawn in black ink on firm white paper and another in red ink on very transparent tracing paper. In plotting the soundings by this method the plan is placed over the black protractor so that one end of the base-line is exactly over the center of the protractor. The other (transparent) protractor is then slid in under the transparent plan and above the black protractor until its center is exactly under the other end of the base-line. Both protractors are, of course, placed so that their zero lines coincide with the base-line or whatever zero line was used by the respective transitmen in the field; and where the black radial line representing the angle read from one end of the base-line cuts the red line representing the corresponding angle at the other end of the base-line is the location of that sounding.

Another method of plotting the soundings upon the original plan without the use of transparent paper as in the method just explained is to use two annular protractors printed on firm paper or transparent plastic with graduations on the outer circumference. These are properly set at the respective transit points and the soundings plotted by the intersection of two threads or straight-edges pivoted at the centers and swung to the corresponding angles of each sounding. Two men, one for each protractor, may plot the located soundings very rapidly by this method by first numbering the locations faintly in pencil and afterward putting on the depths in ink. The size of protractors required will depend upon the size of the sheet used, but for most work a diameter of 14 inches will be found sufficient.

7-41. The Three-point Problem. Where the soundings are located by sextant angles taken from the boat their positions may be quickly plotted as follows. On a piece of tracing cloth or other transparent material lay off the measured angles *AOB* and *BOC* (Fig. 7-15). Lay the tracing cloth down on the map and move it about until the three lines *AO*, *BO*, and *CO* pass through the corresponding signals *A*, *B*, and *C* as plotted on the map. The point *O* may then be pricked through to indicate the position of the sound-

ing. A more rapid method of plotting the soundings is to use an instrument called a *three arm protractor*. These are available in metal or in plastic (Fig. 7-16). The latter have shorter arms than the metal type, but have the advantages of transparency and light weight. Referring to Fig. 7-16, the two arms *C* and *B* are

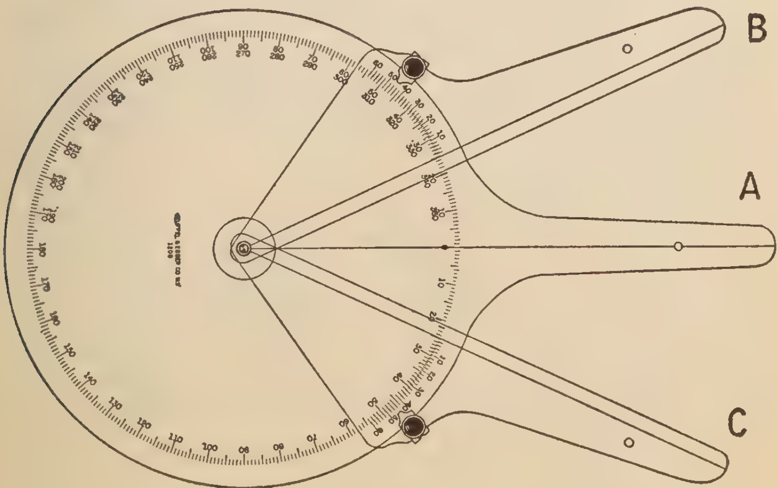

FIG. 7-16. THREE-ARM PROTRACTOR.

movable, while the middle arm *A* is fixed so that its center line is at the 0° mark of the circle. This circle is graduated from 0° each way to 360°. The two movable arms are set at the two angles which were measured simultaneously with the sextants. The protractor is then laid on the plan and moved about until the three arms pass through the plotted positions of the three shore signals. When the instrument is in this position its center locates the position of the sounding on the plan, which may be marked by a needle point. Only one position of the center point can be found from which the three arms pass through the plotted signals, except in the case where the three shore points lie in the circumference of a circle which also passes through the center point. If the sounding is near this circle the location of the center point cannot be determined with sufficient accuracy.

The three-arm protractor can be tested for eccentricity as follows. Draw two fine lines exactly perpendicular to each other, lay the center of the protractor over the intersection of these lines, and see if the 90° points on the arc coincide with the lines. Several angles may be laid off on the same piece of paper by the method of tangents or chords (see Vol. I, Arts. 490–4, pp. 567–71) and the intermediate graduations of the arc compared with these lines to test the regularity of the graduations.

7-42. Geometric Solution of Three-point Problem. While the geometric solution of the Three-point Problem is seldom used for

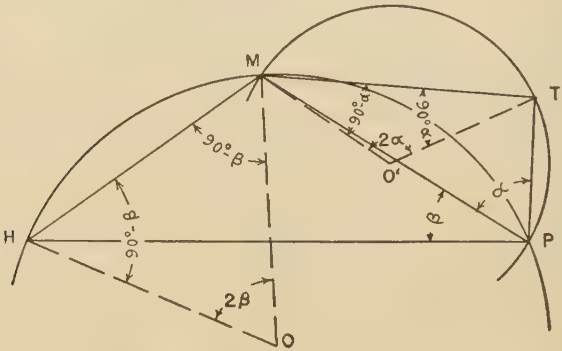

FIG. 7-17. GEOMETRIC SOLUTION OF THREE-POINT PROBLEM.

plotting, on account of the rapidity with which the methods just mentioned can be applied, still its demonstration will doubtless give the student a better understanding of the problem.

In Fig. 7-17, H, M, and T represent the three shore signals. Let the two angles read from the boat be α and β. At both extremities of the line HM lay off an angle equal to $90° - \beta$, the sides of which if extended will intersect at O; and at both extremities of MT lay off an angle equal to $90° - \alpha$, thus obtaining point O'. Then with O as a center describe an arc passing through H and M, and with O' as a center describe an arc through M and T. Where these two arcs intersect, at P, is the position of the boat.

$$\text{Angle } O = 180° - 2(90° - \beta) = 2\beta$$

Similarly $$O' = 2\alpha$$

But $$HPM = \frac{HOM}{2} = \beta$$

And $$MPT = \frac{MO'T}{2} = \alpha$$

The angle β determines the circle HMP, which is the locus of the point P. In like manner the angle α determines the circle MTP as the locus of P, and the intersection of these two circles (at P) is the only point (except M) satisfying both conditions.

The trigonometric solution of the Three-point Problem will be found in Art. 1-57.

7-43. Echo Sounding. In this method high-frequency supersonic sound waves are produced in the boat and transmitted to the bed of the river or ocean; they are then reflected back to the sending instrument. The short intervals of time required for the sound waves to travel to the bottom and back again are measured, converted into feet of depth and recorded by a stylus on a moving strip of profile paper. The process is illustrated in Fig. 7-18. The oscillator which transmits the sound waves and the hydrophone which receives the reflected waves are mounted in the hull of the ship; the recorder is in the cabin. One type of recorder is shown in Fig. 7-19. The depths are traced on a moving roll of coordinate paper, 100 feet long and $6\frac{3}{32}$ inches wide. The window is hinged so that notations may be made in the graph.

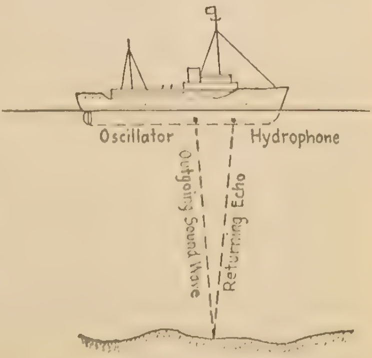

FIG. 7-18. METHOD OF TAKING SOUNDINGS.

Two types of acoustic depth-finders are in general use non-portable and portable.

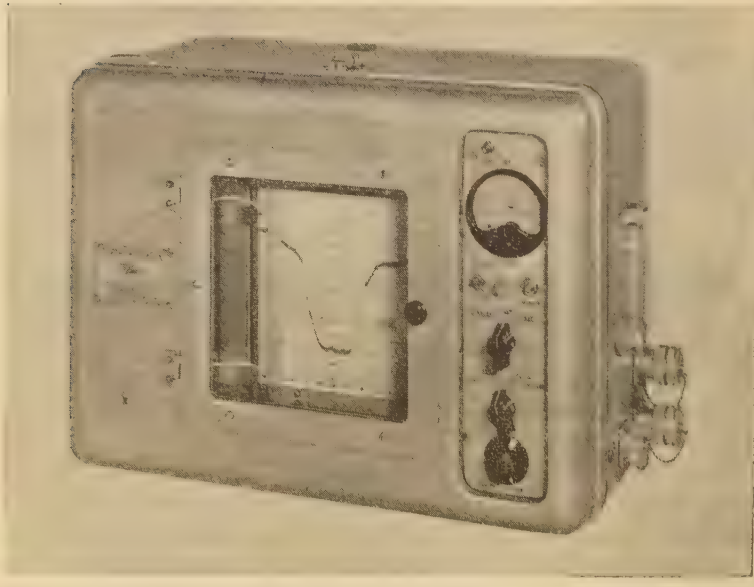

FIG. 7-19. DEPTH RECORDER.
(Courtesy, Edo Corporation.)

The non-portable types are fixed installations placed in large vessels. They will measure soundings to depths of over 200 feet to an accuracy of about 0.2 foot. They are very sensitive and will record reflections from layers of different density, such as the thin layer of brackish water that forms the boundary between fresh and salt waters, and from the surface of flocculated clay layers as well as from the stream or ocean bottom. These separate profiles are of value in studying sedimentation in harbors, rivers and reservoirs.

Portable types may be placed in a small boat or suspended over the side. They will record a depth range of 0 to 65 feet, with adjustments by stages to 230 feet.

Since water transmits sound at slightly different velocities depending upon the water temperature, salinity and concentration of suspended sediment, the calibration of the recording device should be checked frequently. This can be done by suspend-

ing an aluminum T-bar horizontally and with the flat side up at known distances below the surface of the water and checking these known depths with those recorded by the echo sounding device. The storage battery which operates the oscillator should not be allowed to run down because loss of electrical charge will influence the readings. The battery should be charged frequently and a spare or stand-by should be carried in the boat for emergency use. In all installations the presence of air bubbles in the water near the transmitter should be avoided since the air pockets interrupt the transmission of the sound waves through water.

The position of the sounding boat may be located by any of the methods described for lead-line soundings. Shore traverses and triangulation stations must be established and ranges clearly marked. At the instant that a fix is obtained, the chief-of party on board the sounding boat can produce a "fix" line on the recording profile strip by pressing a button. At the same instant sextant angles are read to land stations by two observers on the boat to fix the location of the boat on the range. The designation of the range and any pertinent remarks may be written directly on the recording strip.

Small portable hand radios are also used in this operation. The radios can receive and transmit and have a range of about 10 to 15 miles over open water. The tide observer is equipped with one of these radios, as are the instrumentmen if transits are used. One is also carried in the boat, this radio being larger and more powerful. This radio is kept open at all times in order that the shore stations may communicate with the boat at any time. The tide observer reports any change in tide to the boat. By means of a small adjusting screw on the depth finder the variations in tide may be transferred to the recording strip, adjusting it so that the soundings are recorded automatically to the datum plane to which the soundings are referred. This method eliminates time that formerly had to be employed in arithmetical reducing of soundings.

Depths are plotted crosswise of the recording strip on rulings which are spaced one-half inch apart to represent 5-foot intervals. Distances are plotted lengthwise of the strip, the scale depending upon both the speed at which the recording paper

travels and the speed of the boat. Three rates of travel of the profile paper are usually available, which, together with different boat speeds, afford a wide range of recording scales. The scale selected depends upon the amount of detail required. For many purposes the scale of the finished map is such that only significant depth changes can be shown. In such cases a horizontal scale of 1:2400 could be used with the vertical scale exaggerated to 20 times the horizontal scale. If the detail of sand wave shapes on the bottom is desired, or if the soundings are to be used for computing dredging excavation, a larger horizontal scale may be used with a vertical scale of 1 inch equals 10 feet. Enlarging the scale greatly increases the length of recording strip required.

7-44. Locating a Sounding Ship by Radio Wave Methods. In the sounding of large inland bay areas or distant offshore areas, systems have been developed for locating sounding ship utilizing continuous radio wave reflections. These systems are known under such trade names as Shoran, Hiram, Loran, Decca, Radist, etc.*

A typical system consists essentially of transmitting on a fixed frequency an unmodulated radio wave from the ship to three triangulation stations of known position. On each shore station there is a relay unit consisting of receiver, amplifier, and transmitter. There is also a reference transmitter placed on shore, at any convenient location. This reference station transmits a continuous wave signal. All of the shore apparatus is portable and uses storage batteries for power. Usually there is also a buoy or fixed pile location near the area to be sounded which is used to calibrate the apparatus prior to starting a day's operation.

In order to indicate the position of the ship, it is necessary that all shore stations be placed in operation. The transmitter on the boat then broadcasts a continuous signal on its fixed frequency. This signal is received by each of the three relay stations on shore. Meanwhile, the reference station is also transmitting a wave which is also received by each of the relay stations, where it heterodynes with the signal from the ship and

* See "Electronic Surveying and Mapping" by Simo Laurila, Ohio State University, 1960.

is reflected back to it. The ship has an indicating apparatus which measures the phase count of each station which is an indication of the distance from the particular station. The same phase from another station will intersect the arc of the circle of the distance from the first station in two places. These points lie on the path of a hyperbola at which the various phase counts

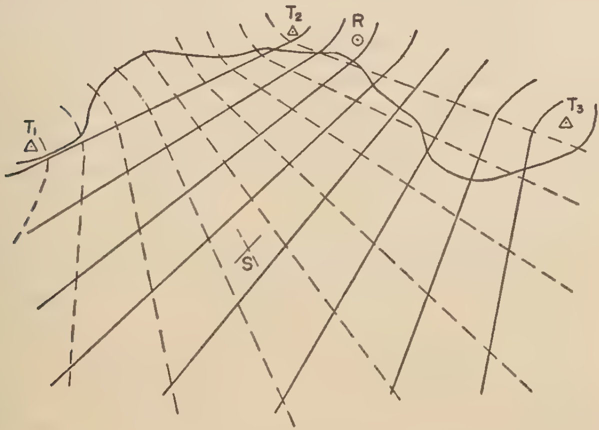

FIG. 7-20. LOCATION OF SHIP BY RADIO WAVES.

intersect. With the intersection of the second and third stations a second set of hyperbolae is formed. The indicating apparatus on ship compares these two sets of signals automatically, giving the phase relationship of the stations. The paths of the hyperbolae indicating the phase counts are plotted on a map of sounding area. The intersection of these hyperbolae give the position of the ship as determined by the relative phase count.

Fig. 7-20 illustrates the system. T_1, T_2 and T_3 are shore relay stations, R is reference station, and S is location of sounding ship. The full lines in the hyperbolic grid system were derived from phase comparisons of stations T_1 and T_2. The dashed lines were similarly derived from stations T_2 and T_3.

The particular hyperbola on which the ship is located may be found for each pair of stations. The intersection of these

hyperbolae establishes the position of the ship within the hyperbola grid.

While this system would seem to be cumbersome, in practice it is quite rapid. Prior to starting a survey shore points are selected with care particularly to insure that the segments of the hyperbolae approaching a straight line would fall within the area to be sounded and that they make a good intersection, with each other, preferably 45° or better. The system has its advantages in that the work can be pursued at any time, as it works in fog, haze, or ēven at night.

7-45. Hydrographic Maps. Hydrographic maps are usually constructed for some special purpose and certain conventional methods of representation are used which are peculiar to this kind of map.

The method of finishing such a map is described in Chapter 9.

7-46. Measurement of Dredged Material. When a harbor or channel is to be dredged the basis of such work is a hydrographic map which has been prepared as described in the previous articles. This map gives the information needed in deciding just where the dredging shall be done and also in estimating the amount of material which must be removed. For dredging the material the contractor is usually paid by the cubic yard; the quantity may be obtained by **measurement in place** or by **scow measurement.**

When the **measurement-in-place** method is used soundings are taken before and again after the dredging work is done, and the volume of the material which has been removed is computed either by the **Borrow-Pit Method** or by the **End Area Method** of cross-sections, as may be the more convenient. Both of these methods are described in Vol. I, Chapter XIII.

Echo soundings may be used to determine the cross sections, but if this method is used it should be agreed to by the contractor and included in the specifications.

If the quantity dredged is determined by scow measurement each pocket of the scow is carefully measured and its capacity computed. When the scows have been loaded with the dredged material the surveyor or inspector makes a note of the number of full pockets, or if they are not full he measures the distance down from the top of the coaming of the scow to where he estimates that

the material in a pocket would come if it were leveled off. The volume of this small rectangular prism is calculated and deducted from the capacity of the corresponding pocket. For each scow in use tables giving the quantity in each pocket at various distances below the coaming are usually prepared for convenience. These scow measurements should be taken just before the tow starts for the dumping ground. When scows remain moored for a day or so before being towed to the dumping ground some of the material in the pockets leaks out through the bottom doors if they are not tightly closed, and such material may find its way back again into the dredged portion of the channel if the scows are moored near the work. In the case of a deck scow where the material is piled on the deck any practical and convenient method may be used to determine the volume of earth, the measurements taken depending upon the shape of the pile.

When the material to be dredged is rock more accurate surveys have to be made. These surveys are usually made by setting a system of intersecting fixed ranges over the rock area. Probings, with a steel rod are then taken to determine the quantity and character of the materials over the rock and the elevation of the rock. For this purpose the Corps of Engineers uses a self propelled barge 50 ft. long by 20 ft. wide. The side of the barge is anchored along the side of one of the sets of fixed ranges and soundings and probings are taken on 10-foot centers from both sides. The probes are either jetted with water or driven to refusal with a 200 pound hammer. In the case of driven probes the number of blows per foot is recorded to determine the relative hardness of the material. The barge is then moved over the entire rock area probing and sounding on 10-foot centers.

After dredging rock, the area is subject to what is called sweeping. This means suspending a light T-Bar to grade and traversing the whole area with the suspended bar. This is necessary to insure that no peaks or pinnacles are left in the dredged area. The bar is kept at grade by making adjustments based on tidal information received from shore via a portable hand radio. Should a high spot be encountered its location may be determined from the fixed ranges or by transit location.

The sweeping bars are light T-Bars each 25 feet long. Two

are used to sweep a 50-foot width. In this operation the 50-foot barge is placed normal to the dredging range and allowed to drift with the tide. Motors are placed on the sides to overcome the effects of wind.

7-47. Measurement of Surface Currents. In certain engineering problems it is important to determine the direction and velocity of the tidal currents in a harbor at various points and at various stages of the tide. This may be done by setting off floats from points whose locations are known or can be determined, and locating these floats from time to time by sextant angles taken between signals on shore. The float used for this purpose should set deep enough in the water to be carried along by the current and not appreciably affected by wind. A simple float can be made of a piece of joist about two feet long with a flag on top and weighted with lead on the lower end so that the top projects but little above the surface. A more conventional float is shown in Fig. 7-21. It consists of a buoyant metal float supporting a pair of zinc vanes mounted at right angles at the end of an adjustable-length brass chain. The flag used on the float for identification should be no larger than necessary, as it may cause the float to be blown off its true course in a strong wind. Floats should be numbered and a proper record made in the note-book, showing the position of each float and the time when it was located. In locating a float the boat is pulled up alongside of it and sextant angles taken just as in locating soundings. Notes as to the stage of the tide and the direction and velocity of wind should be made in connection with the float observations. The results of these measurements may be plotted so as to show the path taken by each float, which, together with the observed times, will give its velocity. For convenience in studying the currents the observations at different stages of tide may be worked up on separate plots.

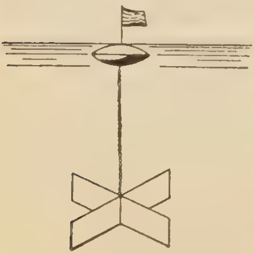

FIG. 7-21.

7-48. Roberts Radio Current Meter. For measurement of currents at sea or in navigable waters, the Coast and Geodetic

Survey uses a current meter attached to a buoy anchored at a known position. The meter is steamed out from the buoy and aligns itself with the direction of the current. The velocity of the flow turns a blade-type impeller which actuates a contacting mechanism inside of the meter. These contacts are communicated through an electrical cable to the buoy where they are broadcasted to a shore station. The buoy contains a battery-operated transmitter and antenna. Two types of signals are transmitted; *V* signals related to the revolutions of the meter, and *D* signals conveying the magnetic direction of the meter alignment. These signals are received at the land station and printed out on a tape against a time scale. The current velocities are obtained by scaling times between *V* recordings and referring them to the meter rating curve. The directions are found by applying a template to *D* signal spacings on the tape.

The transmitters at individual buoys operate on different frequencies so that a number of meters can be read at one land station by alternately tuning in on the different buoys.

7-49. The Wire Drag. When a harbor or a bay is being charted by the use of the ordinary sounding lead there is always much uncertainty about the existence of pinnacle rocks, coral rocks, and the like. These are a great menace to navigation, but they cover such small areas that it is a mere chance if their presence is revealed by the sounding lead.

To locate such obstructions the U. S. Coast and Geodetic Survey has developed an apparatus known as the *wire drag*. This consists of a wire of any suitable length up to 6000 feet which may be set at any desired depth. It is maintained at this depth by means of buoys placed at 300-foot or variable intervals connected to the drag wire by vertical wires whose length may be adjusted. A sinker is attached to the lower end of each buoy wire, those at the ends of the drag being heavier than the others. Floats are attached to the drag wire at 100-foot intervals between buoys (Fig. 7-22).

The drag is drawn over any desired course by means of two power launches. One of these is kept on a desired line, due allowance being made for the effect of the drag, which is towed from the side of the launch; the other launch is controlled by

means of suitable signals. The drag wire between buoys takes a curve which is approximately a parabola. If it strikes an obstruction, it assumes straight lines from the obstruction toward the two launches. The position of the obstruction, at the intersection of the two straight lines of buoys, becomes known as soon as the two launches are located by sextant angles between

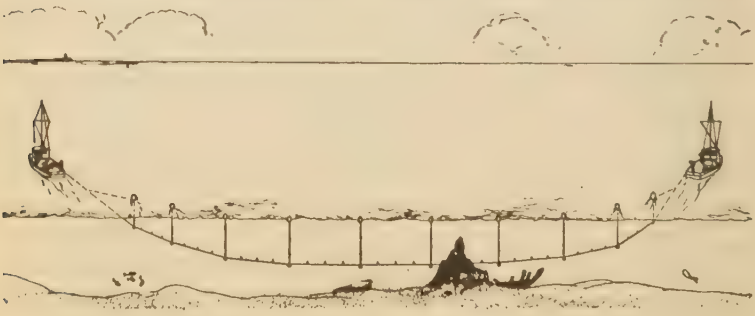

FIG. 7-22. WIRE DRAG.

signals on shore. The point indicated is then examined by means of the sounding lead and the position of least depth is determined. Later a drag is passed over the same spot at a slightly less depth in order to verify the result.

For investigating large deep areas which may have pinnacle rocks projecting from great depths, as in Alaskan waters, a wire sweep of lengths up to 15,000 feet may be used. The buoys in this case are 2500 feet or more apart, the vertical wires are set at a fixed depth and the wire sweep, instead of being horizontal, is allowed to hang in a deep curve between buoys. The drag is used to supplement the work of the sweep.

CHAPTER 8

MEASUREMENT OF THE FLOW OF WATER IN OPEN CHANNELS *

8-1. Definitions. The volume of water flowing in a stream, called the *discharge*, or in some cases called *run-off*, is usually expressed as a rate of flow and in one of the following units.

Second-feet is the unit of flow most commonly used. It is an abbreviation of cubic feet per second and is the quantity of water flowing in a stream one foot wide, one foot deep, and with an average velocity of one foot per second.

Gallons per minute and *gallons per day* are common units of flow in connection with pumping and city water supplies.

The miner's inch, a unit of flow sometimes used by miners and irrigators in the West, is the quantity of water that passes through an orifice one inch square under a *head* (depth of water above the orifice), which varies in the different states, being defined by statute. The California miner's inch, which is probably the most common, is equivalent to $\frac{1}{40}$ of a second-foot.

Second-feet per square mile is the ratio of the discharge at the point considered to the tributary drainage area of the stream above that point, the discharge being expressed in second-feet and the drainage area in square miles. It assumes that the run-off is uniformly distributed, both as regards time and drainage area, which of course is not true. It is a convenient unit for comparing the run-off of different drainage areas.

Besides the above-mentioned units the discharge of a stream

* For a more complete treatment of this subject, see "Stream Flow-Measurements, Records and Their Uses," by Grover and Harrington, John Wiley & Sons, Inc., "Stream Gaging," by W. A. Liddell, McGraw-Hill Book Co., Inc., "Stream-Gaging Procedure — A Manual Describing Methods and Practices of the Geological Survey," Water Supply Paper 888, U. S. Geological Survey, and "Water Measurement Manual — A Manual Pertaining to Measurement of Water for Irrigation Projects," U. S. Department of the Interior, Bureau of Reclamation.

is frequently expressed as an actual quantity of water, the following being common terms of this nature.

Run-off in inches is the depth, in inches, to which the drainage area would be covered if all the water flowing from it in a given period were conserved and uniformly distributed over the surface. It is used for comparing run-off with rainfall, the latter being the average precipitation for the period as determined by rain gage observations, and usually expressed as depth in inches. Run-off in inches is also a convenient unit in computations regarding storage of water in natural or artificial reservoirs.

An *acre-foot* is the quantity of water required to cover an acre to the depth of one foot; it is equivalent to 43,560 cubic feet. One second-foot flowing for 24 hours will deliver approximately 2 acre-feet, or in other words, one second-foot is approximately one acre-inch per hour. This unit is commonly used in connection with storage for irrigation work.

8-2. Theory of Stream Flow. *Steady* flow in a channel is said to occur when the same quantity of water per unit of time passes through each cross-section. While at times the flow in a channel may be said to be nearly or quite steady, it is evident that the natural regimen of stream flow is one of continual change and variation owing to the irregular manner in which water is supplied from adjacent drainage areas and by tributaries. Moreover, many rivers are now controlled by dams, where water is used intermittently, thus causing still further variation in flow. For practical purposes, however, at a given section of river and for short periods of time the flow will usually be found to be nearly steady or else to vary with sufficient regularity to permit of the determination of its average amount.

Uniform flow is a special case of steady flow and would occur if all water cross-sections were alike, so that with the same quantity of water passing each cross-section per second the mean velocity in each section would be the same. This would be the case with a conduit or canal of constant size and slope and whose supply does not vary.

If the stream bed were perfectly smooth and frictionless the water, under constant supply, would tend to flow with accelerated

motion under the action of the force of gravity, but owing to resistances primarily caused by the roughness of the bed and banks this tendency to accelerate is overcome, and the result is approximately steady flow. The resistances include not only surface friction but also the effect of eddies and turbulence of the water.

8-3. The Chezy Formula for Velocity of Flow. The relation between velocity, cross-sectional area, and slope of a stream for different discharges is a complex one. The formula for velocity in general use is that deduced by Chezy, an empirical formula assuming a condition of uniform flow; it is

$$V = C\sqrt{RS}\,* \qquad (8\text{-}1)$$

In this formula V = mean velocity of flow in the cross-section considered, this being equal to the discharge divided by the cross-sectional area, the usual units being discharge in second-feet, area in square feet, and mean velocity in feet per second.

The term R is the *hydraulic mean depth*, or, as more often called, the *hydraulic radius*, and is the ratio of the water cross-section to the wetted perimeter (the portion of the cross-sectional perimeter wet by the flowing water). If the area is expressed in square feet and the wetted perimeter in feet, R will be in feet. For ordinary rivers where the width is many times the depth, it is evident that the hydraulic radius is approximately equal to the average depth, hence the term *hydraulic mean depth.*

The term S is the slope of the water surface in a longitudinal direction and is expressed as the ratio of the fall to the length in which that fall occurs. Both the fall and the length should be expressed in the same unit.

C is a coefficient which varies with the physical characteristics of the stream bed, with the hydraulic radius, and with the slope. Kutter's formula (deduced in 1869 by Ganguillet and Kutter from a study of many gagings of flow) is an empirical formula intended to give values of C in the Chezy formula for different

* For a discussion of this formula see Russell's Hydraulics, 5th Ed., pp. 267–271.

values of R, S, and roughness of bed n. It is as follows (for English units).

$$C = \frac{41.65 + \dfrac{1.811}{n} + \dfrac{0.00281}{S}}{1 + \left(41.65 + \dfrac{0.00281}{S}\right)\dfrac{n}{\sqrt{R}}} \tag{8-2}$$

In this formula n is an abstract number whose value depends only upon the roughness of the channel. The following are the values of n assigned by Kutter to different surfaces.

For a lining of pure cement plaster; planed timber; glazed surface in perfect order, $n = 0.010$.

For unplaned timber in good order; plaster of sand and cement; clean iron and steel surfaces, $n = 0.012$.

For brickwork; well-dressed stonework; iron; cement and terra cotta in perfect condition, $n = 0.015$.

For canals in earth or gravel, straight, well trimmed, in perfect order, and lined with a film of sediment, $n = 0.020$.

For rough rubble; brickwork in bad order; good earth canals in perfect order, $n = 0.0225$.

For canals in earth in average condition, free from stones and weeds; rivers above the average in regimen, alignment, and uniformity of cross-section and slope, and free from detritus, $n = 0.025$.

For canals in rather bad order, having stones or weeds occasionally; rivers in fair order and regimen, $n = 0.030$.

For canals in very bad order, having stones and weeds in large quantities; rivers rather below the average in order and regimen, $n = 0.035$.

It will be seen by analysis of the Kutter formula that for a given slope and hydraulic radius the mean velocity of a stream is nearly proportional to $1:n$; that is, the velocity is inversely proportional to the roughness of the stream bed.

8-4. The Manning Formula. Another well-known formula for determining the velocity of flow in open channels is the Manning formula:

$$V = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} = \frac{1.486}{n} R^{\frac{1}{6}} \sqrt{RS} \tag{8-3}$$

The terms, V, R, S and n, have the same meaning as in the Chezy and Kutter formulas. For the usual range of values of R and S, the value of n to be used in the Manning formula is substantially the same as that used in Kutter's formula.

The solution of the Kutter and the Manning formulas (numerically) involves much work. Tables and diagrams designed to simplify the solution will be found in standard texts on hydraulics.* Table XIII, p. 506, gives values of C for different values of n, R and S for use in the Chezy formula.

Both the Chezy and Manning formulas apply to a condition of **uniform** flow which is rarely attained in ordinary streams. The use of these formulas must be limited to short stretches of river with constant slope and in which there is little variation in the cross section and condition of the bed. The formulas are most applicable to artificial channels where uniform flow prevails.

8-5. The General Nature of Measurements of Stream Flow. The flow of a river is primarily a function of the rainfall upon its drainage area, and is therefore subject to fluctuations. To procure an accurate knowledge of this flow at all times requires frequent and systematic observations and is much more difficult and expensive than the measurement of the amount of water flowing at a given time. With the exception of a few records of the continuous flow of rivers kept by private corporations, this work has been left to government agencies. The gathering and compiling of stream flow data is the responsibility of the Water Resources Division of the U. S. Geological Survey. Records are published in annual series of Water Supply Papers under title "Surface Water Supply of the United States." An estimate of the long-range discharge of a stream that has not been previously gaged may be had by carrying on measurements of flow of the stream for a short time and expand the observations to a longer period by using data available for a stream of similar characteristics in the same locality.

The general subject of determining flow may be divided as follows.

(1) Determination of flow at a given time, or during a short period of time.

(2) Determination of flow during a considerable period of time, covering seasonal and annual fluctuations.

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* See "Handbook of Hydraulics" by H. W. King, McGraw-Hill Book Co., 4th Ed., Section 7.

The instruments and methods used are the same up to a certain point in both of these classes of work, the additional essential in the case of long time records of flow being the recording of the variations in river height by means of some form of gage.

8-6. Instruments for Measuring Differences in Level of Water. Hook Gage. This is an accurate instrument for measuring difference in level of water. It consists essentially of a hook (Fig. 8-1) with a needle point fastened to a vertical movable scale, which by means of a vernier can be read very closely, often to $\frac{1}{10000}$ feet. The hook is lowered just beneath the surface of the water and then slowly raised by a slow-motion screw at the top until the point just touches the surface. Very accurate determinations of water level can be made in this way, the gage being securely clamped to some firm support and the elevation of the zero of the scale determined by careful leveling. The hook gage is frequently used to determine the depth of water flowing over a weir (Art. 8-22), in which case it is first necessary to determine the gage reading for the crest of the weir, or the reading when the water is just level with the crest. This is called the "*zero gage reading*," which is subtracted from other gage readings to obtain the depth of water flowing over the crest.

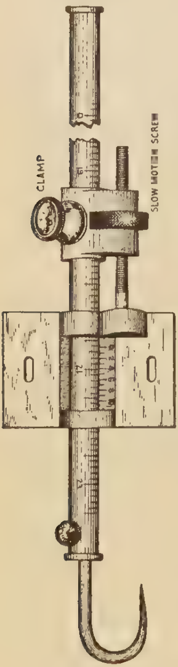

FIG. 8-1. HOOK GAGE.

8-7. Staff Gage. In its various modifications this gage is used most generally for noting changes in water level. In its simplest form this is a smooth board, from 4 to 10 inches wide, set vertically in the water, attached to some stable support, as a pier or wall, and graduated by painted lines, or preferably by metal staples, so that it can be easily read, the distance apart of the units of graduation depending upon the precision desired in gage readings. For ordinary observations of river height a gage of this kind should be graduated to feet and tenths of feet, and the observer should read to the nearest tenth of a foot. For more careful work the tenths are

subdivided to half-tenths, permitting the easy estimation of hundredths; where greater precision is desired every hundredth is shown. More permanent forms of staff gages, made of more durable materials such as enameled metal strips, are used where observations are continued over long periods of time. (See Vol. I, Art. 262a, p. 311.)

8-8. Wire-Weight Gage. This is largely used for observations of river height, especially by the U. S. Geological Survey. It is used in place of the staff gage and has the great advantage that when not in use it is safe from blows by any floating object such as ice and logs. (See Art. 7-25.)

8-9. Float Gages. These are frequently used at power stations where it is desirable to have gage readings made inside the building. A hollow air-tight copper cylinder floats in a pipe of slightly larger diameter set vertically in the water. The float is connected by a wire or chain over pulleys, and a marker attached to the wire moves over a scale where the readings are made.

8-10. Automatic Gages. These are used where a **continuous** record of water level is desired. These are of various kinds, a common form consisting essentially of a drum upon which is fastened a sheet of record paper, and the drum made to revolve by clockwork at a regular rate, the record of water level being transferred by a recording pencil or pen, which is in turn connected by a suitable reducing device with a float which moves up and down with changes in water level. (See Fig. 7-9.)

8-11. Pressure-type Recording Gage. Another form of automatic gage, developed by the U. S. Geological Survey, utilizes a tube filled with a gas fed from a cylinder and bubbling freely into the stream at a set elevation (Fig. 8-2). The pressure on the gas (nitrogen) depends upon the head or depth of water over its outlet. The fluctuations in pressure due to changes in depth of water are detected in a mercury manometer, the variations being translated by means of a servo-mechanism to a graphic recorder, similar to the one shown in Fig. 7-9. This gage may be used for recording either river or tide stages.

The bubbling tube replaces the stilling well, float and intake required in conventional installations (Fig. 8-11), thus reducing the cost of construction and increasing flexibility in the choice of

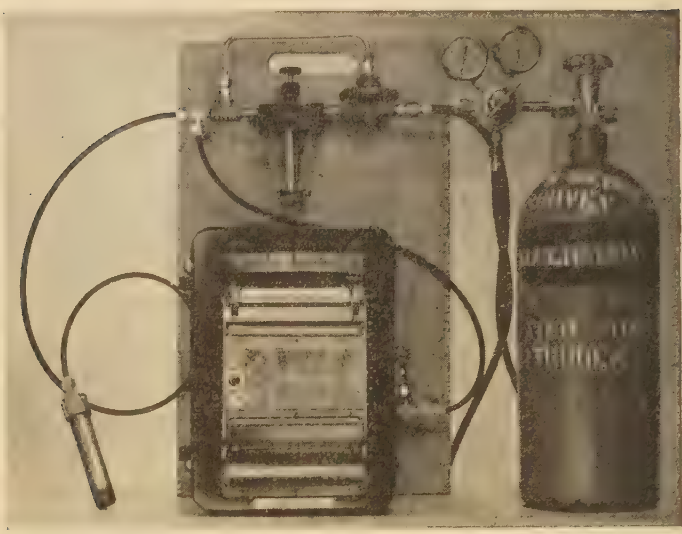

FIG. 8-2. PRESSURE-TYPE RECORDING GAGE.
(Courtesy, U. S. Coast and Geodetic Survey)

location of the gaging station. The entire station can be easily dismantled and relocated at another site, if desired, with little loss of investment.

8-12. Piezometers. Heights of water surface may also be measured with refinement by a *piezometer*, or *manometer*, consisting of a graduated glass tube, which is connected in a vertical position to a pipe, the other end of which is in the flowing water, the top of the glass tube being above the level of the water surface. The *piezometer float gage* is a modification of the piezometer in which a small float and index is used in the vertical tube to enable exact readings on the scale to be made. In either case the connecting pipe should enter the channel of flow exactly at right angles to the direction of flow and be cut off flush with the side of the channel.

8-13. Instruments used for Measuring the Velocity of Flowing Water — Floats. The velocity of water may be measured directly by determining the velocity of a body, such as a float, which moves along with the water. A simple form of surface float, i.e.,

one which is intended to move on or near the surface of a river, consists of a corked bottle with a flag in the top and weighted at the bottom.

The *tube float* is a copper or tin tube weighted at its lower end so that it will stand vertical in the water. It is made long enough to reach nearly to the bottom of the channel and to project a few inches above the water surface. It is evident that a number of tubes of different lengths will be required for different depths of water. *Rod floats* made of wood similar in shape to the tube floats are also used.

Surface floats are used for approximate determinations of velocity, as, for example, in freshet flow where no other method can be used, owing to floating ice, etc., or in making a reconnaissance where only an approximate value for the flow is desired. (See also Art. 7-47.)

Tube and rod floats are best adapted for use in canals or artificial channels, where the depth of water is fairly constant. They have the advantage of being simple in form and construction, of interfering little with the velocity of the water, and of being little affected by silt, floating ice, etc. On the other hand, they are affected somewhat by wind, cannot be used in deep streams or where the bed is rough and irregular, and are expensive to operate.

8-14. Current Meters. The velocity of flow may also be measured **indirectly** by means of a *current meter*, which consists essentially of a wheel with cups or vanes so constructed that the impact of flowing water will cause the wheel to revolve, the number of revolutions being indicated by some counting device.

A successful current meter should possess the following characteristics:—it should be of simple construction, with all delicate parts properly protected; the cleaning and replacements of parts should be readily made; it should be adapted for use in rivers, canals and conduits of all kinds; it should have a simple device for recording the revolutions of the wheel which can be accurately and easily read; it should be so shaped as to present the least possible resistance to the flow of water.

8-15. The Price Meter. The *Price* meter is in most general use; and has been largely developed and used by the U. S.

Geological Survey. It consists of a wheel (Fig. 8-4) made with cups which are fastened to a vertical shaft and turning upon a steel point in a conical bearing which is capable of adjustment at *e*. The wheel and shaft are carried by a yoke *Y*, to which a vane is attached through the shaft *S*, which is in turn supported either by a rod *T* as shown in Fig. 8-4 or by a cable with suspended weight as shown in Fig. 8-3. The lead weight is of stream-line shape so that it will offer low resistance to stream flow.

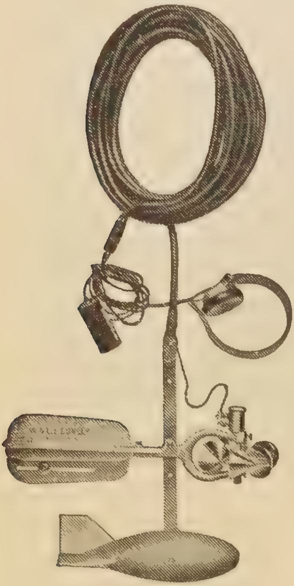

FIG. 8-3. PRICE CURRENT METER.
(Courtesy, W. & L. E. Gurley.)

While the upper end of the wheel shaft is held in place by a bearing surface the weight of the wheel is carried upon the lower point, and a sleeve nut *k* is arranged to be screwed down against the frame, thereby lifting the shaft and bringing the bearing off the point during transportation. The bearings and bucket wheel shaft are of hardened steel.

The meter shown in Fig. 8-4 may be used to count either single revolutions of the meter wheel or every fifth revolution. If single revolutions are desired wire connection is made to binding post *a* which leads through a hard rubber circular insulating nipple *c* to a slender platinum spring *d*. The top end of the shaft of the meter wheel is cut away at *m* and the remaining part bevelled so that with every revolution of the shaft a contact is made and broken with the platinum spring thus completing the electric circuit and giving one buzz for each revolution. If connection is made to binding post *b*, then the circuit is made and broken between the wire *f* and the gear *g* once every five revolutions of the meter wheel, and the buzz will only be heard for each fifth revolution.

A compact form of battery is employed, which is arranged in

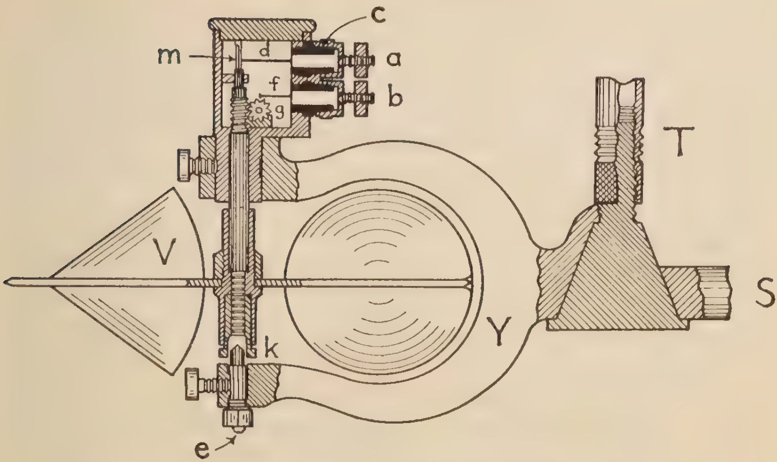

FIG. 8-4. DETAILS OF PRICE CURRENT METER.

a case with a form of "buzzer," or sounder, and fastened to the upper end of the wire cable or rod supporting the meter, the contact wires being properly connected with the poles of the battery. The revolutions of the meter, as indicated by the "buzzer," are counted by the observer.

The cable is employed when the meter is to be used from an overhead cableway or bridge. The cable is also favored when the meter is used from a boat. When used in shallow streams, irrigation ditches and conduits, the rod is used in place of the cable. This is done by wading in the stream. The rod can be rested on the bottom with the meter attached to the rod at the desired depth.

When the cable is used it generally is composed of two insulated electric wires.

The current meter may also be suspended by a single wire, using the ground and water to complete the circuit needed to operate the recording apparatus, which must be of the telephone receiver type in order to be successfully used. The wire is insulated from the body of the meter but connected with the spring in the head of the meter which makes and breaks the circuit. The other end of the wire passes in circuit through a

battery cell and telephone receiver and is connected with a conductor leading to the ground, such as a bridge iron, the station cable, or an extra wire stretched along the bridge. There is also a thin steel cable, with insulated copper conductor core, for use in streams of very high velocities where the ordinary cable would be dragged downstream and consequently the meter would ride at less depth than was intended.

A standard rating table is furnished with each current meter outfit. It applies only to the model in the box. This table is the mean of the ratings of many new meters of this particular design and seldom varies over one per cent from the actual rating of the particular meter.

8-16. Pygmy Type Current Meter. This is a small current meter similar in principle to the Price meter, except that the bucket wheel is 2 inches in diameter instead of 5 inches; and the other dimensions are scaled down in proportion. The meter (Fig. 8-5) is designed particularly for measuring the flow of water in shallow streams, flumes, and canals where the depth of water or the current velocity is too low to permit accurate measurements with larger meters. The Pygmy type is rod-suspended, and not adaptable to cable suspension. It records each revolution only.

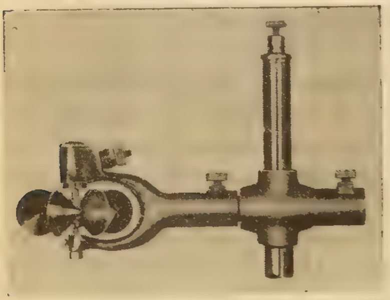

FIG. 8-5. PYGMY CURRENT METER.
(Courtesy, W. & L. E. Gurley.)

8-17. Rating Current Meters. The relation between the revolutions of the meter wheel in a unit of time and the velocity of flow is determined by *rating* the meter, i.e., by moving it along in still water at various speeds and determining the relation between revolutions and velocity. A car to which the meter is attached, either by cable or rod, is commonly used for this purpose, being pushed along a track close to the water's edge.

The values observed are plotted graphically using velocity in feet per second as abscissas and revolutions per second as ordi-

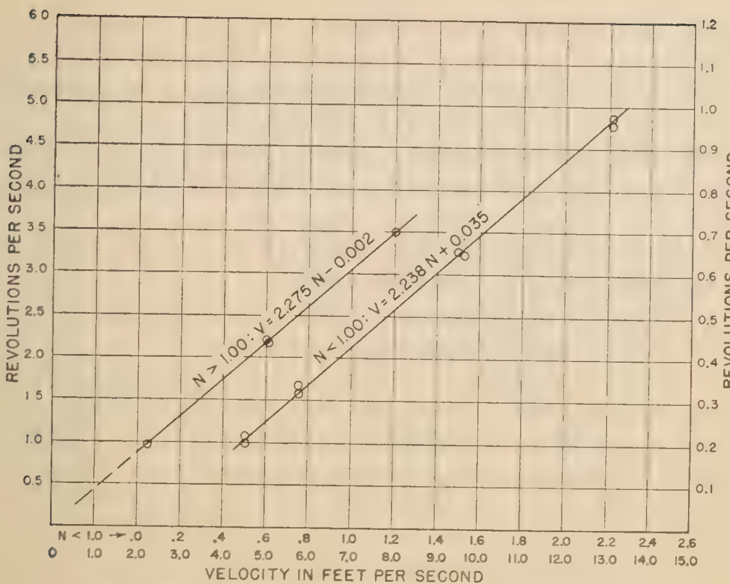

FIG. 8-6. TYPICAL RATING CURVE FOR PRICE METER.

nates. Fig. 8-6 is a typical rating curve for a Price meter. Two lines are drawn, one for velocities below 1.00 ft. per sec. (at right) and one for higher velocities (at left). Equations for the lines are also given. It will be noticed that the slope of the lower velocity line is somewhat different than that of the higher velocity line; this is typical of Price meter rating curves. For convenience a meter rating table is usually made from the plotted curve.

In an emergency a meter can be rated by propelling a boat at uniform speed through a stretch of still water.

For a nominal charge current meters are rated by the U. S. Bureau of Standards, Washington, D. C.

8-18. Use of Current Meters. The current meter is, in general, the most satisfactory instrument for measuring stream flow, since velocities can be accurately determined at nearly all points in the cross-section of the stream, if desired, and over a sufficient length of time to eliminate the effect of pulsations in flow. It

cannot be used where weeds, grass, or floating ice occur, but on the other hand it furnishes the only means of measuring velocity of flow under ice cover. Current meters must be frequently rated, however, and used with considerable care.

8-19. Methods of Measuring Stream Flow. There are three distinct methods of determining the flow in open channels, as follows.

(1) By measurement of slope and cross-section and the computation of flow by the Chezy formula.

(2) By means of a weir, dam or flume observing the head of water on the crest and computing the flow.

(3) By measurements of the mean velocity of current and the area of the cross-section, their product giving the discharge.

8-20. Slope Method of Measuring Stream Flow. The slope method of measuring stream flow requires a straight stretch of river of uniform slope and cross-section. The difference in level of the water surface at two cross-sections, as far apart as practicable, must be carefully determined. This will require the use of hook gages (Art. 8-6) for the best results, and observations for a given section should be made at each shore and their mean taken and used as the elevation of the water surface at that section. The water surface near the middle of a stream is often at a slightly higher level than that at the sides, but if care is taken in selecting a stretch of river in which the distribution of flow is approximately the same at all cross-sections no error of consequence will be involved in slope measurements by locating the hook gages near the shores.

The hook gages are connected by careful lines of levels, and a sufficient number of cross-sections of the stream obtained to give an average value of the area and of the hydraulic radius as explained in Art. 8-3. Assuming a proper value of n for the given conditions, the discharge is then computed by the Chezy formula, using tables or diagram.

The results obtained by the slope method are as a rule only approximate, principally owing to the difficulty in securing favorable conditions of flow, in measuring the slope, and in assigning a proper value of n in Kutter's formula.

When other methods are not convenient this method is of

value in estimating approximately the flood discharge of a stream, frequently by use of high-water marks left after a flood.

8-21. Weir Method of Measuring Stream Flow. A *weir* is a notch in the top of the vertical side of a vessel or reservoir through which water flows. This notch is usually rectangular, with a horizontal lower edge called the *crest*. The upstream edge of the crest of a weir is usually made exactly rectangular and the crest itself thin, so that the water in flowing out will be completely contracted, touching the crest only in the line of its inner edge. In this form the weir is known as a *thin-edged* or *standard weir*, and provides one of the most accurate methods of measuring the flow of water. Where the vertical edges of the notch are at some distance from the sides of the channel of approach, so that the particles of water in passing around the ends of the notch move in a curved path, the weir is said to be one *with end contractions* (Fig. 8-7). In this case the length of the sheet of water passing

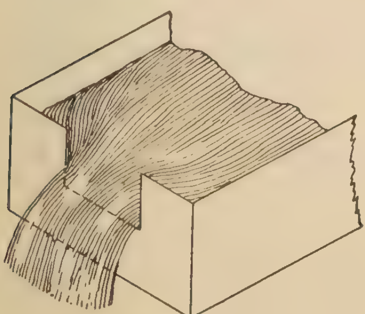

FIG. 8-7. WEIR WITH END CONTRACTIONS.

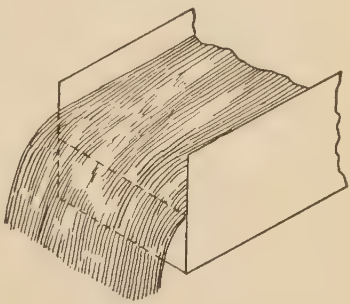

FIG. 8-8. WEIR WITHOUT END CONTRACTIONS.

over the weir is less than the length of the weir. Where the edges of the notch coincide with the sides of the feeding canal, so that the particles of water at the sides pass over without deflection from the verticle planes in which they move, the weir is said to be one *without end contractions* or *with end contractions suppressed* (Fig. 8-8).

The head of water flowing over the crest of the weir is determined by means of a hook gage (Art. 8-6) or by a scale set

sufficiently far upstream to avoid the surface curve formed by the water as it passes over the weir crest (usually 5 or 6 feet) and referred to the crest level. The flow is computed by formula.

8-22. Weir Formulas. There are numerous weir formulas, but those of Hamilton Smith given below for rectangular weirs are convenient in application.

$$Q = cbH^{\frac{3}{2}} \tag{8-4}$$

where Q = discharge in second-feet,
c = a constant based upon various experiments,
b = length of crest of weir in feet,
H = head on crest of weir in feet.

The value of c is taken from the Tables X and XI, pp. 503-4, separate tables being given for suppressed and contracted weirs.

The Francis, the Fteley and Stearns, and the Bazin formulas for weirs are also available.*

Where the channel above the weir is narrow the *velocity of approach* may have to be considered in weir computations. This is the mean velocity of the water in the channel at the section where the hook gage is placed, and tends to increase the discharge.

In the Smith formulas H is **increased** as follows to allow for velocity of approach.

$H_v = H + 1.4h$, for contracted weirs. $H_v = H + 1\frac{1}{3}h$, for suppressed weirs. In these formulas H_v is the head on the crest of the weir corrected for velocity of approach, this being used instead of H in Hamilton Smith formula (8-4).

The term $h = \dfrac{V^2}{2g} = \dfrac{\left(\dfrac{Q}{a}\right)^2}{2g}$, where a is the cross-sectional area of *approach channel* in square feet at the hook gage, V is the mean velocity in this cross-section in feet per second, and g is the acceleration due to gravity (32.16 feet per second per second).

In making this correction V can first be computed neglecting

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* See Water Supply and Irrigation Paper No. 200, U. S. Geological Survey for Weir Experiments, Coefficients, and Formulas; also Chapters 3 and 4 in "Handbook of Hydraulics," 4th Ed., by H. W. King and E. F. Brater, McGraw-Hill Book Co.

velocity of approach, and this approximate value used in computing h, a step which usually involves only slight error. In refined calculations h should be recomputed, using the new value for V, and Q calculated anew.

8-23. Submerged Weirs. In a *submerged* weir the water on the downstream side stands above the crest level. Such a weir sometimes has to be utilized in measurements of flow in connection with works of river improvement, canals, etc., or where the loss of head will preclude the use of an ordinary weir.

Let H be the head above the crest measured by the hook gage in the usual manner on the upstream side of the weir, and H' be the head above the crest of the water on the downstream side of the weir measured by a second hook gage. If the level of the water on the downstream side of the weir is lower than the crest, then the flow is the same as for an ordinary weir provided there is full access of air beneath the sheet of water. As the water level on the downstream side rises, the contraction of the weir crest is suppressed as soon as H' has any appreciable value, and the discharge is therefore increased; but as H' is further increased the discharge is diminished on account of the effect of backwater on the downstream side.

There are several formulas in use for computing the flow over submerged weirs, all based upon experiments which have been almost entirely confined to weirs without end contractions. Herschel's formula, based upon experiments made by Francis and by Fteley and Stearns, is in convenient form. The observed head H is first multiplied by a number n which depends upon the ratio of H' to H and the discharge is obtained from the formula

$$Q = 3.33b(nH)^{\frac{3}{2}} \tag{8-5}$$

Values for n may be found in Water Supply Paper 200, p. 140. They are little affected by submergence up to about $H_1/H =$ 60%, where n is about 0.95. For larger ratios, values of n vary substantially.

8-24. Use of Dams as Weirs. A dam can frequently be utilized as a weir in measuring the flow of a stream, if the physical characteristics of the dam are suitable and the manner of use of water around and through it will permit.

The essentials as regards the dam are (1) sufficient height so that backwater below the dam will not interfere with free fall over it, (2) little or no leakage, (3) level crest, (4) crest and cross-section of some form for which the coefficient c is known in the usual weir formula $Q = cbH^{\frac{3}{2}}$.

Numerous experiments have been made to determine values of C in this formula for different types of broad crested dams, or weirs. Compilation of data regarding C is in Water Supply Paper No. 200, U. S. Geological Survey. As there noted, the formula $Q = 2.64bH^{\frac{3}{2}}$ is applicable to broad crested weirs or dams, whose crests exceed 2 feet in width and having heads from 0.5 feet up to 1.5 or 2 times the breadth of the weir crest. The height and condition of flashboards, if any, must also be taken into account.

If water is used through or around the dam its amount must be measured either by weir or by current meter, or, where turbines of standard make are installed, the quantity may be estimated from the head, gate opening, and speed.*

8-25. Parshall Flume. A device for measuring the flow of water in canals with flat slopes, such as irrigation ditches, is the Parshall Flume.† It consists of a wood, metal or concrete channel structure with a converging section leading to a constricted throat and diverging section leading from it (Fig. 8-9). The floor is sloped downward within the throat section and then upward, which causes the hydraulic jump to occur well downstream from the throat for a wide range of discharges.

The size of the flume is designated by the width (W) of the throat section. There are three groups of flume sizes and proportions: 3-, 6-, 9-inch throat widths; 1- to 8-ft. widths; and larger widths between 10- and 50-ft. The type selected will depend upon the range of discharges to be measured. Fig. 8-9, taken from Water Measurement Manual,† shows plan and profile for throat widths 1 to 8 ft. and capacities 16- to 140-second feet.

Because of the contracted section at the throat, the velocity of

* See Water Supply Paper No. 180, published by the U. S. Geological Survey.

† See Water Measurement Manual, U. S. Dept. of the Interior, Bureau of Reclamation, Chapter III, U. S. Printing Office.

the water flowing through the structure is relatively greater than that in the natural stream or channel, and for this reason the flume is self-cleaning of sediment or debris. The flume should be installed beyond the influence of other control structures or bends in the channel.

The flume may operate as a free-flow, single head device, or under submerged flow. For the first condition only one measurement (H_a) above the contracted section need be made. In the second condition H_b (below the restriction) is also needed. See measuring wells for H_a and H_b in Fig. 8-9.

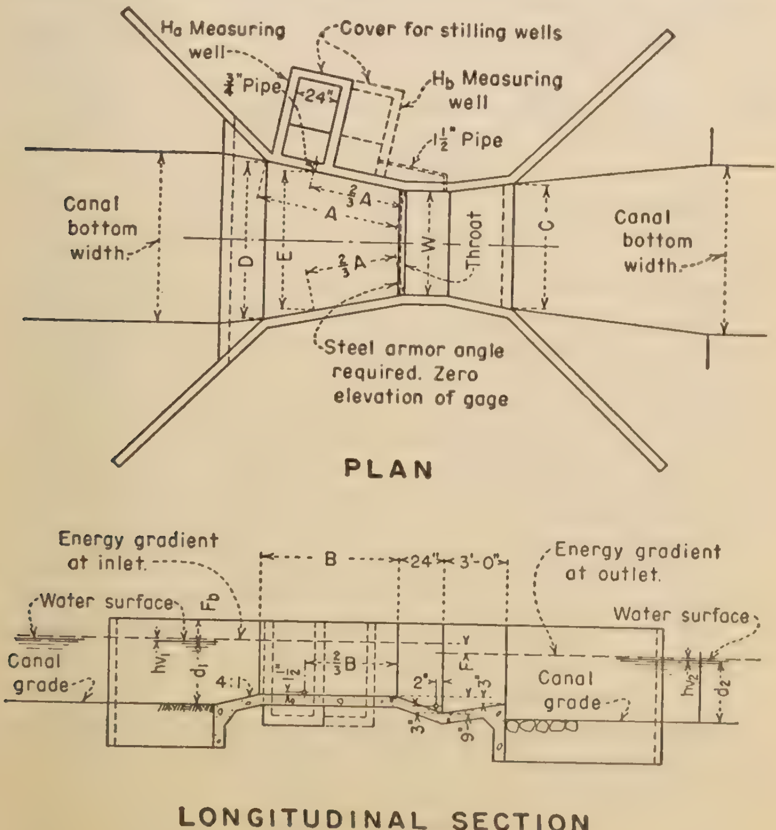

FIG. 8-9. PARSHALL FLUME.

The general formula for free-flow discharge through the Parshall Flume is

$$Q = JH_a^n \tag{8-6}$$

in which Q = discharge in second-feet
J = a coefficient which is a function of the size of the flume
H_a = the upper head, in feet, observed at a point upstream from the crest a distance two-thirds the length of the converging section, and
N = exponent of the head, H_a.

The coefficient J and exponent n vary with the size of the flume.

For flumes of width 1 to 8 feet the formula is

$$Q = 4WH_a^{1.522w^{0.026}} \tag{8-7}$$

in which Q = discharge in second feet
W = size of flume, or width of throat, in feet.

Formulas for free flow can be applied to the submerged condition when the ratio of H_b to H_a does not exceed 0.6 for sizes under 1 ft., 0.7 for sizes 1 to 9 ft. and 0.8 for sizes 10 to 50 ft. The formulas for submerged flow are quite complex. Diagrams for computing discharge for a wide range of flumes under submerged conditions will be found in footnote reference,* Chap. III.

8-26. Submerged Orifices. Where the fall available for measuring water in a channel is low, a submerged orifice may be used. The Bureau of Reclamation uses a vertical sharp-edge, contracted, rectangular submerged orifice for measuring irrigation water. The discharge is computed from the characteristics of the orifice and the difference in elevation of the water surface above and below it. If the velocity of approach is appreciable the velocity head must be taken into account.

The formula for discharge through a submerged orifice when the velocity of approach is negligible is as follows.*

$$Q = 0.61A\sqrt{2gh} \tag{8-8}$$

---

* See Chapter IV, Water Measurement Manual, U. S. Dept. of Interior, Bureau of Reclamation.

in which Q = discharge in second-feet velocity of approach neglected
g = acceleration due to gravity (32.2 feet per second per second)
H = measured head on the orifice in feet, being equal to the difference in elevation of the water surface on the upstream side of the orifice and the water surface on the downstream side, and
A = the area of the orifice in square feet.

When the velocity of approach is appreciable, the following formula should be used for the computation of the discharge;

$$Q' = 0.61 A \sqrt{2g(H + h)} \tag{8-9}$$

where A' = discharge in second-feet, the velocity of approach being taken into consideration, and
h = the head due to velocity of approach, in feet.

The submerged orifice has the disadvantage that it may collect debris, sand and silt upstream from the orifice which will affect the accuracy of the measurement.

8-27. Velocity-Area Method of Measuring Stream Flow. General Requirements. This method is commonly employed in measuring river discharge; it also can be used in measuring flow in artificial channels. It involves determining the cross-section of the river by ordinary surveying methods, and finding the mean velocity at that cross-section by current meter measurements. This cross-section is called the *Measuring Section*. The cross-section where the gage is located is called the *Gaging Section;* it may or may not be at the measuring section. The gaging section must be located upstream from a *Control Section*, which is a place where the stream flow is restricted. The control section serves the purpose of regulating the height of water surface upstream from it; for a given discharge it maintains a constant stream slope and gage height.

Precautions and suggestions regarding the selection of the control, measuring and gaging sections follow.

8-28. Control Section. A control section may be identified by noticing the appearance of the stream above and below that

section; above the control the water surface has a relatively smooth appearance while below the control the water surface is a series of ripples and the flow is much more turbulent.

A control section may be natural such as a sand or gravel bar, a stretch of large boulders or a ledge outcrop extending across the width of a stream; or a control may consist of an artificial obstruction such as a dam or measuring weir. Often the piers and abutments of a bridge, by restricting the flow of the stream, serve as a control, particularly at the higher stages of the river.

Inasmuch as a control section stabilizes the relationship between stage (or gage height) and discharge, it controls the accuracy of the records of discharge, once the station rating curve showing the relationship between stage and discharge has been determined. It is essential, therefore, for consistent results that the control section be permanent.

It is also an advantage to have a control that is sensitive; i.e., a small change in discharge should be reflected by a relatively large change in gage height (stage). Since the gage height is used as an index of the discharge, it is desirable that small variations in the discharge should be readily detected by the gage.

Often a control that is effective at low and medium stages becomes drowned out at times of flood or high stages and a second (oftentimes a third) control further downstream becomes effective, thereby changing the shape of the station rating curve by introducing a discontinuity at the stage where the first control becomes drowned out.

8-29. Measuring Section and Gaging Section. These places should be chosen practically at the same time because they are so interrelated. The measuring section must be selected so that **at all times** the same amount of water passes this section as passes the gaging section. It may be at the same site as the gaging section, or it may be upstream or downstream from the gage. The measuring section does not require a control section; it may be above or below the control section for the gage. The river should be reasonably straight for a distance of 200 to 500 feet above and below the measuring section, and the cross-section should be V-shaped rather than broad and shallow

as this preferred shape tends to give somewhat greater velocities at low flows. The mean velocity at the section should not fall below 0.5 foot per second with Price meters or below 0.2 with the Pygmy type, because they are unreliable for measuring lower velocities. The measuring section should be free from obstructing logs or boulders and the bottom at this section should be fairly smooth.

In order that accurate results of discharge may be obtained certain desirable features should be kept in mind before deciding on the exact location of the gaging section. It should be near the measuring section (usually within 1000 ft.). It should be free from all obstructions such as boulders or sunken logs; the channel cross-section should be such as will give dependable discharge at all stages of the stream; the bed and bank should be stable; there should be no backwater effect due to ice or log jams forming below the section or due to its proximity to the confluence of another stream; the section should be so located that the gage house which encloses the gage is accessible at times of peak flows.

8-30. Variation of Velocity in a Given Cross-Section. The variation of velocity in the cross-section of a stream flowing in an open channel follows in general fairly well-defined laws which are little affected by changes in stage. This makes it possible to determine the mean velocity of flow at a given time by a comparatively small number of observations of velocity, provided these are properly distributed in the cross-section.

In any given vertical section the velocity varies at different depths, and also the mean velocity in vertical sections varies with the distance of the sections from the banks of the stream.

8-31. Velocity in the Vertical. If velocities at different depths in a vertical section are plotted as abscissas and depths as ordinates and a smooth curve drawn averaging the plotted points, it will be found that the resulting curve is approximately a parabola, and for all practical purposes may be considered parabolic. The axis of the parabola will be horizontal and lie in the element corresponding to the maximum velocity; the position of maximum velocity is at a point varying from 0.1 to 0.3 of the total depth below the surface. Fig. 8-10 shows typical vertical velocity

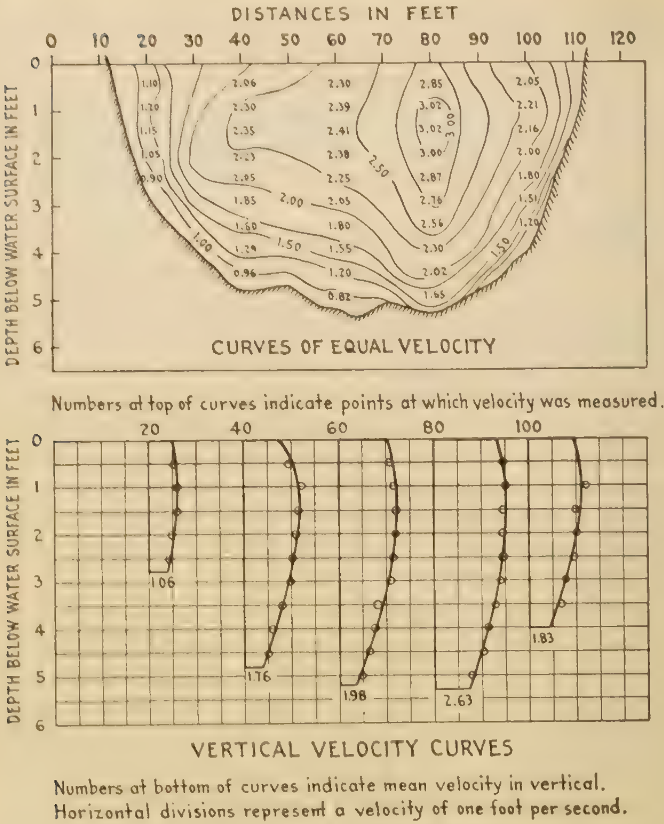

FIG. 8-10. TYPICAL VELOCITY CURVES.

curves at different verticals in a given cross-section; it also shows a plot of curves of equal velocity at the same cross-section. The shape of a vertical velocity curve is influenced by the depth of channel, the condition of the bed, the velocity and direction of the wind, and the distance from the bank. The surface velocity is usually less than the maximum; the minimum velocity is at the bottom and the mean velocity at about 0.6 the total depth. From many experiments it has been found that the average of the velocities at 0.2 and 0.8 of the total depth is very closely the

mean velocity in the vertical. This relationship is widely used in making current meter measurements, as described in Art. 8-37.

8-32. Velocity from Bank to Bank. If the mean velocities in the verticals at different distances out from either shore are plotted as ordinates, and the corresponding distances from the shore as abscissas, a *horizontal mean velocity curve* is obtained. The form of this curve will vary largely with the cross-section. Usually the maximum velocity will be at or near the deepest part of the channel and the minimum velocity at the banks. If piers or other obstructions occur in the section the curve may be quite complex. It is not practicable to find any one vertical section in the stream whose mean velocity will be the mean velocity for the entire cross-section. The lower plot in Fig. 8-10 illustrates the variation in vertical velocity curves obtained at different distances from the bank of a stream. The upper plot in Fig. 8-10 shows the velocity distribution within a cross-section.

8-33. Use of Floats in Determining Velocity and Discharge. The kinds of floats in common use have been described in Art. 8-13. In measuring velocity by means of surface floats record is made of the time taken by the float to pass over a selected stretch of a river, say, from 50 to 200 feet or more in length.

A sufficient number of such velocity determinations is made at different points across the width of the stream to determine the mean velocity of the whole section. By plotting as abscissas the mean position of the floats, as indicated by the distances from the bank, and as ordinates their average velocity, a curve showing the variation in velocity between the banks can be obtained, and from this the mean velocity of the whole cross-section may be determined. The product of this mean velocity and the average cross-sectional area is the discharge. Since the surface velocity is usually greater than the mean velocity it is necessary to multiply the observed results by a coefficient (which varies with conditions but is usually about 0.85) to reduce the observed velocity to the mean velocity in the vertical. (See form of vertical velocity curve, Fig. 8-10.)

Tube or rod floats are intended to give directly the mean velocity in the vertical. The velocities obtained with them,

however, must be corrected slightly owing to the fact that they do not move through that portion of the water near the bottom of the channel, nor at exactly the same velocity as the water itself, so that the true mean velocity is somewhat less than that measured by the tube float.

Francis deduced the following empirical formula for use in such cases.

$$V_m = V_r \left(1.012 - 0.116 \sqrt{\frac{d'}{d}}\right) \tag{8-10}$$

where V_m is the mean velocity desired; V_r is the mean velocity of the tube float; d is the total depth of the stream; d' is the depth of the water below the rod. In using this formula, however, d' must be small in comparison with d.

8-34. Current Meter Determinations of Velocity and Discharge. The use of a current meter in obtaining velocity of stream flow requires some means for placing the meter at any point in the stream cross-section. Where available, a bridge can be used for this purpose, and on account of the height of bridge floor above the water this usually requires that the current meter be suspended by cable. If no bridge is available a boat or canoe may be used, the boat being held in any given position by means of stay lines stretched across the stream at bow and stern. The bow is pointed upstream and the meter either suspended by a cable from an outrigger and pulley, or fastened to a rod which is held vertically in front of the bow of the boat. For more accurate results than can be obtained by the use of a boat a wire cable stretched across the stream with a box or car running on the cable for the observer can be used. (See Fig. 8-11.) Where the stream is not too deep excellent results may be obtained by wading and holding the meter fastened to a rod. In any case it is necessary to lay off, transverse to the axis of the stream, stations spaced at equal distances apart, the interval depending upon the width of the stream, and beginning with some suitable reference point on the bank. These stations may be marked with paint or chalk if on a bridge, or by a tagged line where a boat or cable is

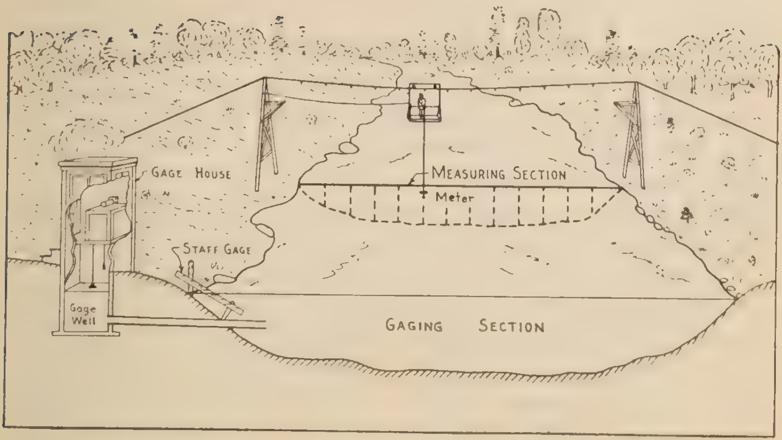

Fig. 8-11. Stream Measurement Station.

used or when measurements are made by wading. Either a gage should be read at the beginning and end of the work or the distance measured to the water surface from some fairly permanent reference point, so as to determine the stage of the river at which the gaging is made. Soundings can usually be made with the meter by graduating the rod used for supporting it, or, in case the meter is suspended by a cable, by using a cloth tape and reference mark on the bridge, boat, or car, taking these readings usually to the nearest 0.1 of a foot. The distance apart of these soundings and their accompanying velocity observations will depend upon the width of the stream, varying from 2 feet or less, with very narrow streams, to 25 feet or more, where very wide. To maintain accuracy of the method it is desirable to space the measurements so that not more than 5% of the discharge occurs in any one measuring section.

8-35. Methods for Velocity Observations in the Vertical. There are three general methods in use for determining the mean velocity in a vertical section:

(1) Multiple Point
(2) Single Point
(3) Integration

In the MULTIPLE-POINT METHOD the meter is held at two or more points in the vertical, an observation being made at each point by counting the number of revolutions of the meter for a given number of seconds, usually not less than 40 nor more than 70, one observation at a point being sufficient. There are several multiple-point methods in use; the most accurate, called the *vertical velocity curve method*, consists in taking a sufficient number of observations in the vertical to enable the vertical curve to be plotted, from which the mean velocity is obtained by scaling or by planimeter. A quicker method, called the 0.2 *and* 0.8 *method*, usually accurate within a small percentage, is to observe the velocity at 0.2 and 0.8 total depth, respectively, the mean of these being the mean velocity in the vertical. This is the method now in most general use by the U. S. Geological Survey.

In the SINGLE-POINT METHOD the meter is held at the depth of the thread of mean velocity, or at some depth for which the coefficient for reducing to mean velocity has been determined. The most common method of this kind is the 0.6 *depth method*, this being approximately the depth of the thread of mean velocity, and gives the mean velocity with sufficient accuracy in most cases.

In another single-point method called the *subsurface method*, the meter is held just below the surface, usually about 1 foot, and a coefficient, usually about 0.90, is applied to obtain the mean velocity. This is a very useful method where the current is so swift as to interfere with holding the meter at a greater depth, and is much used during high-water stages.

In the INTEGRATION METHOD the meter is moved at a slow, uniform speed, from the surface to the bottom and back again to the surface, noting the number of revolutions and the time taken in the operation. The Price meter is not as well adapted to this method as are the other forms, because a vertical movement of the meter through the water increases the revolutions of the wheel and thus give records velocities which are too large.

8-36. Computation of Discharge. In computing the discharge from the observed velocities the cross-section is considered as divided into vertical strips, the width of these strips varying, as previously noted, depending upon the width of the stream and

amount of the discharge. The depth at each vertical is measured and the velocities at both the 0.2 and 0.8 depth observed. The average of these observed velocities gives the mean velocity in the vertical. Where the depth is insufficient the velocity at the 0.6 depth is observed and assumed to be the mean velocity in the vertical. If the stream is very shallow, a pygmy-type meter may be used. Formerly the mean velocity for any given strip was obtained by taking the average of the mean velocities in the two verticals bounding the strip and multiplying this velocity by the area of the strip, assumed to be equal to its width multiplied by the average of the depths at the two verticals bounding it. In the later practice of the U. S. Geological Survey it is assumed that the mean velocity in a vertical applies to a section extending half way to the next vertical in each direction. The average velocity in a vertical is then multiplied by an area equal to the width of this strip times the depth of the vertical used to obtain the strip discharge. The summation of these individual discharges across the stream gives the total discharge of the stream.

Figs. 8-12 and 8-13 illustrate standard forms of notes for a discharge measurement. Fig. 8-12 contains the location, date of measurement, dimensions of measuring section, discharge, meter number and method of measurement, the gage height, weather conditions and the observers appraisal of the reliability of the measurement (good to within 5%). Note that gage recorder readings were checked against a gage float in the well and against a staff gage outside (which required 0.1 ft. correction for comparison with the inside gage).

The horizontal angle coefficient is required if the velocity readings are not normal to the measuring section, since a normal condition is assumed in computing discharge. The small numbers in the right margin of Fig. 8-13 are coefficients to be applied to the velocities where the direction of flow as indicated by the position of the meter, makes an angle with the normal to the measuring section. The coefficient to apply can be found by holding the notes sheet so that the small circle at the left margin under "angle Coefficient" is over the meter and the left edge of the sheet is in line with the measuring cross-section (i.e., the notes sheet is turned 90° from its usual reading position). For

9-275-F
Jan. 1956

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

DISCHARGE MEASUREMENT NOTES

Meas. No. 258
Comp. by J. D. L.
Checked by L. A. S.

Lamprey River near New Market, N. H.

Date April 18, 1960 Party J. D. Linney

Width 104' Area 658 s.f. Vel. 1.09 G. H. 4.18 Disch. 719

Method 2-.8 No. secs. 26 G. H. change -.005 in 1 hrs. Susp. 30 C-5

Method coef. 1.00 Hor. angle coef. 1.00 Susp. coef. 1.005 Meter No. 2351

GAGE READINGS				
Time		Recorder	Inside	Outside
11:35		4.18	4.182	4.27
S 11.55		4.18		
F 12.55		4.175		
1:05		4.175	4.172	4.27
Weighted M. G. H.		4.18	4.18	4.27
G. H. correction				-.10
Correct M. G. H.		4.18	4.18	4.17

Date rated 8-15-57 Used rating for rod .... susp. Meter 0.5 ft. above bottom of wt. Tags checked ....

Spin before meas. O. K. after O. K.

Meas. plots 3 % diff. from .... rating

Wading, cable, ice, boat, upstr., downstr., side bridge 250 feet, mile, above, below gage, and ....

Check-bar, chain found —

changed to .... at ....

Correct ....

Levels obtained —

Measurement rated excellent (2%), good (5%), fair (8%), poor (over 8%), based on following conditions: Cross section Smooth

Flow Uniform Weather Cloudy - showers

Other .... Air — °F@ ....

Gage Operating O. K. Water — °F@ ....

.... Record removed .... Intake flushed U L

Observer None

Control Clear

Remarks Tested tape-gage index

G. H. of zero flow — ft.

16—58354-4

Fig. 8-12. Discharge Measurement Notes.
(Courtesy, U. S. Geological Survey.)

L.E.W 11:55 River at:— near Newmarket, N.H.

Angle coefficient	Dist. from initial point	Width	Depth	Observation depth	Revolutions	Time in seconds	Velocity: At point	Velocity: Mean in vertical	Adjusted for hor. angle or --------	Area	Discharge
	102	3.5	0					0		0	0
	95	6.0	5.1	.2	15	46	.74	.66	–	30.6	20.2
				.8	10	40	.58				
	90	4.5	6.3	.2	15	43	.79	.62		28.4	17.6
				.8	10	51	.46				
	86	4.0	6.7	.2	20	40	1.12	.84		26.8	22.5
				.8	10	42	0.55				
	82	4.0	6.5	.2	25	44	1.28	1.06		26.0	27.6
				.8	15	40	0.85				
	78	4.0	7.5	.2	25	41	1.37	1.10		30.0	33.0
				.8	20	54	0.84				
	74	4.0	7.6	.2	30	46	1.46	1.10		30.4	33.4
				.8	15	46	0.74				
	18	4.0	6.1	.2	25	45	1.25	1.05		24.4	25.6
				.8	15	40	0.85				
	14	4.0	5.5	.2	25	47	1.20	1.08		22.0	23.8
				.8	20	47	0.96				
	10	4.5	4.9	.2	20	44	1.02	0.92		22.0	20.2
				.8	15	42	0.81				
	5	5.0	4.0	.2	15	40	0.85	0.55		20.0	11.0
				.8	5	49	0.25				
	0	3.5	2.6	.2	15	56	0.61	0.60		9.1	5.5
				.8	10	40	0.58				
	-2	1.0	0					0		0	0
	R.E.W.										
	12:55	104.0								658.1	715.8
											X1.005
											719.4

FIG. 8-13. CURRENT METER MEASUREMENTS AND COMPUTATION OF DISCHARGE.

(Courtesy, U. S. Geological Survey.)

example, if the meter takes a position in the current so that its direction cuts through the number .94, then this coefficient should be applied in computing the discharge at that point. Since no divergence from the normal was observed in measurements shown in Fig. 8-13, the first column in Fig. 8-13 is blank and the coefficient listed in Fig. 8-12 is 1.00.

A partial cross section of the river where measurements were made is shown in Fig. 8-14.

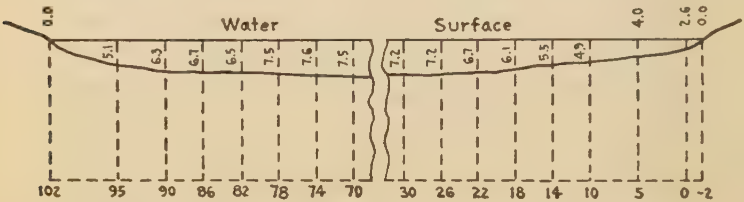

FIG. 8-14. CROSS-SECTION OF RIVER WHERE MEASUREMENTS SHOWN IN FIGS. 8-12 AND 8-13 WERE TAKEN.

8-37. Measurements of Flow of Ice-Covered Streams.* The use of weirs or dams in measuring the flow of streams in winter is frequently impossible unless the crest can be kept clear of ice. Measurements of flow by means of the current meter can usually be made through holes cut in the ice, except where accumulations of anchor and needle ice prevent. The methods suitable for current meter velocity observations under ice cover are:

(1) By taking vertical velocity curves.

(2) By the 0.2 and 0.8 method, which gives practically as good results for ice cover as for open section.

(3) By observations at 0.5 depth (i.e., below bottom of ice), and the application of a coefficient of 0.88, this usually giving the mean velocity within a few per cent.

8-38. Measurement of Flow in Artificial Channels. In the case of canals, flumes, etc. any of the methods previously described for streams may be used where the flow is not affected by draft through wheels or gates. For example, where water is being

* See U. S. Geological Survey Water Supply Paper No. 337, by W. G. Hoyt.

drawn from the lower end of a canal or head bay, through water wheels, the flow is no longer like that in an ordinary channel, the distribution of velocity in the vertical and horizontal being quite different. Float measurements may still be made, but the shorter current meter methods will not in general give good results. Under such conditions the best method of obtaining velocity by current meter is to use the vertical velocity curve method or integration method.

8-39. Methods of Estimating Stream Flow During a Period of Time. Estimates of stream flow covering a period of time may be made.

(1) By means of weirs or dams, observing daily the gage heights on the crest.

(2) By the velocity method, using either current meter or floats for occasional measurements of flow at different stages, and some form of gage for obtaining a record of daily stage of the river and to which the discharge measurements can be referred.

The gage heights for daily stage in any of the above methods should be taken in sufficient number to give an average determination for the day. Usually a reading in the morning and at night will suffice, but where large fluctuations occur on account of the pondage at dams, etc., more readings may be necessary.

The most common method for acquiring long range flow data is by use of automatic gage recorders and periodic velocity measurements with a current meter. Automatic recording gages are described in Arts. 7-24 to 7-26. A new type developed by the U. S. Geological Survey is designed to print out the gage readings on punched tape so that they may be fed into an electronic computer to obtain discharge. Once the relationship is established between discharge and gage height at a gaging station, the flow for any stage can be closely estimated. This is done by reference to a station rating curve.

8-40. Station Rating Curve. The individual discharge measurements at a given gaging station are plotted against gage heights producing a station rating curve. The plot is usually made to logarithmic scales, so that the curve will plot very nearly as a straight line. This is so because the relation between dis-

charge and depth of flow is essentially parabolic (i.e., varying as some power greater than one). This relation has been substantiated by plots at many stations and may be demonstrated theoretically by a consideration of the formula for discharge over a rectangular weir (Art. 8-22), which approximates the control at many gaging stations.

The formula is

$$Q = cbH^{\frac{3}{2}}$$

in which Q is the discharge, c is a constant based on experimental data, b is the width of the crest of the weir and H is the depth of flow over the weir. For any particular control, c and b are constant, hence the discharge Q varies as the three-halves power of H, the height above the crest or height above zero flow. An inspection of the formulas for flow over a triangular weir and in open channels will also reveal a parabolic relation between discharge and depth of flow. For actual control sections, the exponent usually varies between the 1.5 and 2.5 power.

If the gage heights are adjusted to read above elevation of "zero flow," the curve can usually be made to plot close to a straight line throughout. The point of zero flow is formed by trial extensions of low part of curve. It is more common, however, to plot the gage heights directly against discharge, as in Fig. 8-15, selecting a scale which will make the curve nearly straight, especially in the upper stages. The principal advantage of the straight-line logarithmic plot is the ease with which it may be extended for extrapolation of discharge at upper stages. The discharge at upper stages is of particular interest in the study of flood flows, yet few if any actual discharge readings are usually available at the higher stages.

The station rating curve in Fig. 8-15 is plotted in two parts, the lower stages being plotted to a larger scale than the upper stages for more accurate reading of lower stages. The discharge-gage height relation is affected by changes in control due to shifting of the stream bed or other causes. Consequently a rating curve is subject to constant modifications or change. The curve in Fig. 8-15 has been adjusted several times over a period of years. The original plot showed these modifications over a six-year

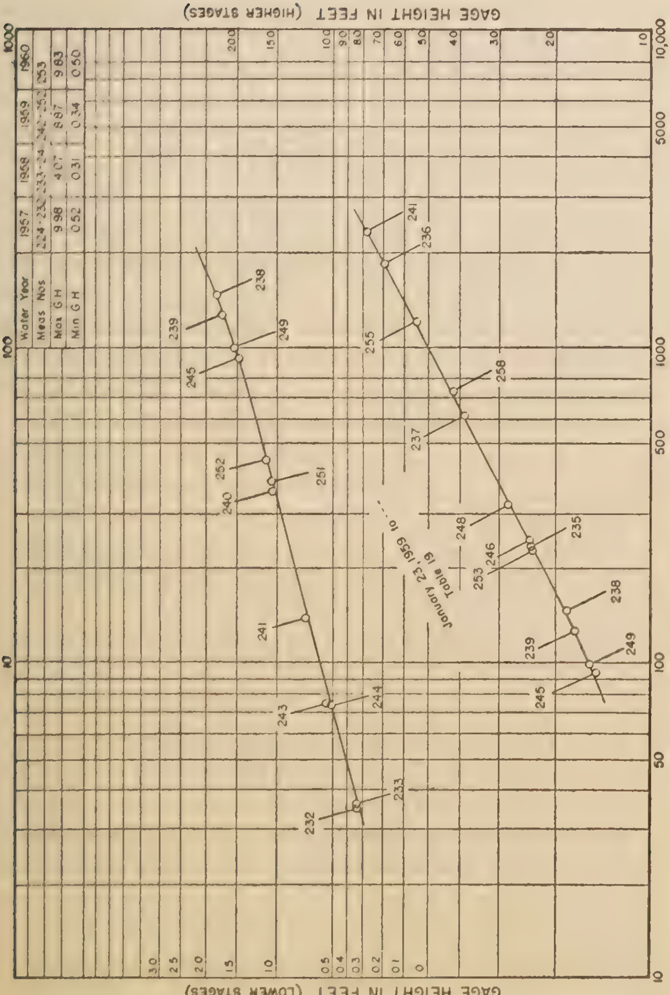

FIG. 8-15. STATION RATING CURVE.
(Compiled from records of U. S. Geological Survey.)

period. To simplify the illustration, only the curve as of latest date is shown, and the fine lines of the grid have been omitted. The small circles with measurement numbers attached show the plotted points of actual measurements. The measurement shown in Figs. 8-12 and 8-13 is plotted at point 258. A tabulation in upper right corner of Fig. 8-15 shows the high and low gage readings for several "water years," which in the New England region, run from October 1 to September 30. This period is chosen in order that the effect on run-off of storage in the form of snow will all be included within one twelve-month period.

From the station rating curve, a rating table is prepared which gives the discharge for every 0.1 foot gage height, and, for the lower stages, for every 0.01 foot. In the preparation of this table the data are "smoothed" by making use of the property of the parabolic type curve; i.e., that second differences are either equal or follow a uniformly varying pattern.

Using the daily records of gage heights and the station rating curve (or the rating table constructed from it) the discharge may be obtained over any desired period of time.

Discharge data for streams and rivers in the United States are published annually by the U. S. Geological Survey in "Surface Water Supply of the United States."

8-41. Streams with Shifting Beds. Where a river bed changes rapidly in condition, owing to the unstable character of materials comprising it, or owing to frequent floods, the station rating curve will be continuously changing in form and frequent measurements of velocity and discharge will be necessary to determine its shape. A plot of the stream cross section, mean velocity of stream and discharge on one diagram is useful in studying the effect of changes in the stream bed. It will also reveal that the maximum discharge during a flood usually occurs some time before the highest stage is reached. The weir method of estimate, where feasible, is preferable to the velocity method on such streams.

8-42. Estimates of Flow in Winter. When the control section is free of ice, continuous estimates of flow may be made using the normal rating curve. When the control is not free of ice, the gage-discharge relation is not reliable, and the rating

curve cannot be used. Satisfactory records of flow can be obtained at dams or weirs, if care is taken to keep the ice back from the crest. Under conditions of continuous ice cover and freedom from anchor and needle ice, discharge measurements, using the current meter, may be made through holes bored through the ice, provided the depth of ice cover is taken into account. These discharge measurements will plot on the normal station rating curve so long as the control is free of ice, and there is no backwater.

8-43. Use of Measurements and Estimates of Flow. Single measurements of flow of rivers are not, as a rule, of value in determining the regimen of flow, unless at extreme stages, such as very high water or extreme drought. It is these extremes of stage that especially affect the design and operation of water power plants and use of water. Detached measurements are useful, however, for determining leakage through a dam, division of water between different users, efficiency of water wheels, etc.

Estimates of flow of rivers, to be of conclusive value in estimating water power and water supply, must be carried over several years and embrace as widely varying conditions of flow as possible. The stream flow records of the U. S. Geological Survey will in many cases provide estimates of flow extending over several years' time at one or more points in the drainage area in question. If an estimate of flow is desired at a location for which no records are available, a control section may be chosen and gage readings taken over a period of several months. Sufficient discharge measurements are also needed to establish a rating curve. This short record of flow may then be extended to a longer period by assuming the discharge to follow the trend established for a long period at another station on the same stream or at a station on an adjacent stream with similar characteristics.

PROBLEMS

1. The elevations of the water surface in a canal at points 800 feet apart are 63.842 and 63.560 feet respectively. The mean cross-section is approximately a trapezoid with a width of 72 feet at the water surface and with banks sloping 2 horizontal to 1 vertical. The depth of water in the middle part of the channel is 3.00 feet. The bottom and banks of the canal are of earth in average condition. Compute the rate of flow in the canal.

2. Compute the discharge over a sharp crested weir 6 feet wide. The head on the weir is 0.46 feet. The upstream channel is broad compared with the width of the weir.

3. What is the straight-line equation of a meter which makes 35 revolutions per minute in a current of 1.42 feet per second velocity, and 90 revolutions per minute in a current of 3.51 feet per second velocity? At what velocity does the equation indicate that the wheel will cease to turn? The equation has the form $v = an + b$ where v is velocity of flow in feet per second, n is the number of revolutions per second of the meter, and a and b are constants.

4.

Dist. from left bank	0	10	20	30	40	50	60	68
Total depth at section	0	2.6	3.5	4.3	4.6	3.2	2.2	0
Velocity at Surface	0	1.00	1.70	2.04	2.40	1.99	0.90	0
" .2 Depth		1.20	2.30	2.39	2.81	2.22	1.10	
" .4 "		1.30	2.24	2.38	2.80	2.00	1.18	
" .6 "		1.16	1.90	2.05	2.56	1.70	1.05	
" .8 "		1.05	1.30	1.55	2.08	1.32	0.94	
" Bottom		0.80	0.90	0.98	1.55	0.96	0.70	

Distances in feet and velocities in feet per second.

(a) Plot the cross-section of the stream. Horizontal scale, 1 inch = 10 feet and vertical scale, 1 inch = 2 feet.

(b) Plot on the cross-section, curves of equal velocity for 0.5, 1.0, 1.5, 2.0 and 2.5 feet per second.

(c) Plot velocity curves in the vertical at the 10, 20, 40 and 60-foot points in the cross-section. Use a velocity scale of 1 inch = 2 feet per second, and a depth scale of 1 inch = 2 feet.

(d) Determine from the plot at the 40-foot section the magnitude and position of the maximum velocity in the vertical. Determine also the magnitude and position of the mean velocity in the vertical. Show these on the plot in (c).

(e) Compare the result for mean velocity in (d) with that obtained by the .2 and .8 method.

5. Compute the discharge from the notes given in Problem 4. Use the .2 and .8 method and arrange the computations as shown in Fig. 8-13.

6. Referring to Parshall flume (Art. 8-25), the head above the contracted section is 1.20 ft., and the width of the throat of the flume is 4.00 ft. Find the discharge.

7. A submerged rectangular orifice 1.0 × 1.5 ft. in an irrigation canal is flowing under a head of 6 inches. What is the rate of discharge?

CHAPTER 9

MAP PROJECTIONS
TOPOGRAPHIC AND HYDROGRAPHIC MAPS

9-1. Map Projections.* A map is a representation of any portion of the surface of the earth, on a plane surface, for the purpose of showing on a convenient scale the relative positions of points and natural features on the earth. The earth, according to Clark's spheroid of reference (1866), resembles an ellipse (Fig. 9-1)

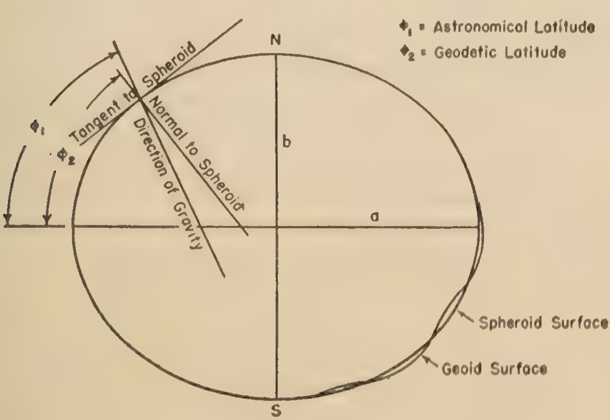

FIG. 9-1. SHAPE OF EARTH'S SURFACE.

with semi-major axis, (a) of 6,378,206.4 meters and semi-minor axis (b) of 6,356,583.8 meters, with eccentricity, $e = \sqrt{\frac{a^2 - b^2}{a^2}}$. Since the Earth is composed of numerous materials of differing densities, the direction and the force of gravity varies from point to point over the surface of the Earth. Whereas a spheroid can

* For a discussion of the relative accuracy of the different projections and historical notes see Special Publication No. 68, U. S. Coast and Geodetic Survey.

be considered as a perfect mathematical figure, the actual sea-level surface of the Earth is more accurately represented by a figure known as a geoid, which takes into account the various local factors but still closely approximates the spheroid as shown in right quadrant of Fig. 9-1. Positions on the face of the Earth must therefore be determined by taking into account the results of several types of observations; force and direction of gravity, astronomic, and others.

Since no curved surface can be developed into a true plane for the purpose of map construction, it becomes necessary to consider what compromises can be made that best serve the desired purposes for the particular map. The narrower the strip of the earth's surface which it is desired to portray on a plane, the smaller will be the distortion involved in the process. From this it can be seen that strips in an east and west, and others in a north and south direction can be chosen to suit the local purposes. (Other strips, in a diagonal direction, can also be used.) Any map showing topographic details is essentially an approximation of the terrain. Its accuracy is controlled by certain points whose positions have been accurately determined. The greater the number of such points the greater is the accuracy of the map as a whole. Since on a survey extending over a large area the positions of the points of control are usually defined by means of spherical coordinates (latitude and longitude) it is customary to show meridians and parallels on the map and to plot the positions of these controlling points from their latitudes and longitudes, the rest of the map being filled in with reference to these controlling points.

Any representation of a portion of a spherical surface on a plane is necessarily distorted, the amount of the distortion depending upon the area mapped. One of the first problems in map making, then, is to find some mode of projection which will make it possible to show a portion of the earth's surface on a plane with the minimum amount of distortion.

Various forms of projection have been devised, each one suited to some special purpose. Some of these projections are purely geometric, while others are arbitrary. On very small areas the distortion is small and the different kinds of projection give

nearly the same results, but for very large areas, such as that of a continent or a hemisphere, the different projections give widely different results.

For more than a century the U. S. Coast and Geodetic Survey has engaged in geodetic operations which determined the geodetic positions (the latitudes and longitudes) of thousands of monumented points distributed throughout the country. These latitudes and longitudes are on an ideal figure, a spheroid of reference which closely approaches the sea-level surface of the earth. By mathematical processes the positions of the grid lines of the various State coordinate systems have now been determined for all the States. These several grid systems have, of course, been determined with respect to the meridians and parallels on the spheroid of reference. A point that is defined by stating its x- and y-coordinates on the State grid of that particular area. If either position is known, the other can be derived by formal mathematical computation (Art. 1-49). Likewise, the geodetic length and azimuth between two positions can be transformed into a grid length and azimuth by mathematical operations. Or the process may be reversed when the grid values are known and the geodetic values are desired. Since the grid values for all States are available in Special Publications, and formulae and methods of computations are fully treated in other Special Publications.*

9-2. Properties of Map Projections. The ideal map projection would possess the following properties: (1) areas should be directly comparable over the entire map (*equal-area* projection); (2) the shapes of smaller features should be preserved although the shape of entire countries may be distorted (*conformal* projection); (3) distances on the map between any pair of points should be in constant scale ratio (impossible to fulfill, but a projection can be prepared giving true distances from any one point to all others on the globe); (4) great circles (shortest distance between two points on the surface of the globe) should be represented by straight lines on the projection (*gnomonic* projec-

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* See U. S. Coast and Geodetic Survey Special Publications, 193, 194, 195, 235, and 251.

tion); (5) positions by latitude and longitude should be easily found or plotted (e.g., Mercator projection). Since no one projection can satisfy all of the above conditions, the choice of a projection depends upon the purpose of the map. There are other projections, such as the polyconic, which are compromises and possess no single property listed above.

The property of equal area would be especially important for maps used in making certain types of economic studies such as land use studies.

The true definition of a conformal projection is that the scale ratio at a point be the same in all directions. That is, the scale will vary over the map but at any one point, the ratio will be independent of the azimuth. As a corollary, lines at an angle to each other on the earth at a given point will appear at the same angle on the map. Therefore, meridians and parallels always intersect at right angles on a conformal projection. This property is especially important when applying map projections to the establishment of coordinates for surveying purposes, such as the transverse Mercator and the Lambert conformal projections (Arts. 1-63 and 1-64) because angles as measured on the ground will be the same on the projection.

By using a conformal map projection as the base for a State coordinate system and limiting one dimension of the area which is to be covered by a single grid, two things are accomplished: first, on a conformal map projection, angles are preserved. This means that, at a given point, the difference between geodetic and grid azimuths of very short lines is a constant, and angles on the Earth formed by such lines are truly represented on the map. For practical purposes of land surveying, this condition holds for distances up to about 10 miles. For longer lines, the difference varies, and the correction to be applied to an observed (geodetric) angle to obtain a corresponding grid angle is the difference of the corrections to the azimuths of the lines, separately derived. Second, the limitation in the width of the projection or grid permits a control of deviations of grid lengths from geodetic lengths. When the width of an area covered by a single grid is 158 miles, the extreme difference between geodetic and grid lengths will be $\frac{1}{10,000}$ of the length of a line, which is quite satisfactory for most land surveys.

State coordinate systems have now been adopted for all of the States (including Alaska and Hawaii) and grids have been prepared for all of them in pamphlet form in Special Publications by the U. S. C. and G. Survey. These publications contain the necessary formulae and tables of factors for converting latitudes and longitudes to grid values and vice versa. These systems are on the Lambert Conformal, the Transverse Mercator, or the Oblique Mercator type of projection, depending on the general configuration of the State (Art. 1-62).

Map projections may be divided into two general classes: first, those which are true projections or perspectives and which may be constructed by graphical methods alone; and second, those which must be derived analytically to possess one of the properties listed above or to satisfy some compromise of those properties.

9-3. Orthographic Projection. In orthographic projection the eye is supposed to be looking along a line which is perpendicular to the plane of the map. This is the ordinary system of projection used in architectural and engineering drawings, where

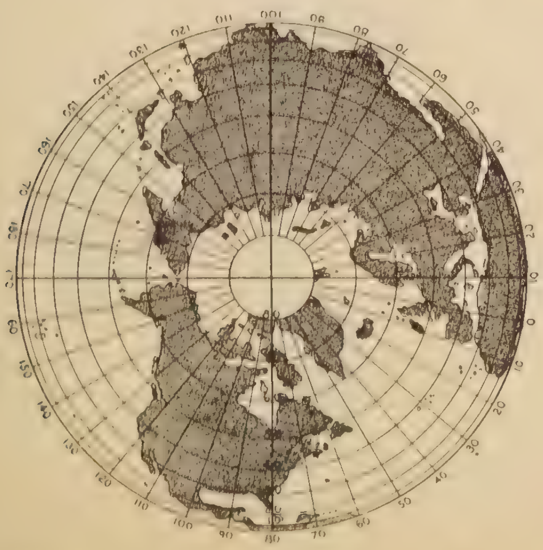

FIG. 9-2. ORTHOGRAPHIC PROJECTION ON THE PLANE OF THE EQUATOR.

objects are shown in plan, and front and side elevations. In map making this method is used chiefly in representing hemispheres. This projection shows the map greatly contracted near the edges, as will be seen by an examination of Fig. 9-2. Such a projection could be used to show the details of those regions only which are near the middle of the map.

9-4. Stereographic Projection. In stereographic projection the eye is assumed to be on the surface of the sphere at the pole of a great circle, whose plane is the plane of projection, and in the opposite hemisphere from that which is to be mapped. The points on the hemisphere to be mapped are projected on this plane by straight lines drawn from these points to the position of the eye. This projection is used to represent a hemisphere where it is desired to show details near the edge of the map, since the area near the edge is greatly expanded as compared with the central portion, as will be seen by comparing the length of 10 degrees of latitude near the equator (Fig. 9-3) with 10 degrees near the pole.

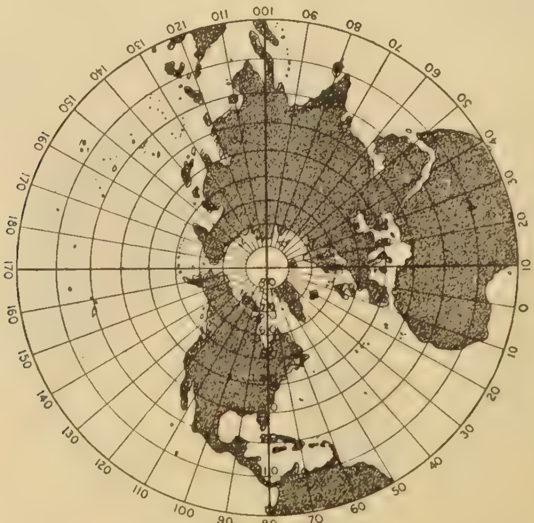

FIG. 9-3. STEREOGRAPHIC PROJECTION ON THE PLANE OF THE EQUATOR.

9-5. Gnomonic Projection. In the gnomonic projection the area to be mapped is projected on a plane tangent to the sphere, and the eye is supposed to be at the center of the sphere. The characteristic of this projection is that all great circles appear on the map as straight lines, since great circles are projected on the map by planes passing through the position of the eye. This projection is much used in constructing charts for "great circle sailing," the shortest route * between points on the globe being shown as a straight line.

The simplest chart to construct on this projection is the "polar chart," in which the plane of projection is tangent to the earth at the pole. All parallels of latitude then appear on the chart as circles, whose centers are at the pole. The radius of any circle is equal to $R \cot \phi$ where R is the radius of the sphere, and ϕ is the latitude of the parallel. The length of a degree of latitude becomes greater as the latitude itself decreases, this spacing increasing so rapidly toward the equator that the polar chart cannot be conveniently extended to the tropics. The meridians all appear on the chart as straight lines radiating from the pole. In using this chart to obtain the position of a great circle it is only necessary to draw a straight line between the two points in question and this will be the great circle desired. The *vertex*, or point where the track comes nearest the pole, may be determined at once from the chart. The latitudes and longitudes of any points, and the bearing of any portion of the line, may be taken directly from this chart. Fig. 9-4 shows a chart constructed on a plane tangent to the equator in longitude 80 degrees west. The parallels on this chart are not circular curves as in the polar chart. It will be seen that the meridians are necessarily straight on all charts constructed by the gnomonic projection.

9-6. The Mercator Projection. The Mercator projection (Fig. 9-5) is a modification of the simple cylindrical projection. The latter is constructed by projecting the meridians and parallels onto the surface of a cylinder which is tangent to the earth at the equator, and then developing this cylinder on the map. The

* On the assumption that the earth is a sphere the great circle is the shortest route between two points.

meridians and parallels all appear as straight lines; the meridians are equally spaced, while the distances between the parallels increase toward the poles. In the Mercator chart, which is much used in navigation, the parallels are so spaced that the ratio of the length of a minute of latitude to that of a minute of

FIG. 9-4. GNOMONIC PROJECTION ON PLANE TANGENT AT THE EQUATOR.

longitude in any part of the map is the same as the ratio existing at the corresponding point on the sphere. Hence the bearing of one point from another as shown by this chart is that course which a vessel would have to steer continuously in order to sail from one point to the other. A straight line on the chart cuts all of the meridians at the same angle. This curve on the earth's surface which corresponds to a straight line on the Mercator chart is called the *loxodrome*, or *rhumb line*. The approximate distance between them may be obtained by using as a scale unit the length of the minute of latitude for a latitude half-way

between the two given points. For the spacing of the parallels of latitude see Bowditch, American Practical Navigator, Table 5, and Chapter II. On account of the varying length of the degrees of latitude all areas near the pole are greatly expanded, while those at the equator are not distorted.

FIG. 9-5. MERCATOR'S PROJECTION.

9-7. Conic Projection. In the simple conic projection a cone is conceived to be tangent to the middle parallel of the map, the apex of the cone being therefore in the earth's axis produced. This cone is developed on the plane of the map as follows. A vertical line is chosen as the central meridian of the map and a point on this line is chosen as the middle latitude. The radius of this middle parallel of latitude is then laid off on the side toward the pole, giving the position of the apex of the cone on the central meridian, which is the center of a series of circles representing the parallels of latitude. This radius equals $N \cot \phi$

where N is the normal * and ϕ is the latitude of this parallel. A circle is then drawn representing the middle parallel of latitude. Beginning at the middle parallel, distances are laid off on the central meridian which are proportional to the degrees of latitude on the earth's surface. The middle parallel itself is also subdivided into degrees of longitude which are proportional to the

FIG. 9-6. SIMPLE CONIC PROJECTION.

degrees of longitude on the earth's surface for this particular latitude. The meridians are all shown as straight lines drawn from the apex of the cone to the points laid off on the middle parallel. The remaining parallels of latitude are circles through the points laid off on the central meridian, the center in each case being the apex of the cone. The parallels and meridians will therefore intersect at right angles in all parts of the map. The

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* The normal is the distance from the parallel of latitude to the axis of the spheroid, measured along a vertical line. If the earth may be considered as a sphere, then the radius of the circle representing the parallel is $R \cot \phi$, where R is the radius of the sphere.

length of the degree, however, on all parallels except the middle one, is evidently slightly in error. (See Fig. 9-6.) The distortion in this projection is so slight that it becomes appreciable only on very large areas.

A modification of this projection which is sometimes used consists in assuming a cone whose surface intersects that of the sphere near the middle portion of the map. This will be found to produce slightly less distortion than where the tangent cone is used.

9-8. Bonne's Projection. Bonne's projection (Fig. 9-7) is a modification of the simple conic projection. Each of the concentric parallels of latitude is divided into degrees of longitude proportional to those on the sphere. The parallels are shown as circles, the radius being equal to $R \cot \phi$ (when the earth is regarded as a sphere) as in the conic projection. Hence the central meridian and every parallel of latitude is divided as on the sphere. The distortion is but slight and distances on the map can be scaled quite accurately. This projection is chiefly used in France.

FIG. 9-7. BONNE'S PROJECTION.

9-9. Polyconic Projection. In the polyconic projection the surface is developed on a series of cones, a different cone being used for each parallel of latitude. Each parallel, then, is developed independently on a cone whose apex is somewhere in the prolongation of the earth's axis. In this form of projection the degrees of latitude are laid off their true lengths on the central meridian, but it will be seen later that they are slightly too large near the east and west edges of the map, because the circles representing the parallels are not concentric as in the conic and in Bonne's projection. The angles of intersection of the meridians and parallels are always nearly true right angles. In fact, on a map of an area as large as that of the United States the unaided eye cannot easily detect the errors in these angles. An investigation of the errors in this projection shows that there is very little distortion and that distances can be scaled accurately enough to satisfy all of the requirements of a map. The polyconic projection was first used by the U. S. Coast and Geodetic Survey and has been adopted by nearly all of the Government surveys for certain kinds of maps.

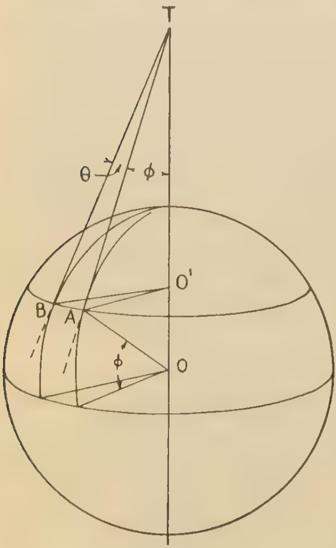

FIG. 9-8.

In each of the cones used in this projection the angle between the axis of the cone and an element will be seen from Fig. 9-8 to equal the latitude ϕ of the parallel in question. The side of the tangent cone, TA, is found by the equation

$$TA = N \cot \phi$$

where N is the length of the normal and ϕ is the latitude. In a sphere N would of course equal the radius AO. TA is the radius of the circle representing the developed parallel. If θ is the angle

at T between two points AB on the parallel and the difference in longitude of A and B is $\Delta\lambda$ then

$$\theta = \Delta\lambda \sin \phi$$

(see Vol. I, p. 177, and Art. 1-55, of this volume).

Since the radius of curvature of the parallels is long it is not convenient to construct these circles by compass; they are therefore usually constructed by plotting the intersections of the meridians and parallels by means of their rectangular coordinates.

In Fig. 9-9, A represents the intersection of a meridian and a parallel. In order to compute the coordinates we have

$$\begin{aligned} x &= TA \sin \theta \\ &= N \cot \phi \sin \theta \\ &= N \cot \phi \sin (\Delta\lambda \sin \phi) \qquad (9\text{-}1) \end{aligned}$$

and

$$\begin{aligned} y &= TA \operatorname{vers} \theta \\ &= \frac{x \operatorname{vers} \theta}{\sin \theta} = x \tan \tfrac{1}{2}\theta \\ &= x \tan \tfrac{1}{2}(\Delta\lambda \sin \phi) \qquad (9\text{-}2) \end{aligned}$$

Tables have been computed from formulas (9-1) and (9-2) giving the coordinates in meters for different latitudes and for different distances east or west from the central meridian of the map. Tables XIV and XV, giving these coordinates for a limited area, are extracted from a larger table in Coast and Geodetic Survey, Special Publication No. 5.

In laying out a map by the polyconic projection a central meridian is first drawn and the true distances between the parallels laid off (see Tables XII and XIII). Lines are then drawn at right angles to this central meridian through the points laid off. The abscissas of the intersections of meridians and parallels desired are then laid off on these perpendiculars

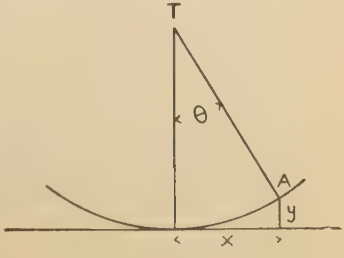

FIG. 9-9.

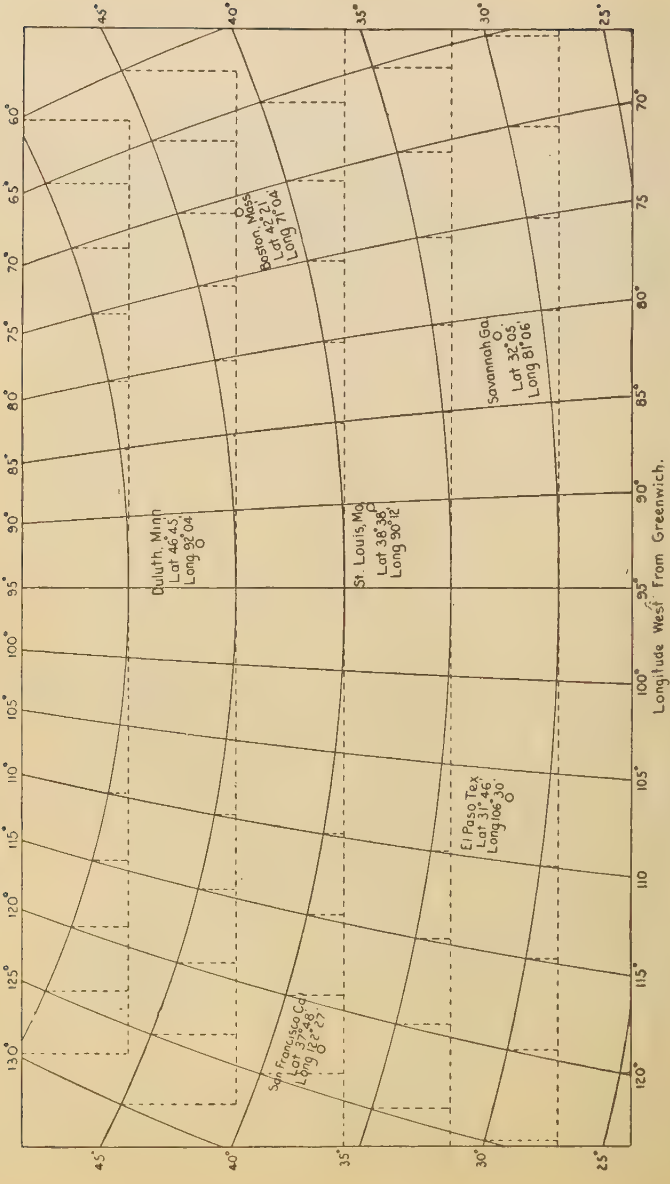

Fig. 9-10. Polyconic Projection of an Area Including the United States.

and the ordinates are measured off at right angles on the side toward the pole. (See Fig. 9-10.) After these points of intersection have all been determined smooth curves are drawn through them representing meridians and parallels. Since the parallels are true circles on the map it will sometimes be convenient to draw these circles by means of curve templets of proper radii.

The interval between consecutive meridians and parallels should be small enough so that the inclosed areas are practically rectangular. This not only facilitates the plotting of triangulation points but insures the accuracy of the map. In finishing the map it may be found convenient to omit some of the lines used in the plotting, in order that the details may not be confused by so large a number of lines passing through them. The interval should be the same in all parts of the map, i.e., if a 5-minute interval is adopted every 5-minute parallel and every 5-minute meridian should be shown. Portions of these lines may be omitted, if necessary, in order to avoid confusion at certain points, or to avoid drawing a line through a title or a note. In some maps only the intersections of meridians and parallels are preserved by means of very short lines which do not interfere with the detail shown on the map, and from which the original lines can be reproduced at any time.

In constructing a polyconic projection for small areas, such as for plane-table sheets, the meridians and parallels will be found almost straight. An examination of tables used for plotting the polyconic projection will show that on a scale of $\frac{1}{10,000}$ the projection which falls within the limits of a normal plane-table sheet is practically rectangular.

The topographic atlas sheets of the U. S. Geological Survey are prepared on the polyconic projection, the most common scales of which are $\frac{1}{24,000}$, $\frac{1}{31,680}$, and $\frac{1}{62,500}$. By inspecting samples of these sheets, it will be noticed that the outlines vary slightly from the rectangular. This is especially prominent in the upper left- and right-hand corners as a circumscribing rectangle is often shown on the map. One minor disadvantage of using this projection is noticed when a series of maps are fitted together. Fig. 9-11 shows a block of nine sheets matched everywhere along the north and south edges, and Fig. 9-12

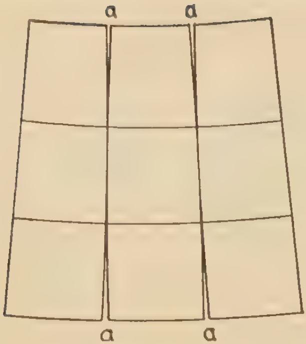

FIG. 9-11.

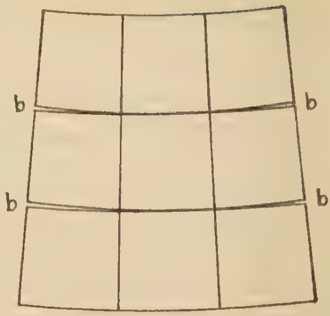

FIG. 9-12.

shows the same block matched along the east and west edges. In the former there will be gores at the places marked *a* and in the latter, there will be gores at the places marked *b*. A skillful map mounter could make the junctions appear practically perfect, but the adjustments become increasingly difficult as the extent of the mounting is increased.

9-10. Lambert Conformal Projection. The Lambert conformal projection is a modification of the simple conic projection. Instead of constructing a projection having a single standard parallel on a tangent cone the scale of the map is reduced in such a proportion as to give a true scale along two selected standard parallels. This projection has the property that small areas on the map are similar in shape to the corresponding areas on the globe; in other words it is *conformal.* Like the simple conic, the scale ratio is large for latitudes far outside the standard parallels; it may be kept reasonably small between the standard parallels by a proper selection of latitudes for these parallels. Along the standard parallels the scale is exact, regardless of distance. The standard parallels are usually chosen so that $\frac{2}{3}$ of the area to be shown lies within these parallels; the remainder is distributed about equally above and below. Fig. 9-13 shows a selection of parallels for the mapping of the United States on the Lambert conformal projection together with the scale ratios (ground distance $\times$ scale of map $\times$ scale ratio = distance on projection).

This projection is therefore well adapted for mapping areas having a long east and west dimension, but is not so good as the polyconic for areas in which the principal dimension is north and south. Another practical advantage of this projection in common with other conic projections is that maps of adjoining areas will fit along the meridians because the meridians are drawn straight; in the polyconic the meridians are curved and maps cannot be joined along the meridians. A disadvantage of the Lambert projection is that when the standard parallels have been selected and the tables computed, local maps cannot be made from these tables except by referring to the same two standard parallels; it is not therefore as flexible as the polyconic in this respect.

In constructing the Lambert projection a central meridian is selected and the two standard parallels are laid out by means of rectangular coordinates. The meridians are drawn straight between plotted points and extended to the limits of the map. The other parallels are put in by laying off proportional distances between the standard parallels, the proper spacings being given in the tables. Tables for constructing this projection will be found in Special Publication No. 52, U. S. Coast and Geodetic Survey. Further information concerning the projection will be found in Special Publication No. 47.

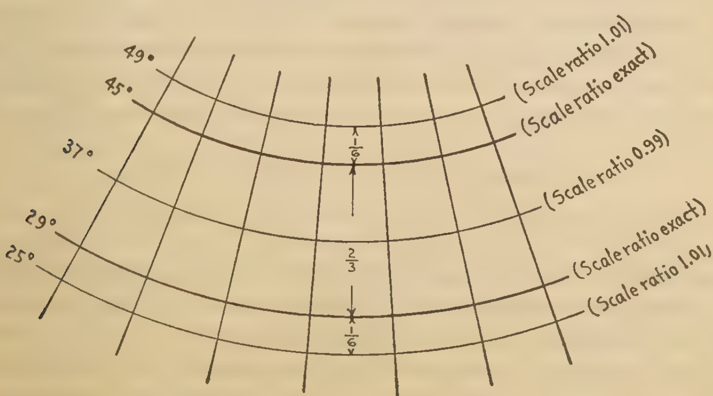

FIG. 9-13.

9-11. Plotting Maps. The first step in constructing a map is to precisely plot the control points upon which the survey was based. These are plotted with reference to the particular map projection or coordinate system being used. The topographic details, whether obtained by aerial or ground surveys, are tied to this control. The process of obtaining a map from aerial surveys is described in Art. 6-12 to 17. Some methods used with ground survey data are described below. A more complete description will be found in Vol. I, Chapters XVI and XVII.

9-12. Plotting Triangulation. The triangulation points are plotted on the map by measurements from the meridians and parallels which have been laid out by one of the methods described in Arts. 9-1 to 9-10. The linear distance of each triangulation station from the nearest meridian and from the nearest parallel is first computed and these two distances are laid out in the same way as when plotting a traverse point by the method of rectangular coordinates. For example, if a triangulation station has a latitude of $42° 12' 43''.94$ and a longitude of $71° 06' 52''.64$ and the meridians and parallels have been drawn on the map for every minute of latitude and longitude, it would only be necessary to compute the number of feet or meters in the seconds of the latitude and longitude and to plot the point by means of these distances. The distance in meters of any point from the meridian and from the parallel may be found by means of tables given in Special Publication No. 5 of the U. S. Coast and Geodetic Survey. In these tables are lengths of arcs of the meridian and of the parallel, expressed in meters, for different latitudes. In tables it will be found that for latitude $42° 13'$,

$$43''.94 \text{ of latitude} = 1355.7 \text{ meters}$$

and

$$52''.64 \text{ of longitude} = 1207.4 \text{ meters}$$

The necessary measurements for plotting the triangulation point are indicated in Fig. 9-14. If desired the distances in meters for $16''.06$ and $07''.36$ could have been found and the point plotted from the nearer meridian and parallel. The positions of these stations on the map are then checked by scaling the lengths of the triangle sides. The interval between consecutive

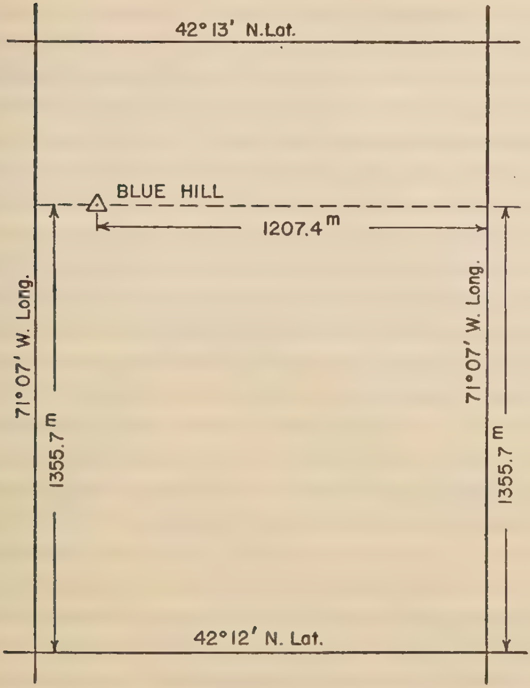

FIG. 9-14. PLOTTING A TRIANGULATION POINT.

meridians and parallels should be such that the figure enclosed can safely be treated as a rectangle when plotting the triangulation points.

9-13. Rectangular Coordinates. In city surveys or other surveys of limited extent where all points are to be referred to a pair of rectangular coordinate axes instead of to the actual meridians and parallels some meridian near the middle of the area to be mapped is chosen as the axis of Y, and all directions are referred to this primary meridian. All of the other so-called "meridians" are not true north lines but are parallel to this primary meridian line. As the axis of X is a straight line through the initial point perpendicular to the axis Y it is evident that neither the axis of X nor any of the lines parallel to it is a true east and west line. The positions of the triangulation points

are plotted by means of their known distances from the two axes, just as the triangulation points of larger surveys are plotted by their distances from the actual meridians and parallels.

9-14. Plotting Details on the Map. All points on the map, whether instrument stations or minor details, should be plotted with reference to the triangulation points, which are already on the map, in order that the accuracy of the map as a whole may be properly controlled. All of the triangulation stations should therefore be plotted before proceeding to plot any of the details. When traverses have been run they may, if desired, be plotted on the final map directly from the field notes or by means of coordinates. In laying out these traverses they should begin at one of the triangulation points even though the fieldwork did not start at any triangulation point, and the plotting of all traverses should be checked by closing on a triangulation point, or, in case the traverse does not close on a triangulation point, by calculating the coordinates of the end point. If the accuracy of a traverse is checked by closing on a plotted triangulation point this also checks the fieldwork, whereas checking by calculated coordinates checks only the plotting. If traverses have been run by means of deflection angles, as is often the case, it will be advisable to calculate the azimuths by means of these deflection angles for the purpose of plotting. The points on the traverse may be laid out from calculated coordinates if the traverse has already been calculated and adjusted to the triangulation (see Art. 1-4). The details may be filled in by means of a protractor and scale. None of the details should be plotted, however, until the traverse has been found to check.

The transit points may be plotted directly from the notes by the usual methods, and if the position of any triangulation point (with which the traverse is connected) as located by the plotting of the traverse fails to coincide with the established position of the triangulation station, this error must be distributed through the traverse by shifting the transit points on the map in such a way as to make the traverse close and to alter the lengths and azimuths of the lines as little as possible; the position of a triangulation point should never be changed simply because a traverse fails to check its location. If the error of closure,

however, is so large as to indicate a mistake in the work rather than an accumulated error, the mistake should be discovered and corrected before proceeding.

9-15. Assembling Maps. It is frequently necessary to assemble several small maps or field sheets of the same or different scales on one large map. In this case, the triangulation is first plotted on the large sheet. The details may then be transferred to this sheet by means of a pantograph (Vol. I, Art. 458, p. 545) or by making photographic reproductions to the large map scale and tracing these on to the large map so that they fit the triangulation points. If the pantograph is used it should be so adjusted that when the tracing point moves from one triangulation station to another on the original sheet the pencil point will move between the corresponding plotted points on the map.

9-16. Finishing Topographic Maps. The method of finishing the map will depend upon whether the original map is to be used as the final product or for making duplicates by means of photographic reproduction.

If the map is to be used for engineering purposes it should be finished with regard to preserving the accuracy of the surveys and the positions of the instrument points rather than to its general appearance. Meridians and parallels, or coordinate lines, should be carefully preserved as a check on the change in scale due to shrinkage of the paper. If the map is for general (public) use especial attention should be paid to making clear everything of a technical character which would not be readily understood by one who is unfamiliar with maps.

It is common practice to make the field sheets on a larger scale than that intended for the published map; in many cases the field sheets are made twice the size of the final map. The weight of lines and size of letters used on the office map will be governed by the amount of the intended reduction. It requires considerable experience to properly prepare a map for reduction, partly because the appearance of such a map is so different from that of the final map to which the draftsman is accustomed. The only way for the beginner to accomplish good results in this work is to first determine what the sizes of the lines and letters are to be on the final map and then to lay out the lines

and letters exactly twice, three times, etc. larger so that they will reduce to the proper size when photographed down. Such a drawing will not look like ordinary drawings; the lines and letters will appear bold and the letters spaced too far apart and too far from the lines to which they refer. After a little practice the draftsman will acquire the ability to judge the appearance of the final plan from that of the original. The tendency of beginners in making drawings for reduction is to crowd the letters, to make them too small, and to place them too close to the lines.

A topographic map should be finished in such a manner that it will convey the desired information and can be readily interpreted. Although the extensive use of different colors on maps is not to be recommended it is sometimes necessary, in order to distinguish readily between land and water surfaces, to use at least one color beside black. In some cases water surfaces are shown in blue and in other cases the land is shown by a flat tint of yellow. When a tint is used on any map it should be a thin wash, so that there will be just enough color to show the desired distinction; deep colors injure the appearance of the map.

9-17. Scales. A scale should always be shown on a map, both for convenience in scaling distances and for detecting errors due to changes in the dimensions of the paper. If, as it is assumed, the paper expands or contracts in exactly the same proportion in all its parts and in all directions, and if a scale is drawn on the map when the plotting is begun, this scale will always give correct distances. Since, however, maps of large size seldom do change so uniformly a complete elimination of such errors can only be effected by observing the changes in the spacing between consecutive meridians or parallels, or between lines of a rectangular coordinate system, and by making due allowance for these charges when scaling distances.

Errors in scale due to paper shrinkage may be avoided if the plan is drawn or traced on a polyester drafting film base which is stable in scale.

For convenience the scale is often shown on a map in different units of measurement, e.g., on many of the charts issued by the Government scales are shown giving distances in statute miles and in kilometers, and in some cases in nautical miles. Not

infrequently two scales are given on the same map, one showing miles, and the other showing distances in feet.

The most common form of scale consists of two parallel horizontal lines, drawn close together, with vertical lines of subdivision drawn between them, and spaced so as to give the desired units, the alternate spaces being inked in black. These spaces represent some large unit (such as a mile, a kilometer, or a thousand feet), except the one at the extreme left end, which is subdivided into tenths of a unit or other convenient fraction; these small spaces are also shown in alternate black and white like the main divisions of the scale. The vertical lines of subdivision are numbered for convenience in taking distances off the scale. A convenient arrangement of the scale is to mark the zero point at the right end of this subdivided space so that all spaces to the right of zero are long and those to the left are short spaces. In taking a distance from the scale with a pair of dividers, for instance, one point of the dividers is placed at the vertical line of the scale marked with the desired number of miles, feet, or other unit; keeping the right-hand point of the dividers in this position the left-hand point may be set at the division giving the desired decimal or other fraction. In this form of scale it is necessary to estimate the fractional parts of the smallest space.

It is customary to state the scale of the map even if this is also shown by the scales just described. This may be done by giving the number of feet or miles to one inch, or it may be stated as a fraction whose numerator is unity, for example $\frac{1}{12000}$ (1,000 feet to one inch). This method has the advantage that distances can be taken from the map in any desired unit of measurement without reference to the particular unit which was used in constructing the map, so that a person who is accustomed for example to the metric system only could take off distances in meters, with a metric scale, from a map which has been made with a foot scale. It is desirable that the scale adopted should be one that gives a simple fraction such as $\frac{1}{1000}$, $\frac{1}{10000}$, $\frac{1}{20000}$, rather than such numbers as $\frac{1}{960}$, $\frac{1}{1800}$, $\frac{1}{4800}$, which correspond to 80 feet, 150 feet, and 400 feet to one inch respectively. Such scales as $\frac{1}{1000}$, $\frac{1}{10000}$, etc. are especially con-

venient when using a metric scale, because the metric scale has a decimal subdivision. The arbitrary selection of a scale, however, is not always practicable, and in many cases this matter of choosing a simple scale would be of minor importance.

When plans are to be reduced to a smaller scale, the numerical scale must be changed to the new scale or only a graphical scale used.

9-18. Conventional Signs for Topographic Maps. Topographic conventional signs are used to represent the form of the surface and such physical features as roads, buildings, cultivated fields, forest growth, rivers, etc. The conventional signs which have come into general use are those which have been adopted and extensively used by the Government surveys. In the United States the Geological Survey maps have been prepared primarily as a basis for a geological study of the country, while the Coast Survey maps are chiefly for the benefit of navigators. The character of the conventional signs used on these maps consequently varies according to the purposes for which the maps were made.

A folder describing the maps available from the U. S. Geological Survey is attached to the back cover of this book. The process of producing these maps is described and samples of typical maps and conventional signs are illustrated.

9-19. Landscape Architect's Plans. The surveyor prepares for the landscape architect a topographic map depicting the shape of the existing ground by contours, the contour intervals usually being 1 or 2 feet. Such a map should also show the locations of trees, shrubbery, buildings, roads and other artificial features. For example, on these maps the trunks of trees are often represented by black dots with the tree name shown by an abbreviation and the diameter marked in inches beside them. The approximate foliage spread of the trees may be shown to scale by circles drawn in dashed lines. Limits of existing tree and shrub growth are sometimes shown by dashed lines or by irregular foliage lines.

The architect's plan, which is superimposed on the surveyor's map, depicts the architect's proposed changes in the shape of the ground by contours drawn in full lines (Fig. 9-15). The re-

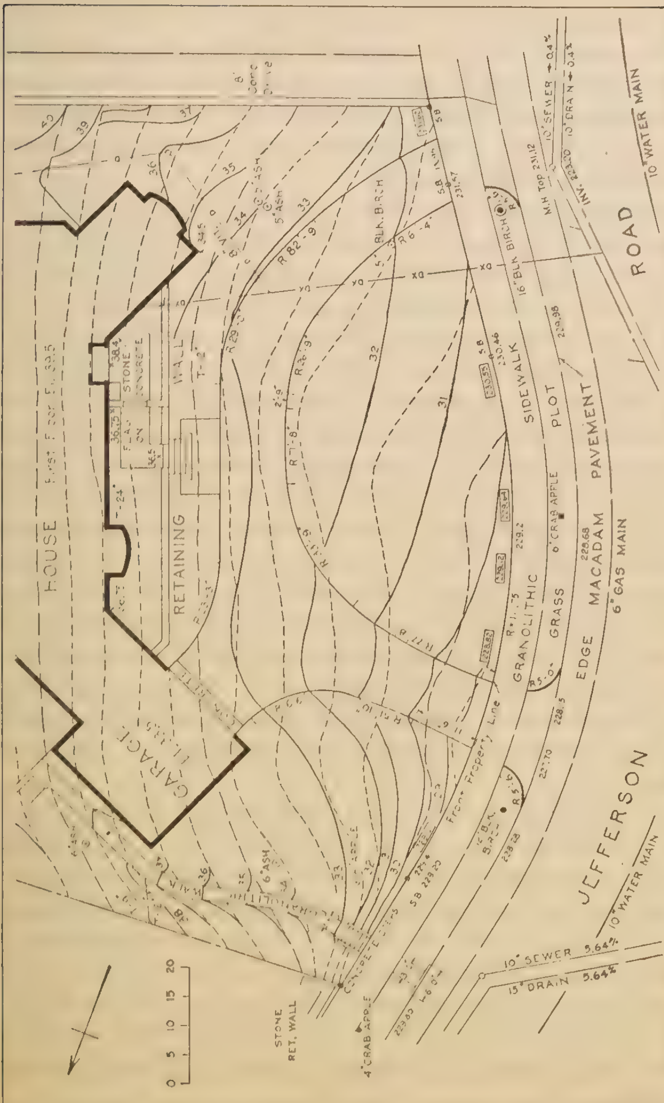

FIG. 9-15. LANDSCAPE ARCHITECT'S PLAN SUPERIMPOSED UPON SURVEYOR'S MAP.

(By permission of Olmsted Bros.)

lation between the existing and proposed graded surfaces can be seen by comparing the two sets of contours. Estimate of grading quantities may be made by planimetering the areas enclosed by existing and proposed contours, and applying the average end area method as described in Vol. I, Art. 413, p. 494. The proposed full-line contours are also used by the surveyor in placing grade stakes for the new work.

The architect's plan will show proposed elevations of drains, catch basins, water lines and building steps, all of which must be installed before the grading is completed. In Fig. 9-15 the symbol T-12″ refers to the thickness of loam or topsoil.

9-20. Hydrographic Maps. The usual method of finishing a hydrographic map is similar to that used for a topographic map. Such maps show, in addition to the shore line, the topography along the shores and of the submerged portions. Just how much of the topography of the surrounding country will be required depends upon the use which is to be made of the map. If, for example, it is prepared for a study of a wharf project the location of streets and buildings in the vicinity should appear on the plan. A hydrographic chart for the purposes of navigation will include merely a sufficient amount of topography to show any landmarks which may be of use to the navigator, such as lighthouses, church spires, or other conspicuous objects. It is well to leave plotted on the finished map a number of the transit stations, so that the survey can be easily connected with other maps or adjacent surveys.

Where the soundings are represented they are usually given in feet and tenths and lettered in black, the number representing the depth of the water below the datum. Where the datum is mean low water, those soundings which are below the datum are lettered in black, while those above the datum may be shown in some other color. The figures are usually written so that the decimal point is at the exact position of the sounding. On the finished map it will not be necessary in most cases to show all of the soundings; only enough of them are lettered to give all necessary information.

From the plotted soundings, underwater contour lines are sketched in the same manner as the contour lines on any topo-

graphic map, the contour interval depending upon the amount of detail required. On charts of the U. S. Coast and Geodetic Survey, depths may be indicated in either feet or fathoms. Numbers or dot-dash symbols are used to indicate depths together with different shades of blue coloring. Marsh and beach areas are shown in green and land areas in yellow. A complete description of hydrographic symbols and abbreviations used on nautical charts is given in "Chart No. 1" available from the U. S. Department of Commerce, Coast and Geodetic Survey.

In finishing hydrographic maps the high-water line should be the heaviest line on the map, the original pencil line being carefully followed; the low-water line should be the next heaviest line on the map. The conventional signs for sand, ledge, etc., are used in representing the low-water line. Swamps are not limited by any definite line drawn on the map but the area is covered by the conventional symbol; this symbol should, however, be made to follow very closely the limiting line sketched on the original sheet. Signs for marsh, grass, etc., should always be parallel to the bottom of the map. Lighthouses, buoys, etc., located on the chart should be shown by their conventional signs or, if preferred, by lettering the name of the object. The color and number of buoys should be indicated.

On the navigation chart of a small river the soundings should be recorded in feet and tenths, and contours drawn every 3 or 6 feet. The direction of current should be shown by an arrow, and sometimes the kind of material forming the bottom is lettered in the proper place on the plan. Rapids and water falls are also indicated.

For dredging work in a harbor the soundings are usually recorded in feet and tenths and the contours are sketched with one-, three-, or six-foot intervals.

CHAPTER 10

ERROR ANALYSIS

10-1. Error Sources. Measurements made of physical quantities are never exact. The precision of the measurement may be improved by repeated measurements, and certain errors corrected or eliminated by surveying techniques, but the exact or true value is never attained. Consequently, quantities computed from measured quantities will never be exact.

Three general classes of error may be present in a measurement: *systematic*, *random* and *mistakes*. *Systematic errors* are those whose magnitude and algebraic sign are definitely related to some condition. Changes in conditions cause corresponding changes in the magnitude, and perhaps the sign, of the resulting error. When conditions remain constant, the resulting errors are sometimes called *constant* or *cumulative* errors. An example of such an error is one resulting from measuring a line with a tape that is too short. The error is introduced each time the tape length is recorded and accumulates for each tape length measured. Systematic errors can be removed by evaluating them and applying the proper correction. In some cases they can be eliminated through use of proper instrumental techniques.

Random errors are those for which it is equally probable that the sign is plus or minus, and also that small errors are more likely to occur than large ones. They are also known as *accidental* or *compensating* errors. They tend to compensate according to laws of chance. The theory of probability can be applied to determine the most likely error resulting from a series of measurements. An example of such an error is the failure of a tapeman to hold the end of his tape exactly over the point. Sometimes he may hold it beyond the mark, and sometimes short of the mark.

Mistakes or blunders are human errors which must be eliminated by careful work and constant checking. No measurement should be accepted as free from mistakes unless measured at least

twice or combined in a closed figure which provides a geometric check. Common types of mistakes are reading a scale backward, misplacing a decimal point, transposition of integers, incorrect recording of field notes, miscounting tape lengths, etc. Before any analysis or adjustment of errors is undertaken mistakes must be eliminated. Techniques for avoiding and detecting mistakes are stressed throughout the text.

Another classification of errors is by cause, such as *natural errors* resulting from climatic changes such as variation in temperature affecting a tape length; *personal errors* caused by inability of an observer to perceive a dimensional value exactly; and *instrumental errors* such as resulting from imperfect adjustment of the instrument. Natural and instrumental errors are usually constant or systematic; personal errors are more often random. They may be systematic, however, if an observer follows a consistent tendency to always judge an observed value too large (or too small).

Although quantities cannot be measured exactly, often certain conditions exist which must be met exactly such as that the three angles of a plane triangle add to 180°, and that the sum of angles taken around a point must add to 360°. Likewise, any elements of the triangle such as the length of the sides, computed from other elements should always yield the same value. In level circuits the difference in elevation between two fixed bench marks can have only one true value. These conditions must be met in the adjustment and distribution of errors.

This chapter will deal with some of the techniques used in the analysis of error sources and the propagation of their effects. Proofs will be presented only to the degree required to enhance the reader's understanding of the subject. Derivations may be found in references listed at the end of this chapter.

10-2. Direct and Indirect Measurements. When the value of a quantity is determined by measuring the quantity itself, a direct measurement has been made. When the magnitude is determined by computing its value from other directly measured quantities an indirect measurement has been made. For example, a distance measured using a steel tape is a direct measurement. Sometimes such a direct measurement is not possible, as across a

river. In this case, other measurements may be made, and the desired quantity computed from them. For example, the unknown distance could be made one side of a triangle in which another side and its adjacent angles can be measured directly, and the unknown distance computed from these direct measurements.

Errors in indirect measurements must be taken into account in determining resultant errors in computed quantities, as demonstrated in Arts. 10-4 and 10-11.

10-3. Systematic Errors. The sign and magnitude of a systematic error are theoretically subject to evaluation. Throughout the text, sources of systematic error have been pointed out and methods given for eliminating or compensating for these types of error. In some cases, the error can be avoided by special measuring techniques, in other cases corrections can be applied directly to the measured quantity, based on secondary observations. Consider the case of instrumental error introduced when the line of sight of a level is not perpendicular to the vertical axis of the instrument. As the level is rotated about its vertical axis, the line of sight will sweep out a vertical cone. Every sight, then, will be in error by the tangent of the angle of inclination times the distance between the level and the leveling rod. Every leveling reading may be corrected for this error if the distance to rod and the angle of inclination of line of sight are known. The process involves applying a computed correction to the directly measured quantity. Note that (1) the error *source* is the inclination of the line of sight (2) the magnitude and sign of the error can be computed and (3) the angle of inclination and distance must be measured in order to compute the magnitude and sign of the error.

Actually, as demonstrated in Art. 3-13, the effect of this systematic error is compensated for in the leveling process by always making the backsights and foresights of equal lengths. The condition is imposed that equality of backsights and foresights be obtained.

10-4. Mathematical Treatment of Systematic Errors. The treatment of systematic errors may be generalized as follows: Let Q represent a physical quantity whose value is to be determined. Assume that Q is a function of several quantities such as

$q_1, q_2, q_3, \cdots q_n$ as follows:

$$Q = f(q_1, q_2, q_3, \cdots q_n) \tag{10-1}$$

These quantities may be directly or indirectly measured; but if indirectly measured, they will be some function of the directly measured quantities. In measuring $q_1, q_2, q_3, \cdots q_n$, the measured values, $m_1, m_2, m_3, \cdots m_n$ are obtained, such that

$$q_1 = m_1 + v_1,\ q_2 = m_2 + v_2,\ q_3 = m_3 + v_3 \cdots q_n = m_n + v_n$$

where $v_1, v_2, v_3, \cdots v_n$ are the measurement errors, then Q may be expressed as

$$Q = f(m_1 + v_1, m_2 + v_2, m_3 + v_3, \cdots m_n + v_n). \tag{10-2}$$

If $v_1, v_2, v_3, \cdots v_n$ are small relative to $m_1, m_2, m_3, \cdots m_n$, respectively, then the effect on Q of $v_1, v_2, v_3, \cdots v_n$ individually may be approximated by

$$\frac{\partial Q}{\partial q_1} v_1, \frac{\partial Q}{\partial q_2} v_2, \frac{\partial Q}{\partial q_3} v_3, \cdots \frac{\partial Q}{\partial q_n} v_n$$

Equation (10-2) then becomes

$$Q = f(m_1, m_2, m_3, \cdots m_n) + \frac{\partial Q}{\partial m_1} v_1 \frac{\partial Q}{\partial m_2} v_2 \frac{\partial Q}{\partial m_3} + \frac{\partial Q}{\partial m_n} \tag{10-3}$$

If $\Delta Q_1, \Delta Q_2, \Delta Q_3, \cdots \Delta Q_n$ are defined as being the changes in Q resulting from $v_1, v_2, v_3, \cdots v_n$, then

$$\Delta Q_1 = \frac{\partial Q}{\partial m_1} v_1,\ \Delta Q_2 = \frac{\partial Q}{\partial m_2} v_2,\ \Delta Q_3 = \frac{\partial Q}{\partial m_3} v_3,\ \Delta Q_n = \frac{\partial Q}{\partial m_n} v_n$$

and Equation (10-3) becomes

$$Q = f(m_1, m_2, m_3, \cdots m_n) + \Delta Q_1, + \Delta Q_2 + \Delta Q_3 \cdots + \Delta Q_n \tag{10-4}$$

This relation provides the basis for studying and analyzing the effect of systematic errors in measured quantities on the value of the quantity indirectly determined, and for tracing the propagation of systematic errors.

The application of the above expressions may be explained by the following example. Consider the problem of computing the unknown elements of a triangle. In the triangle shown in Fig. 10-1, the quantities A, B and a have been measured.

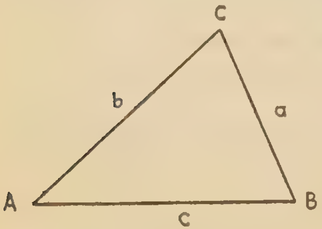

FIG. 10-1.

Side b may be computed from the law of sines:

$$b = a\frac{\sin B}{\sin A}$$

If the measured values of a, A and B contain the errors Δa, ΔA and ΔB, then the object is to determine the resulting propagated error in a computed value of any of the other elements of the triangle.

The error in b resulting from the errors in a, A and B is

$$\Delta b = \frac{\partial b}{\partial A}\Delta a + \frac{\partial b}{\partial B}\Delta A + \frac{\partial b}{\partial a}\Delta B$$

Differentiating b with respect to each of the measured quantities, considering the others as constant

$$\frac{\partial b}{\partial a} = \frac{\sin B}{\sin A}$$

$$\frac{\partial b}{\partial A} = -a \sin B\frac{\cos A}{\sin^2 A}$$

$$\frac{\partial b}{\partial B} = a\frac{\cos B}{\sin A}$$

Finally,

$$\Delta b = \frac{\sin b}{\sin A}\Delta a - a\frac{\sin B \cos A}{\sin^2 A}\Delta A + a\frac{\cos B}{\sin A}\Delta B$$

Example: Measured values are $a = 700.00$ ft., $A = 2° 00'$, $B = 70° 00'$, and errors are $a = +0.10$ ft., $A = +02'$ (0.000581 radians) and $B = -01'$ (0.000291 radians).

Then

$$b = a\frac{\sin B}{\sin A} = 700.00 \times \frac{.93969}{.66913} = 983.04 \text{ ft.}$$

and

$$\Delta b_a = \frac{\sin B}{\sin A}\Delta a = \frac{.934}{.669} \times 0.10 = 0.14 \text{ ft.}$$

$$\Delta b_A = -\frac{a \sin B \cos A}{\sin^2 A}\Delta A$$

$$= -700\left(\frac{.940 \times .743}{(.669)^2}\right) 0.00581 = -0.63 \text{ ft.}$$

$$\Delta b_B = a\frac{\cos B}{\sin A}\Delta B = -700\left(\frac{.342}{.669}\right) 0.000291 = 0.10 \text{ ft.}$$

Adding these values algebraically:

$$\Delta b = +0.14 - 0.63 + 0.10 = -0.39 \text{ ft.}$$

The most serious source of error in the computed value of b resulted from ΔA, since it contributed the largest component of error.

10-5. Distribution of Random Errors. The laws of probability can be applied to random errors to compute the probable error or closeness of approximation to the true value in any one measurement or series of measurements.

If the errors obtained from a series of measurements of the same quantity under like conditions (same observer, same instrument, same climatic conditions, etc.) are plotted as abscissa (− to the left and + to the right) versus number of occurrences as ordinates, a frequency distribution curve results as shown in Fig. 10-2. The errors are obtained by calculating the arithmetic mean and computing the algebraic difference between the arithmetic mean and each observation.

The curve in Fig. 10-2 reveals the following characteristics of random errors:

1. Errors of small magnitude are more frequent than errors of large magnitude.

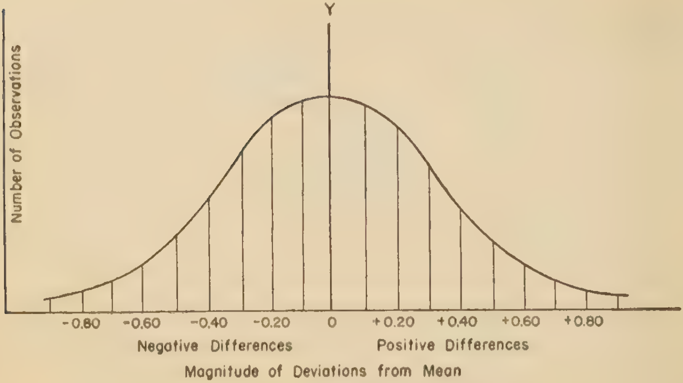

FIG. 10-2. CURVE OF ERROR DISTRIBUTION.

2. Positive and negative errors of equal magnitude are equally likely to occur.

3. The probability of very large errors occurring is small.

If an infinite number of measurements is made of the same quantity under like conditions, a theoretical error curve may be derived based on probability concepts. In this case the occurrences of error are plotted as decimals of the total occurrence, as shown in Fig. 10-3, and the resulting curve is called a Normal Distribution Curve. It has the following properties:

(1) Each ordinate gives in decimals the probability of errors of that magnitude occurring

(2) The sum of probabilities is unity

(3) The total area under the curve is unity

(4) Between any designated limits of error (+ to −) the area under the curve gives the probability of such a range of error occurring.

The equation of the theoretical curve derived from its properties * is

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2x^2} \tag{10-5}$$

* See Bibliography (1) and (2), page 489.

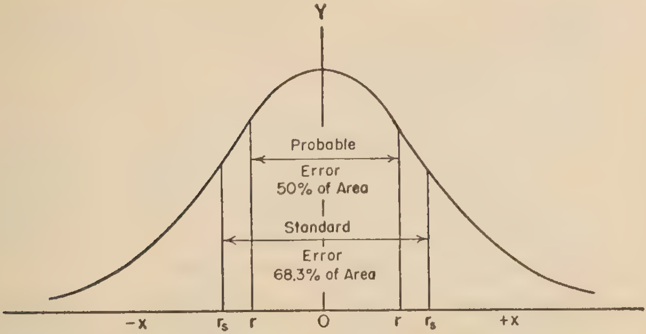

FIG. 10-3. NORMAL DISTRIBUTION CURVE.

in which the value of h depends upon the character of the observations, and e is the base of natural logarithms.

The value of h increases as the accuracy of the measurements increases, and is therefore a measure of precision.

In practice, it is impossible to obtain an infinite number of measurements; however, the curve and the formula derived from the theory may be applied to a finite and even relatively small number of measurements.

10-6. Method of Least Squares. In error analysis and adjustment of measured quantities, frequent reference is made to the " Method of Least Squares." This method is based upon the proposition that the most probable value of a quantity is that which renders the sum of the squares of the residuals a minimum, the residuals being the differences between the most probable value and the observed values, after all systematic errors have been eliminated.

The method of least squares is useful for obtaining the most probable values of measured quantities after all observations have been taken into account, and for deriving the degree of confidence that can be placed in them. The method may also be employed for determining the relative worth of different measurements of the same quantity and for determining the equation of a curve which best represents the relation between two variables in cases where the exact law connecting them is not known.

The method may also be employed to determine how precisely the components of a series of measurements must be taken in order to obtain a desired accuracy in the final results.

The derivation of the principle of least squares can take on different forms, and its application to error analysis and error adjustment involves complicated mathematical derivations. Certain equations and relationships derived from Least Squares are presented in this text. The reader is referred to standard texts listed in the Bibliography for derivations.

10-7. Most Probable Value. From the theory of probability, it can be demonstrated that the most probable value of a quantity derived from a number of measurements n made under the same conditions is the arithmetic mean M_0.

$$M_0 = \frac{M_1 + M_2 + M_3 \cdots + M_n}{n} \tag{10-6}$$

The *deviation* or *residual*, v, of an observation is defined as the most probable value subtracted from the value of that particular observation. $v_1 = M_1 - M_0$, $v_2 = M_2, - M_0$, $\cdots v_n = M_n - M_0$.

Under these conditions, the algebraic sum of the residuals is zero, since

$$\Sigma v = \Sigma M - nM_0 \quad \text{and} \quad M_0 = \frac{\Sigma M}{n}$$

then

$$\Sigma v = \Sigma M - \Sigma M = 0 \tag{10-7}$$

10-8. Probable Error. The probable error, r, of a single observation is an error such that one-half the errors of the series are greater in magnitude than it and the other half are less in magnitude than it; that is, the probability of an error occurring greater than r is just equal to the probability of one occurring less than r.

The limits of this error are shown graphically in Fig. 10-3 by ordinates equidistant from the Y axis and enclosing an area under the curve equal to 0.5. These limits are sometimes called 50 per cent probability ranges.

The mathematical expression for the probable error is

$$r = \pm 0.6745 \sqrt{\frac{\Sigma v^2}{(n - 1)}} \tag{10-8}$$

The probable error, r_0, of the mean of a series of observations of the same quantity is

$$r_0 = \pm 0.6745 \sqrt{\frac{\Sigma v^2}{n(n-1)}} \tag{10-9}$$

Example: Consider six measurements of line AB.

	M	v	v^2
1	615.42	+0.01	0.0001
2	615.44	+0.03	0.0009
3	615.36	−0.05	0.0025
4	615.39	−0.02	0.0004
5	615.45	+0.04	0.0016
6	615.40	−0.01	0.0001
Mean =	615.41	$\Sigma v = 0.00$	$\Sigma v^2 = 0.0056$

$$r = \pm 0.6745 \sqrt{\frac{0.0056}{5}} = \pm 0.02$$

$$r_0 = \pm 0.6745 \sqrt{\frac{0.0056}{6 \times 5}} = \pm 0.01$$

Thus the final expression for the accuracy with which line AB has been determined is 615.41 ± .01 ft. However, the chance that a seventh measurement of line AB would be within 615.41 ± r or within 615.39 and 615.43 is just equal to the chance that it would not.

It should be pointed out that the accuracy of a single observation and of the most probable value are quite different. The relative accuracy of the most probable value in terms of the probable error is

$$\frac{.01}{615.41} = \frac{1}{615.41} \quad \text{or} \quad \frac{1}{60{,}000}$$

However, the relative accuracy expressed in terms of probable error of an individual observation is

$$\frac{0.02}{615.41} = \frac{1}{12{,}308} \quad \text{or} \quad \frac{1}{12{,}000}$$

Before the above steps in the error analysis are carried out, all systematic errors and mistakes must be eliminated from the readings.

10-9. Standard Error. Another measure of precision commonly used is the Standard Error, or *Standard Deviation*. It is expressed as

$$r_s = \sqrt{\frac{\Sigma v^2}{n}} \tag{10-10}$$

In Fig. 10-3 the probability of the standard error is the area under the curve between $+r_s$ and $-r_s$. For an infinite number of observations the probability of the standard error is 0.6827, or 68%. The expression in formula (10-10) is also called the *root-mean-square*.

10-10. Combined Error of a Series of Measurements. Whenever any quantity is made up of the algebraic sum of several other independent quantities, each being subject to random errors, the combined probable error derived from the Method of Least Squares, is as follows:

$$R = \pm\sqrt{r_1^2 + r_2^2 + \cdots r_n^2} \tag{10-11}$$

As an illustration, assume a distance of, say, 400 ft. made up of short sections, each 100 ft. tape-length long. If the random error in measuring a tape length is ± 0.008 ft., then the probable error in the total distance is

$$R = \pm\sqrt{0.008^2 + 0.008^2 + 0.008^2 + 0.008^2} = \pm 0.016 \text{ ft.}$$

The combined error can also be expressed in the above case as $R = \sqrt{n} \cdot r$, where r is the same for each individual measurement.

In combining two or more sets of measurements to obtain a more accurate value, it is logical to give greater weight to that set which had a small error than to a set which had a larger error, and it is customary to apply the weight inversely proportional to the square of the error. For example, if the probable error in one set was ± 0.01 ft., and in another it was $\pm .03$, nine times the weight would be given to the first as compared with the second precision measurement.

The probable error of the final value may be computed from the formula

$$R = \pm \sqrt{\frac{w_1^2 r_1^2 + w_2^2 r_2^2 + w_3^2 r_3^2 + w_4^2 r_4^2}{(\Sigma w)^2}} \tag{10-12}$$

in which r is the probable error of each set and w is the corresponding weight.

The weighted mean (M_0) is found from the relation

$$M_0 = \frac{w_1 M_1 + w_2 M_2 + w_3 M_3 + w_4 M_4}{w_1 + w_2 + w_3 + w_4} = \frac{\Sigma w \times M}{\Sigma w} \tag{10-13}$$

and the check on the calculation of the weighted mean is the fact that $\Sigma w \times v = 0$.

For example, the line A to B has been measured on four different days with the following precision measurements

	M	r	Weighting of Measurements *	w	$W \times M$ for decimal part only	w^2r^2
I	615.41	± 0.02 ft.	$\frac{M \times .0036}{.0004} = 9M$	9	3.69	0.0324
II	615.40	± 0.03	$\frac{M \times .0036}{.0009} = 4M$	4	1.60	0.0144
III	615.42	± 0.03	$\frac{M \times .0036}{.0009} = 4M$	4	1.68	0.0144
IV	615.41	± 0.02	$\frac{M \times .0036}{.0004} = 9M$	9	3.69	0.0324
			Totals	26	10.66	0.0936

$$\text{Weighted } M_0 = 615 + \frac{10.66}{26} = 615.41 \text{ ft.}$$

$$R = \pm\sqrt{\frac{w_1^2 r_1^2 + w_2^2 r_2^2 + w_3^2 r_3^2 + w_4^2 r_4^2}{(\Sigma w)^2}} = \pm\sqrt{\frac{0.0936}{(26)^2}} = \pm 0.01 \text{ ft.}$$

Final expression for A to B: 615.41 ± 0.01 ft.

$$\text{Precision} = \frac{0.01}{615.41} = \frac{1}{62,000}$$

* In the above problem the inverse square of each probable error is multiplied by such a factor (0.0036) as will give weights of integral whole numbers.

If all the probable errors are equal, however, the final value may be obtained by finding a simple mean of the observations, and by finding the probable error of this mean. The latter may be found by dividing the probable error of one determination by the square root of the number of precision values.

Thus, if all the probable errors of the above series had been ± 0.02 ft., the probable error of the final value would be $\pm \frac{0.02}{\sqrt{4}}$ or ± 0.01 ft.

It will be observed that the probable error reached in a series of measurements of the same distance is less than the probable error reached in one set.

In the case illustrated above, four sets of observations gave a probable error of one half the probable error of one set of observations. Since the probable error of the mean varies inversely as the square root of the number of observations, additional measurements beyond a certain number not only have little effect on the resultant probable error, but increase the cost of the survey.

When finding the most probable value of several sets of readings of an angle, weights are assigned to the sets in direct proportion to the number of repetitions in each set.

10-11. Propagation of Random Errors. For the treatment of the propagation of errors by the principle of Least Squares, consider the expression

$$P = f(Q_1, Q_2, \cdots Q_n) \tag{10-14}$$

where the computed value P is a function of measured quantities $Q_1, Q_2, \cdots Q_n$, and the probable errors of the measured quantities are $\Delta Q_1, \Delta Q_2, \cdots \Delta Q_n$. From the Method of Least Squares, it can be shown that the probable error of P is

$$\Delta P = \sqrt{\left(\frac{\partial P}{\partial Q_1}\Delta Q_1\right)^2 + \left(\frac{\partial P}{\partial Q_2}\Delta Q_2\right)^2 \cdots + \left(\frac{\partial P}{\partial Q_n}\Delta Q_n\right)^2} \tag{10-15}$$

Thus the weight given to the first set is $\frac{1}{0.0004} \times 0.0036 = 9$, and for the second the weight is $\frac{1}{0.0009} \times 0.0036 = 4$, etc. The factor 0.0036 is the least common multiple of the squares of the probable errors.

For a condition involving the product of two quantities, $P = Q_1Q_2$

$$\Delta P = \sqrt{\left(\frac{\partial P}{\partial Q_1}\,\Delta Q_1\right)^2 + \left(\frac{\partial P}{\partial Q_2}\,\Delta Q_2\right)^2} \qquad (10\text{-}16)$$

Differentiating $P = Q_1Q_2$ with respect to Q_1 and Q_2, respectively, we obtain

$$\frac{\partial P}{\partial Q_1} = Q_2 \quad \text{and} \quad \frac{\partial P}{\partial Q_2} = Q_1.$$

Substituting in (10-16), we obtain

$$\Delta P = \sqrt{(Q_2\,\Delta Q_1)^2 + (Q_1\,\Delta Q_2)^2}$$

Example: Referring to Fig. 10-3, it is desired to find the probable error of the area of a triangle computed from the measured lengths of the two sides and the included angle.

The observed quantities are $a = 700.00 \pm 0.05$ ft., $b = 500.00 \pm 0.03$ ft., $C = 60° 00' \pm 30''$. Therefore, $a = 0.05$, $b = 0.03$ and $C = 30''$ or .000145 radians.

The area of the triangle $= \frac{1}{2}ab \sin C$ and the probable error of the area, ΔP, from (10-15) is

$$\Delta P = \sqrt{\left(\frac{\partial P}{\partial a}\,\Delta a\right)^2 + \left(\frac{\partial P}{\partial b}\,\Delta b\right)^2 + \left(\frac{\partial P}{\partial C}\,\Delta C\right)^2}$$

By partial differentiation of area equation

$$\frac{\partial P}{\partial a} = \tfrac{1}{2}b \sin C = \tfrac{1}{2} \times 500 \times .866 = 216$$

$$\frac{\partial P}{\partial b} = \tfrac{1}{2}a \sin C = \tfrac{1}{2} \times 700 \times .866 = 303$$

$$\frac{\partial P}{\partial c} = \tfrac{1}{2}ab \cos C = \tfrac{1}{2} \times 700 \times 500 \times .500 = 87500$$

Then

$$P = \sqrt{(216 \times 0.05)^2 + (303 \times 0.03)^2 + (87500 \times .000145)^2}$$

$$= \sqrt{(10.8)^2 + (9.1)^2 + (12.7)^2} = 19.0 \text{ s.f.}$$

10-12. Adjustment of Observations. In surveying practice measurements are repeated and additional measurements taken to obtain checks against mistakes and to increase precision of measurements.

In a case where more measurements are taken than necessary to define a geometric figure, or several runs of levels are made between two bench marks, redundant values are obtained which do not, except by chance, exactly agree with each other. Therefore, a system of adjustments or corrections must be applied to obtain consistency. The Method of Least Squares provides a means for obtaining a system of adjustments which will meet the requirement of consistency and at the same time produce the most probable values for the adjusted quantities.

The general form of equations for this purpose, derived from the Method of Least Squares are the following, known as *normal equations*.

$$
\begin{aligned}
v_1 \frac{\partial v_1}{\partial z_1} + v_2 \frac{\partial v_2}{\partial z_1} + \cdots v_n \frac{\partial v_n}{\partial z_1} &= 0 \\
v_1 \frac{\partial v_1}{\partial z_2} + v_2 \frac{\partial v_2}{\partial z_2} + \cdots v_n \frac{\partial v_n}{\partial z_2} &= 0 \\
\cdots \qquad \cdots \qquad & \cdots \\
\cdots \qquad \cdots \qquad & \cdots \\
\cdots \qquad \cdots \qquad & \cdots \\
v_1 \frac{\partial v_1}{z_q} + v_2 \frac{\partial v_2}{z_q} + \cdots v_n \frac{\partial v_n}{\partial z_q} &= 0
\end{aligned}
\tag{10-17}
$$

where $v_1, v_2 \cdots v_n$ are residuals of

$z_1, z_2 - -z_q$, the measured quantities.

If the measurements are weighted, the equations have the same form except that the appropriate weight is prefixed to each residual. That is, p_1v_1, p_2v_2, and p_nv_n are substituted for v_1, v_2 and v_n.

To illustrate a simple application of the Least Squares adjustment, consider the following example.

A base line is measured from end to end and found to be 5282.76 ft. long. It is then measured in two sections, the resulting lengths being 2869.58 and 2413.50 ft. It is then measured in three sections, obtaining 1592.27, 2100.19 and 1590.15 ft. All measurements were made under same conditions and are of equal weight. Corrections have been made for all systematic errors.

Three expressions for adjusted length of line may be written in terms of observed values and residuals (v's). These equations are sometimes called *observation equations.*

$$(1) \qquad 5282.76 + v_1$$

$$(2) \qquad 2869.58 + v_2 + 2413.50 + v_3$$

$$(3) \qquad 1592.27 + v_4 + 2100.19 + v_5 + 1590.15 + v_6$$

In this example, the condition must be met that each set of measurements add to the same value.

Equating (1) to (2), and (1) to (3) two *condition equations* are formed as follows:

$$5282.76 + v_1 = 2869.58 + v_2 + 2413.50 + v_3$$

$$(4) \qquad v_1 - v_2 - v_3 - 0.32 = 0$$

$$5282.76 + v_1 = 1592.27 + v_4 + 2100.19 + v_5 + 1590.15 + v_6$$

$$(5) \qquad v_1 - v_4 - v_5 - v_6 + 0.15 = 0$$

A function U may now be written expressing the sum of the squares of the residuals, to be minimized according to principle of Least Squares.

$$(6) \qquad U = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + v_6^2$$

Two of the unknowns in (6) may be eliminated from the relations of the condition equations. Solving (4) and (5) for v_2 and v_4, respectively

$$(7) \qquad v_2 = v_1 - v_3 - 0.32$$

$$(8) \qquad v_4 = v_1 - v_5 - v_6 + 0.15$$

and substituting in (6) the function of U may be expressed in

terms of four unknowns as follows:

$$(9)\quad U = v_1^2 + (v_1 - v_3 - 0.32)^2 + v_3^2 + (v_1 - v_5 - v_6 + 0.15)^2 + v_5^2 + v_6^2$$

Expanding squares in brackets and taking partial differentials of U with respect to each v, and placing equal to zero, the following *normal equations* are obtained.

$$\frac{1}{2}\frac{\partial U}{\partial v_1} = 3v_1 - v_3 - v_5 - v_6 - 0.17 = 0$$

$$\frac{1}{2}\frac{\partial U}{\partial v_3} = 2v_3 - v_1 + 0.32 = 0$$

$$\frac{1}{2}\frac{\partial U}{\partial v_5} = 2v_5 - v_1 + v_6 - 0.15 = 0$$

$$\frac{1}{2}\frac{\partial U}{\partial v_6} = 2v_6 - v_1 + v_5 + 0.15 = 0$$

Solving these equations simultaneously, the following values of the adjustments are obtained.

$$v_1 = +.06 \qquad v_4 = +0.07$$

$$v_2 = -.13 \qquad v_5 = +0.07$$

$$v_3 = -.13 \qquad v_6 = +0.07$$

Adjusted distances equal:

$5282.76 + 0.06 = 5282.82$	$1592.27 + 0.07 = 1592.34$
$2869.58 - 0.13 = 2869.45$	$2100.19 + 0.07 = 2100.26$
$2413.50 - 0.13 = 2413.37$	$1590.15 + 0.07 = 1590.22$
5282.82	5282.82

10-13. Application of Least Squares Method. Since the application of Least Squares to a large system of many interrelated measurements becomes complicated mathematically, approximate methods are often employed which produce satisfactory results with less computation. Examples of such adjustments

are given in Appendices B, C and D. With the development of the electronic computer, the use of the Least Squares adjustment has become practical, especially for special problems requiring a high degree of precision.

Certain cautions should be observed in using the method. It applies only to random errors, and does not reveal systematic errors that may be present in the observations. Incorrect measurements can not be improved or corrected by adjustment procedures. There is no assurance that the adjusted values are always more accurate than those observed. Generally, adjustments are valid when the errors are random in character and are small. The intention of adjustments is to obtain consistency among measurements.

The futility of spending undue time and money in the adjustment of small random errors while at the same time failing to remove a large systematic error, should be emphasized. Likewise the refinements in adjustment should be kept consistent with the accuracy requirements of the survey.

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TABLE I. CORRECTION FOR EARTH'S CURVATURE AND REFRACTION.

(Art. 1-21; Art. 3-15.)

Dist.	Corr.	Dist.	Corr.	Dist.	Corr.	Dist.	Corr.
Feet.	Feet.	Feet.	Feet.	Miles.	Feet.	Miles.	Feet.
100	.000	1000	.02	1	0.6	11	69.4
200	.001	2000	.08	2	2.3	12	82.7
300	.002	3000	.18	3	5.2	13	97.0
400	.003	4000	.33	4	9.2	14	112.5
500	.005	5000	.51	5	14.4	15	129.1
600	.007	6000	.74	6	20.6	16	146.9
700	.010	7000	1.01	7	28.1	17	165.8
800	.013	8000	1.32	8	36.7	18	185.9
900	.017	9000	1.67	9	46.4	19	207.2
1000	.020	10000	2.06	10	57.4	20	229.5
Miles.	Feet.	Miles.	Feet.	Miles.	Feet.	Miles.	Feet.
21	253.1	31	551.4	41	964.7	51	1492.5
22	277.7	32	587.6	42	1012.2	52	1551.6
23	303.6	33	624.9	43	1061.0	53	1611.9
24	330.5	34	663.3	44	1111.0	54	1673.3
25	358.6	35	703.0	45	1162.0	55	1735.8
26	388.0	36	743.7	46	1214.2	56	1799.6
27	418.3	37	785.6	47	1267.7	57	1864.4
28	449.9	38	828.6	48	1322.1	58	1930.4
29	482.6	39	872.8	49	1377.7	59	1997.5
30	516.4	40	918.1	50	1434.6	60	2065.8

Distance	Corr. to Rod-Reading	Distance	Corr. to Rod-Reading
m m	mm	m m	mm
0 to 27	0.0	106 to 112	−0.8
28 to 47	−0.1	113 to 118	−0.9
48 to 60	−0.2	119 to 124	−1.0
61 to 72	−0.3	125 to 130	−1.1
73 to 81	−0.4	131 to 136	−1.2
82 to 90	−0.5	137 to 141	−1.3
91 to 98	−0.6	142 to 146	−1.4
99 to 105	−0.7	147 to 150	−1.5

TABLE II. VALUES OF LOG *m* FOR COMPUTING SPHERICAL EXCESS.
(METRIC SYSTEM.)

(Art. 1-52.)

Latitude	Log m	Latitude	Log m	Latitude	Log m
° ′		° ′		° ′	
18 00	1.40639 − 10	33 00	1.40520 − 10	48 00	1.40369 − 10
18 30	636	33 30	516	48 30	364
19 00	632	34 00	511	49 00	359
19 30	629	34 30	506	49 30	354
20 00	626	35 00	501	50 00	349
20 30	623	35 30	496	50 30	344
21 00	619	36 00	491	51 00	339
21 30	616	36 30	486	51 30	334
22 00	612	37 00	482	52 00	329
22 30	608	37 30	477	52 30	324
23 00	605	38 00	472	53 00	319
23 30	601	38 30	467	53 30	314
24 00	597	39 00	462	54 00	309
24 30	594	39 30	457	54 30	304
25 00	590	40 00	452	55 00	299
25 30	586	40 30	446	55 30	295
26 00	582	41 00	441	56 00	290
26 30	578	41 30	436	56 30	285
27 00	573	42 00	431	57 00	280
27 30	569	42 30	426	57 30	276
28 00	565	43 00	421	58 00	271
28 30	560	43 30	416	58 30	266
29 00	556	44 00	411	59 00	262
29 30	552	44 30	406	59 30	257
30 00	548	45 00	400	60 00	253
30 30	544	45 30	395	60 30	249
31 00	539	46 00	390	61 00	244
31 30	534	46 30	385	61 30	240
32 00	530	47 00	380	62 00	235
32 30	1.40525	47 30	1.40375	62 30	1.40231

(The above table is computed for the Clarke spheroid of 1866.)

TABLE III. LOGARITHMS OF FACTORS FOR COMPUTING GEODETIC POSITIONS.

(Art. 1-55.)

Lat.	Log *A*	Log *B*	Log *C*	Log *D*	Log *E*
° ′	− 10	− 10	− 10	− 10	− 20
18 00	8.509 5862	8.512 2550	0.91816	2.1606	5.7317
10	5836	2474	0.92243	2.1641	5.7337
20	5811	2397	0.92667	2.1675	5.7358
30	5785	2320	0.93088	2.1709	5.7379
40	5759	2243	0.93505	2.1742	5.7400
50	5733	2165	0.93919	2.1775	5.7422
19 00	5707	2086	0.94330	2.1808	5.7443
10	5681	2006	0.94737	2.1840	5.7464
20	5654	1927	0.95142	2.1872	5.7486
30	5627	1847	0.95544	2.1903	5.7508
40	5600	1766	0.95943	2.1934	5.7530
50	5573	1684	0.96339	2.1965	5.7552
20 00	5546	1602	0.96733	2.1996	5.7574
10	5518	1519	0.97123	2.2026	5.7597
20	5490	1435	0.97511	2.2055	5.7619
30	5462	1351	0.97896	2.2084	5.7642
40	5434	1267	0.98279	2.2113	5.7664
50	5406	1182	0.98659	2.2142	5.7688
21 00	5377	1096	0.99037	2.2170	5.7711
10	5348	1010	0.99412	2.2198	5.7734
20	5320	0924	0.99785	2.2226	5.7757
30	5290	0836	1.00156	2.2253	5.7780
40	5261	0748	1.00524	2.2280	5.7804
50	5232	0660	1.00890	2.2307	5.7828
22 00	5202	0571	1.01253	2.2333	5.7851
10	5172	0481	1.01615	2.2359	5.7875
20	5142	0391	1.01974	2.2385	5.7899
30	5112	0301	1.02331	2.2411	5.7924
40	5082	0210	1.02686	2.2436	5.7948
50	5051	0118	1.03039	2.2461	5.7972
23 00	5020	8.512 0026	1.03390	2.2485	5.7997
10	4990	8.511 9934	1.03739	2.2510	5.8021
20	4959	9840	1.04086	2.2534	5.8046
30	4927	9747	1.04431	2.2557	5.8071
40	4896	9653	1.04775	2.2581	5.8096
50	4865	9558	1.05116	2.2604	5.8121
24 00	4833	9463	1.05456	2.2627	5.8146
10	4801	9367	1.05794	2.2650	5.8172
20	4769	9271	1.06130	2.2672	5.8197
30	4737	9174	1.06464	2.2694	5.8223
40	4704	9077	1.06797	2.2716	5.8249
50	4672	8979	1.07128	2.2738	5.8274
60	8.509 4639	8.511 8881	1.07457	2.2759	5.8300

TABLE III. Continued.

Lat.	Log A	Log B	Log C	Log D	Log E
° ′					
25 00	8.509 4639	8.511 8881	1.07457	2.2759	5.8300
10	4606	8783	1.07785	2.2780	5.8326
20	4573	8684	1.08111	2.2801	5.8352
30	4540	8584	1.08435	2.2822	5.8379
40	4507	8484	1.08758	2.2842	5.8405
50	4473	8383	1.09080	2.2862	5.8431
26 00	4439	8283	1.09400	2.2882	5.8458
10	4406	8181	1.09718	2.2902	5.8485
20	4372	8079	1.10036	2.2922	5.8512
30	4337	7977	1.10351	2.2941	5.8539
40	4303	7874	1.10666	2.2960	5.8566
50	4269	7771	1.10979	2.2978	5.8593
27 00	4234	7667	1.11290	2.2997	5.8620
10	4200	7563	1.11600	2.3015	5.8647
20	4165	7458	1.11909	2.3033	5.8675
30	4130	7353	1.12217	2.3051	5.8702
40	4094	7248	1.12523	2.3069	5.8730
50	4059	7142	1.12829	2.3086	5.8757
28 00	4024	7036	1.13132	2.3104	5.8785
10	3988	6929	1.13435	2.3121	5.8813
20	3952	6822	1.13737	2.3137	5.8841
30	3917	6714	1.14037	2.3154	5.8870
40	3881	6607	1.14337	2.3170	5.8898
50	3845	6498	1.14635	2.3187	5.8926
29 00	3808	6389	1.14932	2.3203	5.8955
10	3772	6280	1.15228	2.3218	5.8983
20	3735	6171	1.15522	2.3234	5.9012
30	3699	6061	1.15816	2.3249	5.9041
40	3662	5950	1.16109	2.3264	5.9069
50	3625	5840	1.16401	2.3279	5.9098
30 00	3588	5729	1.16692	2.3294	5.9127
10	3551	5617	1.16981	2.3309	5.9157
20	3514	5505	1.17270	2.3323	5.9186
30	3476	5393	1.17558	2.3337	5.9215
40	3439	5281	1.17845	2.3351	5.9245
50	3401	5168	1.18131	2.3365	5.9274
31 00	3363	5054	1.18416	2.3379	5.9304
10	3325	4941	1.18700	2.3392	5.9334
20	3287	4827	1.18983	2.3405	5.9363
30	3249	4713	1.19266	2.3418	5.9393
40	3211	4598	1.19548	2.3431	5.9423
50	3173	4483	1.19828	2.3444	5.9453
60	8.509 3134	8.511 4368	1.20108	2.3456	5.9484

TABLE III. Continued.

Lat.	Log A	Log B	Log C	Log D	Log E
° ′					
32 00	8.509 3134	8.511 4368	1.20108	2.3456	5.9484
10	3096	4252	1.20387	2.3469	5.9514
20	3057	4136	1.20666	2.3481	5.9544
30	3018	4020	1.20944	2.3493	5.9575
40	2980	3903	1.21220	2.3504	5.9605
50	2940	3786	1.21496	2.3516	5.9636
33 00	2901	3669	1.21772	2.3527	5.9667
10	2862	3551	1.22047	2.3539	5.9698
20	2823	3433	1.22321	2.3550	5.9729
30	2784	3315	1.22594	2.3561	5.9760
40	2744	3197	1.22866	2.3571	5.9791
50	2704	3078	1.23138	2.3582	5.9822
34 00	2665	2959	1.23409	2.3592	5.9853
10	2625	2840	1.23680	2.3602	5.9885
20	2585	2720	1.23950	2.3612	5.9916
30	2545	2600	1.24219	2.3622	5.9948
40	2505	2480	1.24488	2.3632	5.9980
50	2465	2360	1.24756	2.3642	6.0011
35 00	2425	2239	1.25024	2.3651	6.0043
10	2384	2118	1.25291	2.3660	6.0075
20	2344	1997	1.25557	2.3669	6.0107
30	2304	1875	1.25823	2.3678	6.0140
40	2263	1754	1.26088	2.3687	6.0172
50	2222	1632	1.26353	2.3695	6.0204
36 00	2182	1510	1.26617	2.3704	6.0237
10	2141	1387	1.26881	2.3712	6.0269
20	2100	1265	1.27145	2.3720	6.0302
30	2059	1142	1.27407	2.3728	6.0334
40	2018	1019	1.27670	2.3735	6.0367
50	1977	0895	1.27932	2.3743	6.0400
37 00	1936	0772	1.28193	2.3750	6.0433
10	1895	0648	1.28454	2.3758	6.0466
20	1853	0524	1.28715	2.3765	6.0499
30	1812	0400	1.28975	2.3772	6.0533
40	1771	0276	1.29234	2.3779	6.0566
50	1729	0151	1.29494	2.3785	6.0600
38 00	1687	8.511 0027	1.29753	2.3792	6.0633
10	1646	8.510 9902	1.30011	2.3798	6.0667
20	1604	9777	1.30269	2.3804	6.0701
30	1562	9652	1.30527	2.3810	6.0734
40	1521	9526	1.30785	2.3816	6.0768
50	1479	9401	1.31042	2.3822	6.0802
60	8.509 1437	8.510 9275	1.31299	2.3827	6.0836

TABLE III. Continued.

Lat.	Log A	Log B	Log C	Log D	Log E
° ′					
39 00	8.509 1437	8.510 9275	1 .31299	2 .3827	6 .0836
10	1395	9149	1 .31555	2 .3832	6 .0871
20	1353	9023	1 .31811	2 .3838	6 .0905
30	1311	8897	1 .32067	2 .3843	6 .0939
40	1269	8771	1 .32323	2 .3848	6 .0974
50	1227	8644	1 .32578	2 .3852	6 .1008
40 00	1184	8517	1 .32833	2 .3857	6 .1043
10	1142	8391	1 .33088	2 .3861	6 .1078
20	1100	8264	1 .33342	2 .3866	6 .1113
30	1057	8137	1 .33596	2 .3870	6 .1148
40	1015	8010	1 .33850	2 .3874	6 .1183
50	0973	7883	1 .34104	2 .3878	6 .1218
41 00	0930	7755	1 .34358	2 .3882	6 .1253
10	0888	7628	1 .34611	2 .3885	6 .1289
20	0845	7500	1 .34864	2 .3889	6 .1324
30	0803	7373	1 .35117	2 .3892	6 .1360
40	0760	7245	1 .35370	2 .3895	6 .1395
50	0718	7117	1 .35623	2 .3898	6 .1431
42 00	0675	6989	1 .35875	2 .3901	6 .1467
10	0632	6861	1 .36127	2 .3903	6 .1503
20	0590	6733	1 .36379	2 .3906	6 .1539
30	0547	6605	1 .36631	2 .3908	6 .1575
40	0504	6477	1 .36883	2 .3910	6 .1612
50	0461	6348	1 .37135	2 .3913	6 .1648
43 00	0419	6220	1 .37386	2 .3914	6 .1684
10	0376	6092	1 .37638	2 .3916	6 .1721
20	0333	5963	1 .37889	2 .3918	6 .1758
30	0290	5835	1 .38141	2 .3919	6 .1795
40	0247	5706	1 .38392	2 .3921	6 .1831
50	0204	5578	1 .38643	2 .3922	6 .1868
44 00	0162	5449	1 .38894	2 .3923	6 .1905
10	0119	5320	1 .39145	2 .3924	6 .1943
20	0076	5192	1 .39396	2 .3925	6 .1980
30	8 .5090033	5063	1 .39648	2 .3925	6 .2017
40	8 .5089990	4935	1 .39898	2 .3926	6 .2055
50	9947	4806	1 .40149	2 .3926	6 .2092
45 00	9904	4677	1 .40400	2 .3926	6 .2130
10	9861	4548	1 .40651	2 .3926	6 .2168
20	9818	4420	1 .40902	2 .3926	6 .2206
30	9776	4291	1 .41153	2 .3926	6 .2244
40	9733	4162	1 .41404	2 .3925	6 .2283
50	9689	4034	1 .41655	2 .3925	6 .2321
60	8.508 9647	8.510 3905	1 .41906	2 .3924	6 .2359

TABLE III. Continued.

Lat.	Log *A*	Log *B*	Log *C*	Log *D*	Log *E*
° ′					
46 00	8.508 9647	8.510 3905	1.41906	2.3924	6.2359
10	9604	3776	1.42157	2.3923	6.2398
20	9561	3648	1.42409	2.3922	6.2436
30	9518	3519	1.42660	2.3921	6.2475
40	9475	3391	1.42911	2.3920	6.2514
50	9433	3262	1.43163	2.3918	6.2553
47 00	9390	3134	1.43414	2.3917	6.2592
10	9347	3005	1.43666	2.3915	6.2632
20	9304	2877	1.43917	2.3913	6.2671
30	9261	2749	1.44169	2.3911	6.2710
40	9219	2621	1.44421	2.3909	6.2750
50	9176	2493	1.44673	2.3906	6.2790
48 00	9133	2364	1.44926	2.3904	6.2830
10	9091	2236	1.45178	2.3901	6.2870
20	9048	2108	1.45431	2.3898	6.2910
30	9005	1981	1.45683	2.3895	6.2950
40	8963	1853	1.45937	2.3892	6.2990
50	8920	1725	1.46190	2.3889	6.3031
49 00	8878	1598	1.46443	2.3886	6.3071
10	8835	1470	1.46696	2.3882	6.3112
20	8793	1343	1.46950	2.3878	6.3153
30	8750	1216	1.47204	2.3875	6.3194
40	8708	1088	1.47459	2.3871	6.3235
50	8666	0962	1.47713	2.3866	6.3276
50 00	8623	0835	1.47968	2.3862	6.3318
10	8581	0708	1.48223	2.3858	6.3359
20	8539	0581	1.48478	2.3853	6.3401
30	8497	0455	1.48734	2.3848	6.3443
40	8455	0328	1.48989	2.3843	6.3485
50	8413	0202	1.49246	2.3838	6.3527
51 00	8371	8.510 0076	1.49502	2.3833	6.3569
10	8329	8.509 9950	1.49759	2.3828	6.3612
20	8287	9825	1.50016	2.3822	6.3654
30	8245	9699	1.50273	2.3817	6.3697
40	8203	9574	1.50531	2.3811	6.3740
50	8161	9448	1.50789	2.3805	6.3782
52 00	8120	9323	1.51048	2.3799	6.3826
10	8078	9198	1.51307	2.3792	6.3869
20	8036	9074	1.51566	2.3786	6.3912
30	7995	8949	1.51826	2.3779	6.3956
40	7953	8825	1.52086	2.3773	6.4000
50	7912	8701	1.52347	2.3766	6.4043
53 00	7871	8577	1.52608	2.3759	6.4088
10	7829	8453	1.52869	2.3751	6.4132
20	7788	8329	1.53131	2.3744	6.4176
30	7747	8206	1.53393	2.3736	6.4221
40	7706	8083	1.53656	2.3729	6.4265
50	7665	7960	1.53919	2.3721	6.4310
60	8.508 7624	8.509 7838	1.54183	2.3713	6.4355

TABLE IV. CORRECTION TO LONGITUDE FOR DIFFERENCE BETWEEN ARC AND SINE.

(Art. 1-55.)

Log s (−) *	Log diff. (units of 8th decimal place)	Log $\Delta\lambda$ (+) *	Tab. diff.‡	Log s (−) *	Log diff. (units of 8th decimal place)	Log $\Delta\lambda$ (+) *	Tab. diff.‡
3.3756	1	1.8846	5000	4.8403	850	3.3493	124
3.8756	10	2.3846	1505	.8527	900	.3617	118
4.0261	20	.5351	881	.8645	950	.3735	111
.1142	30	.6232	624	.8756	1000	.3846	106
.1766	40	.6856	485	.8862	1050	.3952	101
.2251	50	.7341	396	.8963	1100	.4053	97
.2647	60	.7737	335	.9060	1150	.4150	92
.2982	70	.8072	290	.9152	1200	.4242	89
.3272	80	.8362	255	.9241	1250	.4331	85
.3527	90	.8617	229	.9326	1300	.4416	82
4.3756	100	2.8846	207	4.9408	1350	3.4498	79
.3963	110	.9053	189	.9487	1400	.4577	76
.4152	120	.9242	174	.9563	1450	.4653	74
.4326	130	.9416	161	.9637	1500	.4727	71
.4487	140	.9577	150	.9708	1550	.4798	69
.4637	150	.9727	140	.9777	1600	.4867	67
.4777	160	.9867	131	.9844	1650	.4934	64
.4908	170	2.9998	124	.9908	1700	.4998	63
.5032	180	3.0122	229	4.9971	1750	.5061	61
.5261	200	.0351	207	5.0032	1800	.5122	60
4.5468	220	3.0558	189	5.0092	1850	3.5182	58
.5657	240	.0747	174	.0150	1900	.5240	56
.5831	260	.0921	161	.0206	1950	.5296	55
.5992	280	.1082	150	.0261	2000	.5351	106
.6142	300	.1232	140	.0367	2100	.5457	101
.6282	320	.1372	131	.0468	2200	.5558	97
.6413	340	.1503	125	.0565	2300	.5655	92
.6538	360	.1628	117	.0657	2400	.5747	89
.6655	380	.1745	111	.0746	2500	.5836	85
.6766	400	.1856	106	.0831	2600	.5921	82
4.6872	420	3.1962	198	5.0913	2700	3.6003	79
.7070	460	.2160	181	.0992	2800	.6082	76
.7251	500	.2341	167	.1068	2900	.6158	74
.7418	540	.2508	155	.1142	3000	.6232	140
.7573	580	.2663	145	.1282	3200	.6372	131
.7718	620	.2808	136	.1413	3400	.6503	125
.7854	660	.2944	128	.1538	3600	.6628	117
.7982	700	.3072	149	.1655	3800	.6745	111
.8131	750	.3221	141	.1766	4000	.6856	106
4.8272	800	3.3362	131	5.1872	4200	3.6962	101

* Signs to be applied as shown; signs to be reversed when obtaining corrections to coordinates from differences of longitude.

‡ The tabular difference (in units of the 4th place) is the same for Log s and Log $\Delta\lambda$.

TABLE V. FOR CONVERTING SIDEREAL INTO MEAN SOLAR TIME.

(Increase in Sun's Right Ascension in Sidereal h. m. s.)

(Art. 2-22.)

Mean Time = Sidereal Time − C'.

Sid. Hrs.	Corr.	Sid. Min.	Corr.	Sid. Min.	Corr.	Sid. Sec.	Corr.	Sid. Sec.	Corr.
	m s		s		s		s		s
1	0 9.830	1	0.164	31	5.079	1	0.003	31	0.085
2	0 19.659	2	0.328	32	5.242	2	0.005	32	0.087
3	0 29.489	3	0.491	33	5.406	3	0.008	33	0.090
4	0 39.318	4	0.655	34	5.570	4	0.011	34	0.093
5	0 49.148	5	0.819	35	5.734	5	0.014	35	0.096
6	0 58.977	6	0.983	36	5.898	6	0.016	36	0.098
7	1 8.807	7	1.147	37	6.062	7	0.019	37	0.101
8	1 18.636	8	1.311	38	6.225	8	0.022	38	0.104
9	1 28.466	9	1.474	39	6.389	9	0.025	39	0.106
10	1 38.296	10	1.638	40	6.553	10	0.027	40	0.109
11	1 48.125	11	1.802	41	6.717	11	0.030	41	0.112
12	1 57.955	12	1.966	42	6.881	12	0.033	42	0.115
13	2 7.784	13	2.130	43	7.045	13	0.035	43	0.117
14	2 17.614	14	2.294	44	7.208	14	0.038	44	0.120
15	2 27.443	15	2.457	45	7.372	15	0.041	45	0.123
16	2 37.273	16	2.621	46	7.536	16	0.044	46	0.126
17	2 47.102	17	2.785	47	7.700	17	0.046	47	0.128
18	2 56.932	18	2.949	48	7.864	18	0.049	48	0.131
19	3 6.762	19	3.113	49	8.027	19	0.052	49	0.134
20	3 16.591	20	3.277	50	8.191	20	0.055	50	0.137
21	3 26.421	21	3.440	51	8.355	21	0.057	51	0.139
22	3 36.250	22	3.604	52	8.519	22	0.060	52	0.142
23	3 46.080	23	3.768	53	8.683	23	0.063	53	0.145
24	3 55.909	24	3.932	54	8.847	24	0.066	54	0.147
		25	4.096	55	9.010	25	0.068	55	0.150
		26	4.259	56	9.174	26	0.071	56	0.153
		27	4.423	57	9.338	27	0.074	57	0.156
		28	4.587	58	9.502	28	0.076	58	0.158
		29	4.751	59	9.666	29	0.079	59	0.161
		30	4.915	60	9.830	30	0.082	60	0.164

TABLE VI. FOR CONVERTING MEAN SOLAR INTO SIDEREAL TIME.

(Increase in Sun's Right Ascension in Solar h. m. s.)

(Art. 2-22.)

Sidereal Time = Mean Time + C.

Mean Hrs.	Corr.	Mean Min.	Corr.	Mean Min.	Corr.	Mean Sec.	Corr.	Mean Sec.	Corr.
	m s		s		s		s		s
1	0 9.856	1	0.164	31	5.093	1	0.003	31	0.085
2	0 19.713	2	0.329	32	5.257	2	0.005	32	0.088
3	0 29.569	3	0.493	33	5.421	3	0.008	33	0.090
4	0 39.426	4	0.657	34	5.585	4	0.011	34	0.093
5	0 49.282	5	0.821	35	5.750	5	0.014	35	0.096
6	0 59.139	6	0.986	36	5.914	6	0.016	36	0.099
7	1 8.995	7	1.150	37	6.078	7	0.019	37	0.101
8	1 18.852	8	1.314	38	6.242	8	0.022	38	0.104
9	1 28.708	9	1.478	39	6.407	9	0.025	39	0.107
10	1 38.565	10	1.643	40	6.571	10	0.027	40	0.110
11	1 48.421	11	1.807	41	6.735	11	0.030	41	0.112
12	1 58.278	12	1.971	42	6.900	12	0.033	42	0.115
13	2 8.134	13	2.136	43	7.064	13	0.036	43	0.118
14	2 17.991	14	2.300	44	7.228	14	0.038	44	0.120
15	2 27.847	15	2.464	45	7.392	15	0.041	45	0.123
16	2 37.704	16	2.628	46	7.557	16	0.044	46	0.126
17	2 47.560	17	2.793	47	7.721	17	0.047	47	0.129
18	2 57.417	18	2.957	48	7.885	18	0.049	48	0.131
19	3 7.273	19	3.121	49	8.049	19	0.052	49	0.134
20	3 17.129	20	3.285	50	8.214	20	0.055	50	0.137
21	3 26.986	21	3.450	51	8.378	21	0.057	51	0.140
22	3 36.842	22	3.614	52	8.542	22	0.060	52	0.142
23	3 46.699	23	3.778	53	8.707	23	0.063	53	0.145
24	3 56.555	24	3.943	54	8.871	24	0.066	54	0.148
		25	4.107	55	9.035	25	0.068	55	0.151
		26	4.271	56	9.199	26	0.071	56	0.153
		27	4.435	57	9.364	27	0.074	57	0.156
		28	4.600	58	9.528	28	0.077	58	0.160
		29	4.764	59	9.692	29	0.079	59	0.162
		30	4.928	60	9.856	30	0.082	60	0.164

TABLE VII. MEAN REFRACTIONS IN ZENITH DISTANCE AND SUN'S PARALLAX IN ALTITUDE

(Art. 2-24, 2-25.)

Barometric Pressure 29.6 ins. Temperature 50 F.

Apparent Altitude	Refraction (−)	Sun's Par. (+)	Apparent Altitude	Refraction (−)	Sun's Par. (+)	Apparent Altitude	Refraction (−)	Sun's Par. (+)
° ′	′ ″	″	° ′	′ ″	″	°	′ ″	″
7 30	6 53	8.8	12 0	4 25	8.7	25	2 3	8.1
7 40	6 45	8.8	12 30	4 15	8.7	26	1 58	8.0
7 50	6 37	8.8	13 0	4 5	8.7	27	1 53	7.9
8 0	6 30	8.8	13 30	3 56	8.7	28	1 48	7.9
8 10	6 22	8.8	14 0	3 47	8.6	29	1 44	7.8
8 20	6 15	8.8	14 30	3 39	8.6	30	1 40	7.7
8 30	6 8	8.8	15 0	3 32	8.6	32	1 32	7.6
8 40	6 2	8.8	15 30	3 25	8.6	34	1 25	7.4
8 50	5 55	8.8	16 0	3 19	8.6	36	1 19	7.2
9 0	5 49	8.8	16 30	3 13	8.5	38	1 14	7.0
9 10	5 43	8.8	17 0	3 7	8.5	40	1 9	6.8
9 20	5 38	8.8	17 30	3 1	8.5	42	1 4	6.6
9 30	5 32	8.8	18 0	2 56	8.5	44	1 0	6.4
9 40	5 26	8.8	18 30	2 51	8.4	46	0 56	6.2
9 50	5 21	8.8	19 0	2 46	8.4	48	0 52	6.0
10 0	5 16	8.8	19 30	2 42	8.4	50	0 48	5.7
10 20	5 6	8.8	20 0	2 37	8.4	55	0 40	5.1
10 40	4 57	8.7	21 0	2 29	8.3	60	0 33	4.4
11 0	4 48	8.7	22 0	2 22	8.2	65	0 27	3.8
11 20	4 40	8.7	23 0	2 15	8.2	70	0 21	3.0
11 40	4 32	8.7	24 0	2 9	8.1	80	0 10	1.5
12 0	4 25	8.7	25 0	2 3	8.1	90	0 0	0.0

Apparent altitude (observed vertical angle) : v
Sun's true altitude : h = v − refraction + parallax
Star's true altitude : h = v − refraction

True refraction : mean refraction x coefficient for barometric pressure x coefficient for temperature.

TABLE VIII. COEFFICIENTS TO APPLY TO MEAN REFRACTIONS FOR VARIATIONS IN BAROMETER AND TEMPERATURE

(Art. 2-24.)

Barometer (Ins.)	Elevation above sea level (Feet)	Coefficient	Barometer (Ins.)	Elevation above sea level (Feet)	Coefficient	Temperature (Fahr.)	Coefficient
30.5	−451	1.03	25.1	4,859	0.85	−10°	1.13
30.2	−181	1.02	24.8	5,186	.84	0°	1.11
30.0	00	1.01	24.5	5,518	.83	+10°	1.08
29.9	+ 91	1.01	24.2	5,854	.82	20°	1.06
29.6	366	1.00	23.9	6,194	.81	30°	1.04
29.3	643	.99	23.6	6,538	.80	40°	1.02
29.0	924	.98	23.3	6,887	.79	50°	1.00
28.7	1,207	.97	23.0	7,239	.78	60°	.98
28.4	1,493	.96	22.7	7,597	.77	70°	.96
28.1	1,783	.95	22.4	7,960	.76	80°	.94
27.8	2,075	.94	22.1	8,327	.75	90°	.93
27.5	2,371	.93	21.8	8,700	.74	100°	.91
27.2	2,670	.92	21.5	9,077	.73	+110°	.90
26.9	2,972	.91	21.2	9,460	.72		
26.6	3,277	.90	20.9	9,848	.71		
26.3	3,586	.89	20.6	10,242	.70		
26.0	3,899	.88	20.3	10,642	.69		
25.7	4,215	.87	20.0	11,047	.68		
25.4	4,535	.86					

TABLE IX. VALUES OF C FOR USE IN THE CHEZY FORMULA $V = C\sqrt{RS}$

(Art. 8-3.)

Slope	R	n .020	n .025	n .030	n .035	n .040	n .045	n .050	n .055	n .060
.0001	3.28	91	73	60	52	46	40	36	33	30
	10	111	92	78	69	62	55	50	46	42
	20	122	102	89	79	71	65	60	55	51
	50	134	114	100	91	83	76	71	67	63
	100	140	121	108	98	91	84	79	74	70
.0002	10	108	89	76	67	60	53	49	45	41
	20	117	98	85	76	68	61	57	53	49
	50	126	108	94	85	78	71	66	62	58
	100	131	113	99	90	83	77	72	68	64
.0004	10	107	88	75	66	59	53	48	44	41
	20	115	96	83	73	66	60	55	51	48
	50	123	104	91	82	75	68	63	59	56
	100	127	108	96	87	80	73	68	64	61
.0010	10	105	87	74	65	58	52	47	44	40
	20	113	94	81	72	65	59	54	50	47
	50	120	101	89	79	72	66	61	57	54
	100	124	105	94	85	77	71	66	62	59
.010	10	105	86	74	65	58	51	47	43	40
	20	112	93	80	71	64	58	53	49	46
	50	119	100	87	78	71	65	60	56	53
	100	122	104	91	82	75	69	65	61	58

NOTE. — For R = 3.28 feet, n constant, C is constant for all values of slope. For slopes greater than 0.01, or fall of 52.8 feet per mile, C remains nearly constant.

From "River Discharge," by Hoyt and Grover.

TABLE X. HAMILTON SMITH'S COEFFICIENTS FOR WEIRS WITH CONTRACTION SUPPRESSED AT BOTH ENDS, FOR USE IN THE FORMULA $Q = cbH^{\frac{3}{2}}$.

(Art. 8-22.)

H = head in feet	b = length of weir, in feet								
	19	15	10	7	5	4	3*	2*	0.66*
0.1	3.515	3.515	3.520	3.520	3.526	......	......	......	3.611
.15	3.440	3.445	3.445	3.451	3.451	3.461	3.472	3.488	3.542
.2	3.397	3.403	3.408	3.408	3.413	3.429	3.435	3.450	3.510
.25	3.371	3.376	3.381	3.386	3.392	3.403	3.413	3.429	3.494
.3	3.349	3.354	3.360	3.365	3.376	3.386	3.403	3.418	3.483
.4	3.322	3.328	3.333	3.344	3.360	3.371	3.386	3.403	3.478
.5	3.312	3.317	3.322	3.338	3.354	3.371	3.386	3.408	3.478
.6	3.306	3.312	3.317	3.333	3.354	3.371	3.392	3.413	3.483
.7	3.306	3.312	3.317	3.338	3.360	3.376	3.397	3.424	3.494
.8	3.306	3.317	3.322	3.344	3.365	3.386	3.408	3.441	3.510
.9	3.312	3.317	3.328	3.354	3.375	3.397	3.418	3.451	......
1.0	3.312	3.322	3.338	3.360	3.386	3.408	3.429	3.467	......
1.1	3.317	3.328	3.344	3.371	3.397	3.419	3.445	......	......
1.2	3.317	3.333	3.349	3.381	3.403	3.429	3.456	......	......
1.3	3.322	3.338	3.360	3.386	3.413	3.440	3.467	......	......
1.4	3.328	3.344	3.365	3.392	3.424	3.445	......	......	......
1.5	3.328	3.344	3.371	3.403	3.429	3.456	......	......	......
1.6	3.333	3.349	3.376	3.408	3.435	3.461	......	......	......
1.7	3.333	3.349	3.381	3.413	......	......	......	......	......
2.0	......	......	......	......	......	......	......	......	......

* Approximate.

(From Water Supply and Irrigation Paper No. 200, U. S. Geological Survey.)

TABLE XI. HAMILTON SMITH'S COEFFICIENTS FOR WEIRS WITH TWO COMPLETE END CONTRACTIONS, FOR USE IN THE FORMULA $Q = cbH^{\frac{3}{2}}$.

(Art. 8-22.)

H = head in feet	b = length of weir, in feet										
	0.66	1*	2	2.6	3	4	5	7	10	15	19
0.1	3.381	3.419	3.456	3.478	3.488	3.494	3.494	3.499	3.504	3.504	3.510
.15	3.312	3.344	3.392	3.408	3.413	3.419	3.424	3.424	3.429	3.435	3.435
.2	3.269	3.306	3.349	3.365	3.371	3.376	3.376	3.381	3.386	3.392	3.392
.25	3.237	3.274	3.322	3.333	3.338	3.344	3.349	3.354	3.360	3.360	3.365
.3	3.215	3.253	3.296	3.306	3.312	3.322	3.322	3.333	3.338	3.338	3.344
.4	3.183	3.215	3.258	3.274	3.280	3.285	3.290	3.301	3.306	3.312	3.317
.5	3.156	3.189	3.237	3.247	3.253	3.264	3.269	3.280	3.290	3.295	3.301
.6	3.140	3.172	3.215	3.231	3.237	3.247	3.253	3.269	3.280	3.285	3.290
.7	3.130	3.156	3.199	3.210	3.226	3.231	3.242	3.258	3.274	3.280	3.285
.8	.....	.....	3.183	3.199	3.215	3.221	3.231	3.247	3.269	3.274	3.280
.9	.....	.....	3.167	3.189	3.199	3.210	3.226	3.242	3.258	3.269	3.274
1.0	.....	.....	3.156	3.172	3.183	3.199	3.215	3.231	3.253	3.264	3.269
1.1	.....	.....	3.140	3.162	3.172	3.189	3.205	3.226	3.242	3.258	3.264
1.2	.....	.....	3.130	3.151	3.162	3.178	3.194	3.215	3.237	3.253	3.264
1.3	.....	.....	3.114	3.135	3.151	3.167	3.199	3.205	3.231	3.247	3.258
1.4	.....	.....	3.103	3.124	3.140	3.156	3.178	3.199	3.221	3.242	3.258
1.5	.....	.....	.....	3.114	3.130	3.151	3.167	3.189	3.215	3.237	3.253
1.6	.....	.....	.....	3.103	3.114	3.140	3.162	3.183	3.210	3.231	3.247
1.7	.....	.....	.....	.....	.....	.....	.....	3.178	3.205	3.226	3.247
2.0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Approximate.

(From Water Supply and Irrigation Paper No. 200, U. S. Geological Survey.)

TABLE XII. LENGTHS OF DEGREES OF THE MERIDIAN *

(Art. 9-9.)

Latitude	Meters	Latitude	Meters	Latitude	Meters
0°	110 567.2	30°	110 848.5	60°	111 414.5
1	567.6	31	865.7	61	431.5
2	568.6	32	883.2	62	448.2
3	570.3	33	901.1	63	464.4
4	572.7	34	919.2	64	480.3
5	575.8	35	937.6	65	495.7
6	579.5	36	956.2	66	510.7
7	583.9	37	975.1	67	525.3
8	589.0	38	110 994.1	68	539.3
9	594.7	39	111 013.3	69	552.9
10	601.1	40	032.7	70	565.9
11	608.1	41	052.2	71	578.4
12	615.8	42	071.7	72	590.4
13	624.1	43	091.4	73	601.8
14	633.0	44	111.1	74	612.7
15	642.5	45	130.9	75	622.9
16	652.6	46	150.6	76	632.6
17	663.3	47	170.4	77	641.6
18	674.5	48	190.1	78	650.0
19	686.3	49	209.7	79	657.8
20	698.7	50	229.3	80	664.9
21	711.6	51	248.7	81	671.4
22	725.0	52	268.0	82	677.2
23	738.8	53	287.1	83	682.4
24	753.2	54	306.0	84	686.9
25	768.0	55	324.8	85	690.7
26	783.3	56	343.3	86	693.8
27	799.0	57	361.5	87	696.2
28	815.1	58	379.5	88	697.9
29	831.6	59	397.2	89	699.0
30	110 848.5	60	111 414.5	90	111 699.3

* These lengths of a degree of the meridian extend 0° 30′ north and 0° 30′ south of the given latitude.

TABLE XIII. — MERIDIONAL DISTANCE IN METERS FROM WHOLE DEGREE PARALLEL.

(Art. 9-9.)

Lat.	Minutes from Whole Degree Parallel									
	1′	2′	3′	4′	5′	6′	7′	8′	9′	10′
25°	1846.1	3692.3	5538.4	7384.6	9230.7	11076.9	12923.0	14769.2	16615.4	18461.5
26	1846.4	3692.8	5539.2	7385.6	9232.0	11078.4	12924.8	14771.2	16617.7	18464.1
27	1846.7	3693.3	5540.0	7386.6	9233.3	11080.0	12926.7	14773.3	16620.0	18466.7
28	1846.9	3693.8	5540.8	7387.7	9234.6	11081.6	12928.5	14775.5	16622.5	18469.4
29	1847.2	3694.4	5541.6	7388.8	9236.0	11083.2	12930.5	14777.7	16624.9	18472.2
30	1847.5	3695.0	5542.4	7389.9	9237.4	11084.9	12932.4	14779.9	16627 4	18475.0
31	1847.8	3695.5	5543.3	7391.1	9238.9	11086.7	12934.4	14782.2	16630.0	18477.9
32	1848.1	3696.1	5544.2	7392.3	9240.3	11088.4	12936.5	14784.6	16632.7	18480.8
33	1848.4	3696.7	5545.1	7393.4	9241.8	11090.2	12938.6	14787.0	16635.4	18483 8
34	1848.7	3697.3	5546.0	7394.6	9243.3	11092.0	12940.7	14789.4	16638.1	18486.8
35	1849.0	3697.9	5546.9	7395.9	9244.9	11093.9	12942 8	14791.8	16640.8	18489.9
36	1849.3	3698.5	5547.8	7397.1	9246.4	11095.7	12945.0	14794.3	16643 6	18493 0
37	1849.6	3699.2	5548.8	7398.4	9248.0	11097.6	12947.2	14796.8	16646.5	18496.1
38	1849.9	3699 8	5549.7	7399.6	9249.6	11099.5	12949.4	14799.4	16649.3	18499.3
39	1850.2	3700.5	5550.7	7400.9	9251.2	11101.4	12951.7	14801.9	16652.2	18502.5
40	1850.5	3701.1	5551.7	7402.2	9252.8	11103.4	12953.9	14804.5	16655.1	18505.7
41	1850.9	3701.7	5552.6	7403.5	9254.4	11105.3	12956.2	14807.1	16658.0	18509.0
42	1851.2	3702.4	5553.6	7404.8	9256.0	11107.3	12958.5	14809.7	16661.0	18512.2
43	1851.5	3703.1	5554.6	7406.1	9257.7	11109.2	12960.8	14812.4	16663.9	18515.5
44	1851.9	3703.7	5555.6	7407.4	9259.3	11111.2	12963.1	14815.0	16666.9	18518.8
45	1852.2	3704.4	5556.6	7408.8	9261.0	11113.2	12965.4	14817.6	16669.9	18522.1
46	1852.5	3705.0	5557.6	7410.1	9262.6	11115.2	12967.7	14820.3	16672.8	18525.4
47	1852.8	3705.7	5558.5	7411.4	9264.3	11117.1	12970.0	14822.9	16675.8	18528.7
48	1853.2	3706.3	5559.5	7412.7	9265.9	11119.1	12972.3	14825.5	16678.7	18531.9
49	1853.5	3707.0	5560.5	7414.0	9267.5	11121.1	12974.6	14828.1	16681.7	18535.2
50	1853.8	3707.7	5561.5	7415.3	9269.2	11123.0	12976.9	14830.7	16684.6	18538.5

TABLE XIV. COÖRDINATES OF CURVATURE. (METERS.)

(Art. 9-9.)

Long	Latitudes.							
	26°		27°		28°		29°	
	X	Y	X	Y	X	Y	X	Y
1′	1668.7	0.1	1654.3	0.1	1639.4	0.1	1624.0	0.1
2	3337.3	0.4	3308.5	0.4	3278.8	0.4	3248.0	0.5
3	5006.0	1.0	4962.8	1.0	4918.2	1.0	4872.0	1.0
4	6674.6	1.7	6617.1	1.7	6557.6	1.8	6496.1	1.8
5	8343.3	2.7	8271.4	2.7	8197.0	2.8	8120.1	2.9
6	10011.9	3.8	9925.7	3.9	9836.4	4.0	9744.1	4.1
7	11680.6	5.2	11579.9	5.4	11475.7	5.5	11368.1	5.6
8	13349.2	6.8	13234.2	7.0	13115.1	7.2	12992.1	7.3
9	15017.9	8.6	14888.5	8.8	14754.5	9.1	14616.1	9.3
10	16686.6	10.6	16542.8	10.9	16393.9	11.2	16240.1	11.5

Long.	30°		31°		32°		33°	
1′	1608.1	0.1	1591.8	0.1	1574.9	0.1	1557.6	0.1
2	3216.3	0.5	3183.5	0.5	3149.8	0.5	3115.2	0.5
3	4824.4	1.1	4775.3	1.1	4724.8	1.1	4672.8	1.1
4	6432.6	1.9	6367.1	1.9	6299.7	1.9	6230.3	2.0
5	8040.7	2.9	7958.9	3.0	7874.6	3.0	7787.9	3.1
6	9648.8	4.2	9550.6	4.3	9449.5	4.4	9345.5	4.4
7	11257.0	5.7	11142.4	5.8	11024.4	6.0	10903.1	6.0
8	12865.1	7.5	12734.2	7.6	12599.4	7.8	12460.7	7.9
9	14473.2	9.5	14325.9	9.7	14174.3	9.8	14018.3	10.0
10	16081.4	11.7	15917.7	11.9	15749.2	12.1	15575.9	12.3

Long.	34°		35°		36°		37°	
1′	1539.8	0.1	1521.5	0.1	1502.8	0.1	1483.6	0.1
2	3079.6	0.5	3043.0	0.5	3005.5	0.5	2967.1	0.5
3	4619.3	1.1	4564.5	1.1	4508.3	1.2	4450.7	1.2
4	6159.1	2.0	6086.0	2.0	6011.1	2.1	5934.2	2.1
5	7698.9	3.1	7607.5	3.2	7513.8	3.2	7417.8	3.3
6	9238.7	4.5	9129.0	4.6	9016.6	4.6	8901.4	4.7
7	10778.5	6.1	10650.5	6.2	10519.3	6.3	10384.9	6.4
8	12318.3	8.0	12172.0	8.1	12022.1	8.2	11868.5	8.3
9	13858.0	10.1	13693.5	10.3	13524.8	10.4	13352.1	10.5
10	15397.9	12.5	15215.0	12.7	15027.6	12.8	14835.6	13.0

TABLE XIV. COÖRDINATES OF CURVATURE. (METERS.)

Long.	Latitudes.							
	38°		39°		40°		41°	
	X	Y	X	Y	X	Y	X	Y
1′	1463.9	0.1	1443.8	0.1	1423.3	0.1	1402.3	0.1
2	2927.8	0.5	2887.6	0.5	2846.5	0.5	2804.6	0.5
3	4391.7	1.2	4331.4	1.2	4269.8	1.2	4206.9	1.2
4	5855.6	2.1	5775.2	2.1	5693.0	2.1	5609.2	2.1
5	7319.6	3.3	7219.0	3.3	7116.3	3.3	7011.5	3.3
6	8783.5	4.7	8662.9	4.8	8539.6	4.8	8413.7	4.8
7	10247.4	6.4	10106.7	6.5	9962.8	6.5	9816.0	6.6
8	11711.3	8.4	11550.5	8.5	11386.1	8.5	11218.3	8.6
9	13175.2	10.6	12994.3	10.7	12809.3	10.8	12620.6	10.8
10	14639.1	13.1	14438.1	13.2	14232.6	13.3	14022.9	13.4

Long.	42°		43°		44°		45°	
1′	1380.9	0.1	1359.1	0.1	1336.8	0.1	1314.1	0.1
2	2761.8	0.5	2718.1	0.5	2673.6	0.5	2628.3	0.5
3	4142.7	1.2	4077.2	1.2	4010.4	1.2	3942.5	1.2
4	5523.5	2.2	5436.2	2.2	5347.2	2.2	5256.6	2.2
5	6904.4	3.4	6795.3	3.4	6684.0	3.4	6570.8	3.4
6	8285.3	4.8	8154.3	4.9	8020.8	4.9	7884.9	4.9
7	9666.2	6.6	9513.4	6.6	9357.7	6.6	9199.1	6.6
8	11047.1	8.6	10872.4	8.6	10694.5	8.6	10513.2	8.6
9	12428.0	10.9	12231.5	10.9	12031.3	10.9	11827.4	10.9
10	13808.8	13.4	13590.5	13.5	13368.1	13.5	13141.5	13.5

Long.	46°		47°		48°		49°	
1′	1291.1	0.1	1267.6	0.1	1243.8	0.1	1219.6	0.1
2	2582.2	0.5	2535.3	0.5	2487.6	0.5	2439.1	0.5
3	3873.3	1.2	3802.9	1.2	3731.4	1.2	3658.7	1.2
4	5164.4	2.2	5070.5	2.2	4975.2	2.1	4878.3	2.1
5	6455.5	3.4	6338.2	3.4	6219.0	3.3	6097.9	3.3
6	7746.6	4.9	7605.8	4.8	7462.8	4.8	7317.5	4.8
7	9037.6	6.6	8873.5	6.6	8706.6	6.6	8537.0	6.6
8	10328.7	8.6	10141.1	8.6	9950.4	8.6	9756.6	8.6
9	11619.8	10.9	11408.7	10.9	11194.2	10.9	10976.2	10.8
10	12910.9	13.5	12676.4	13.5	12437.9	13.4	12195.8	13.4

TABLE XV. COÖRDINATES OF CURVATURE. (METERS.)

(Art. 9-9.)

Long.	Latitudes 25°		30°		35°	
	X	Y	X	Y	X	Y
5°	504 645	9 307	482 288	10 523	456 261	11 421
10	1 008 603	37 215	963 658	42 074	911 379	45 656
15	1 511 190	83 685	1 443 193	94 591	1 364 214	102 619
20	2 011 722	148 656	1 919 982	167 977	1 813 632	182 168
25	2 509 518	232 038	2 393 116	262 089	2 258 507	284 102
30	3 003 900	333 718	2 861 694	376 749	2 697 724	408 168

Long.	40°		45°		50°	
5°	426 757	11 972	393 996	12 160	358 224	11 978
10	852 171	47 852	786 492	48 594	714 847	47 859
15	1 274 904	107 525	1 175 994	109 162	1 068 277	107 482
20	1 693 628	190 805	1 561 019	193 635	1 416 934	190 581
25	2 107 023	297 430	1 940 103	301 690	1 759 262	296 785
30	2 513 790	427 063	2 311 802	432 918	2 093 731	425 619

TABLE XVI. — FOR DETERMINING RELATIVE STRENGTH OF FIGURES IN TRIANGULATION (UNIT = 6th DECIMAL PLACE)

°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
10	428	359																					
12	359	295	253																				
14	315	253	214	187																			
16	284	225	187	162	143																		
18	262	204	168	143	126	113																	
20	245	189	153	130	113	100	91																
22	232	177	142	119	103	91	81	74															
24	221	167	134	111	95	83	74	67	61														
26	213	160	126	104	89	77	68	61	56	51													
28	206	153	120	99	83	72	63	57	51	47	43												
30	199	148	115	94	79	68	59	53	48	43	40	33											
35	188	137	106	85	71	60	52	46	41	37	33	27	23										
40	179	129	99	79	65	54	47	41	36	32	29	23	19	16									
45	172	124	93	74	60	50	43	37	32	28	25	20	16	13	11								
50	167	119	89	70	57	47	39	34	29	26	23	18	14	11	9	8							
55	162	115	86	67	54	44	37	32	27	24	21	16	12	10	8	7	5						
60	159	112	83	64	51	42	35	30	25	22	19	14	11	9	7	5	4	4					
65	155	109	80	62	49	40	33	28	24	21	18	13	10	7	6	5	4	3	2				
70	152	106	78	60	48	38	32	27	23	19	17	12	9	7	5	4	3	2	2	1			
75	150	104	76	58	46	37	30	25	21	18	16	11	8	6	4	3	2	2	1	1	1		
80	147	102	74	57	45	36	29	24	20	17	15	10	7	5	4	3	2	1	1	1	0	0	
85	145	100	73	55	43	34	28	23	19	16	14	10	7	5	3	2	2	1	1	0	0	0	0
90	143	98	71	54	42	33	27	22	19	16	13	9	6	4	3	2	1	1	1	0	0	0	0
95	140	96	70	53	41	32	26	22	18	15	13	9	6	4	3	2	1	1	0	0	0	0	
100	138	95	68	51	40	31	25	21	17	14	12	8	6	4	3	2	1	1	0	0	0		
105	136	93	67	50	39	30	25	20	17	14	12	8	5	4	2	2	1	1	0	0			
110	134	91	65	49	38	30	24	19	16	13	11	7	5	3	2	2	1	1	1				
115	132	89	64	48	37	29	23	19	15	13	11	7	5	3	2	2	1	1					
120	129	88	62	46	36	28	22	18	15	12	10	7	5	3	2	2	1						
125	127	86	61	45	35	27	22	18	14	12	10	7	5	4	3	2							
130	125	84	59	44	34	26	21	17	14	12	10	7	5	4	3								
135	122	82	58	43	33	26	21	17	14	12	10	7	5	4									
140	119	80	56	42	32	25	20	17	14	12	10	8	6										
145	116	77	55	41	32	25	21	17	15	13	11	9											
150	112	75	54	40	32	26	21	18	16	15	13												
152	111	75	53	40	32	26	22	19	17	16													
154	110	74	53	41	33	27	23	21	19														
156	108	74	54	42	34	28	25	22															
158	107	74	54	43	35	30	27																
160	107	74	56	45	38	33																	
162	107	76	59	48	42																		
164	109	79	63	54																			
166	113	86	71																				
168	122	98																					
170	143																						

(Art. 1-6.)

GREEK ALPHABET.

LETTERS	NAME
Α, α,	Alpha
Β, β,	Beta
Γ, γ,	Gamma
Δ, δ,	Delta
Ε, ϵ,	Epsilon
Ζ, ζ,	Zeta
Η, η,	Eta
Θ, θ,	Theta
Ι, ι,	Iota
Κ, κ,	Kappa
Λ, λ,	Lambda
Μ, μ,	Mu
Ν, ν,	Nu
Ξ, ξ,	Xi
Ο, ο,	Omicron
Π, π,	Pi
Ρ, ρ,	Rho
Σ, σ, ς,	Sigma
Τ, τ,	Tau
Υ, υ,	Upsilon
Φ, φ,	Phi
Χ, χ,	Chi
Ψ, ψ,	Psi
Ω, ω,	Omega

APPENDIX A.

ADJUSTMENT OF TRAVERSES

Traverses generally are of two types, one that extends in a linear direction and may or may not be connected at its ends to fixed positions, and a second type that forms a closed loop. There are often a number of interconnected loops that form a network which is tied into fixed positions at two or more points. A method of adjustment of this latter type will be described.

The chief objection to any method of traverse adjustment, other than by least squares, is the difficulty of dividing the discrepancy between the angle and length measurements. It is normally necessary to make adjustments to both. A modification of the Dell Method of Level Adjustment is adaptable to the adjustment of traverse networks of limited extent. Large or complex networks may not be readily susceptible to such adjustment as the corrections obtained tend to diverge from probable values and hence may be misleading. Such networks should be adjusted by "The Circuit-Reduction Method" (known more generally as the "Braaten Method") which has been developed for Level adjustment and is adaptable to Traverse adjustment also; this method is described in detail in U. S. Coast and Geodetic Survey Special Publication No. 240. It is more complex than the Dell Method but gives results which agree with those determined by the method of least squares.

In the Dell Method it is assumed that each angle in a loop or circuit will share equal corrections (except for the angles at loop junction points) and that the length corrections will be in direct proportion to the lengths themselves. The angles at junction points must meet the special condition that they total to 360°. Hence it is customary to split the correction for this particular angle into two halves and apply the halves (with alternate signs) on each of the two sides of the leg or link leading to the junction, as indicated in Fig. A-3.

Procedure. It is desirable to draw two sketches of the network loops approximately to scale as in Figs. A-1 and A-2. Fig. A-1 indicates the number of legs along each section (a section being

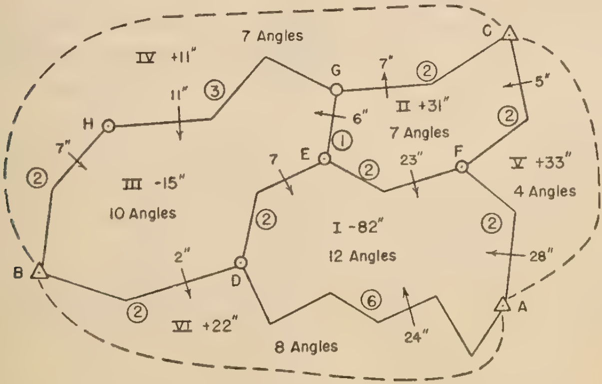

FIG. A-1. ADJUSTMENT OF TRAVERSE ANGLES.

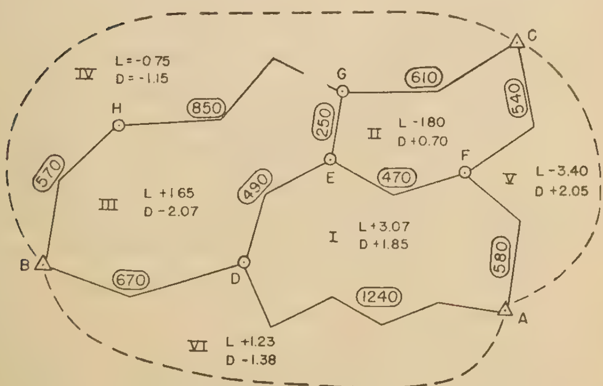

FIG. A-2. ADJUSTMENT OF LATITUDE AND DEPARTURE CLOSURES.

the series of legs between lettered junction or other intermediate points in the network), the circuit numbering and the angular error of closure of each loop, with appropriate sign. Subsequent to computations, the amount of correction for each section is shown by arrows crossing the section lines. Fig. A-2 indicates the lengths of the individual legs, the total length of each circuit,

and the errors, with appropriate sign, of the latitudes and departures of each circuit. Fig. A-2 is useful for computation purposes and illustrates the fact that the two adjustment processes must be carried-out in sequence, and not simultaneously. That is, after the angles have been adjusted, the computations are then made throughout each loop to determine the closures in latitudes and departures.

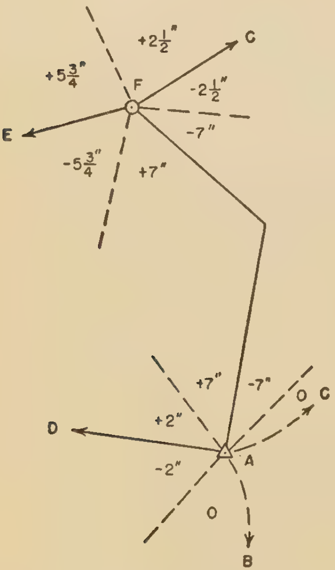

Fig. A-3. Distribution of Angle Corrections at Junctions.

In making adjustment computations it is customary to proceed around each loop in a clockwise direction, and to prepare a form for computing the adjustments in a consistent manner. Fig. A-4 illustrates the angle adjustment procedure.

At the left of the tabulation the sections are listed in clockwise order by circuits. The numbers of station points in each section are also listed for each circuit, both by number and by percentage of total stations in the circuit. The closing error for each circuit is recorded adjacent to the circuit designation in the "Section" column. The computation of adjustments begins with circuit I and proceeds as follows:

(1) Closing error for circuit I is distributed to the sections in that circuit according to percentages of stations in each section, and results are recorded in the left side of column I.

(2) These connections are then distributed with sign reversed to like section designations in other circuits in column I. For example, the $+41$ correction for *AD* in circuit I is entered opposite *DA* in circuit VI as -41.

(3) The corrections distributed in (2) are then added algebraically for each circuit and recorded opposite proper circuit

Circuit	Section	No. Stas.	% of Circuit	Corrections to Sections from adjustments within the Circuit and carry-over adjustments from adjacent circuits.									Final Adjustment	Adopted Value
				I	II	III	IV	V	VI	II	III	I		
	AD	6	50	+41					-18			+2	-1	+24
	DE	2	16	+13		-5					-1			+ 7
	EF	2	17	+14	+5					+3		+1		+23
	FA	2	17	+14				+12				+1	+1	+28
I	-82	12	100	+82 0	+5 +5	-5 0	0 0	+12 +12	-18 -6	+3 -3	-1 -4	+4 0	0 0	→ Σ
	FE	2	28	-14	-5					-3		-1		-23
	EG	1	14		-2	-3				-1				-6
	GC	2	29		-5		+1			-3				-7
	CF	2	29		-5			+12		-3			+1	+5
II	+31	7	100	-14 +17	-17 0	-3 -3	+1 -2	+12 +10	+10	-10 0	0	-1 -1	+1 0	→ Σ
	ED	2	20	-13		+5					+1			-7
	DB	2	20			+5			-6				-1	-2
	BH	2	20			+5	+1				+1			+7
	HG	3	30			+8	+1				+1		+1	+11
	GE	1	10		+2	+3				+1				+6
III	-15	10	100	-13 -28	+2 -26	+26 0	+2 +2	+2	-6 -4	+1 -3	+3 0	0	0 0	→ Σ
	CG	2	28		+5		-1			+3				+7
	GH	3	43			-8	-1				-1		-1	-11
	HB	2	29			-5	-1				-1			-7
	BC	0	—											—
IV	+11	7	100	+11	+5 +16	-13 +3	-3 0	0	0	+3 +3	-2 +1	+1	-1 0	→ Σ
	CA	0	—											—
	AF	2	50	-14				-12				-1	-1	-28
	FC	2	50		+5			-12		+3			-1	-5
V	+33	4	100	-14 +19	+5 +24	+24	+24	-24 0	0	+3 +3	+3	-1 +2	-2 0	→ Σ
	AB	0	—											—
	BD	2	25			-5			+6				+1	+2
	DA	6	75	-41					+18			-2	+1	-24
VI	+22	8	100	-41 -19	-19	-5 -24	-24	-24	+24 0	0	0	-2 -2	+2 0	→ Σ
Σ Corrections				0	0	0	0	0	0	0	0	0	0	(Check)

FIG. A-4. TABULATION OF TRAVERSE ANGLE ADJUSTMENT.

designation on the left side of column. These are then combined algebraically with circuit correction directly to the left and the result recorded on the right side of the same vertical column. For example, in column I opposite designation VI, −41 is combined with +22 giving −19.

(4) Proceeding to the right in Fig. A-4, the net correction for circuit II (−17) is distributed in column II, in the same manner as described for circuit I. The process is repeated in columns III, IV, V, and VI, until corrections have been distributed to all six circuits.

(5) In the first full distribution, residual corrections will invariably remain, such as in Fig. A-4 we find −6 in circuit I, +10 in circuit II and −4 in circuit III. The distribution process is therefore repeated to reduce and ultimately eliminate residual values. However, it is unnecessary to repeat all of the work. Usually in the second run circuits are adjusted in the order of largest net imbalance. (Circuit II, then III and I in Fig. A-4.)

(6) The final adjustment can be made by inspection, as in "Final Adjustment" column in Fig. A-4. Since all sections appear twice in the tabulation, they must have the same amount of correction, one plus and one minus. No corrections are placed along the sections joining fixed points. Checks are available throughout the computations, as the sum of all corrections in each vertical column must be zero.

The angles having thus been adjusted, new bearings of the various legs are found and computations made to determine the latitude and departure errors of each of the circuits (Fig. A-2). The process of finding the several amounts of corrections is the same as before with the one exception that now the *lengths* of the legs in each case are used, instead of the number of angles, wherever it is required to prorate the correction of each section in the circuit. Upon completion of this phase, which requires two sets of computations, one for latitudes and one for departures, the coordinates of each station may next be found, which should check by whatever route computed. With the coordinates thus found, it is merely a matter of reverse computation to obtain the final adjusted bearings of all legs in the network, together with their adjusted lengths.

APPENDIX B.

ADJUSTMENT OF TRIANGULATION.

Whenever a single direct observation is made to determine the magnitude of a quantity, such as an angle or a distance, the result must necessarily be accepted as the true value. If several measurements of the same quantity are made, then no one result can be accepted as correct, but the most probable value, as shown by the measurements, is the arithmetical mean of all of the results. When the measurements, instead of being made directly upon the quantity desired, are made upon one or more functions of this quantity, then the most probable value of the quantity sought must be determined by applying the principles of the Method of Least Squares. (Art. 10-6.)

If any geometric relation exists among different measured quantities, such, for example, as that existing among the angles of a triangle, this relation may be used to check the accuracy of the measurements. If all three angles of the triangle are directly measured, and if in addition it is known that the sum should be 180° plus the spherical excess, then there is more data than is absolutely necessary to determine the three angles. This leads at once to a discrepancy because the measurements are imperfect and never will exactly fulfil the geometric condition. The difference between the true sum and that found from the measurements is due to errors of observation and is known as the **error of closure.** The distribution of this error so as to give the most probable values of the angles is called **adjusting the triangle.** The complete adjustment of a triangulation system so as to remove the discrepancies in the measurements is based upon the Method of Least Squares. While the exact methods of making these adjustments are usually long, there are certain approximate adjustments which are easy to apply and which are useful in cases where a complete adjustment is not required.

The adjustment of the triangulation system consists of (1) the **station adjustment,** or correction of the angles at each station so

as to satisfy any geometric conditions existing among the measured angles, and (2) the **figure adjustment,** or adjustment of each figure (triangle, quadrilateral, etc.) so as to make it a perfect geometric polygon. The figure adjustment consists of two parts, the angle adjustment and the side adjustment. Both of these adjustments are made by correcting the observed angles, but one set of corrections arises from the failure of the triangles to "close," and the other from the fact that even if the triangles close perfectly the length of a side of the figure when computed from a base through two different routes may have different values. The proper corrections to the angles will remove these discrepancies so that finally the triangles will all close perfectly and the lengths of all of the lines will be consistent with the angles, and furthermore the values will be those rendered most probable by the existing measurements.

Station Adjustment. — If all of the angles at one station have been measured independently, as is generally the case when a repeating instrument is used, and if, in addition, an angle has been measured which "closes the horizon," then there exists the rigid geometric condition that the sum of the angles around the point must equal 360°. Assuming that the angles have all been equally well measured, the discrepancy would be removed by distributing the error equally among the measured angles regardless of their magnitude, the closing angle being considered as one of the measured angles and being given its correction like the others. If instead of closing the horizon the sum-angle, or angle from first to last signal, is measured, the adjustment is made in a similar manner except that the algebraic sign of the correction to the sum-angle is reversed.

Figure Adjustment — Triangle. — The simplest case of figure adjustment is that of a single triangle. In this case there is an angle adjustment but no side adjustment. The only geometric relation existing is that the sum of the three measured angles should equal 180° plus the spherical excess of the triangle. The adjustment is made by dividing the error of closure equally among the three angles, the corrections being applied in such a way as to make the corrected sum agree with the true sum.

Quadrilateral. — When a quadrilateral is to be adjusted so

as to remove all discrepancies and to give the most probable values of the angles the adjustment may be conveniently made by the following approximate method.*

Let $ABCD$ (Fig. B-1) be the quadrilateral in which eight angles have been measured, numbered as shown. Let CD be the base; then by means of angles 3 and 4 + 5, point A may be located. The angle 6 is not necessary in locating A, and hence an adjustment is necessary to make $3 + 4 + 5 + 6 = 180° + e''$. Angles 5 and 6 + 7 locate B. The measurement of 8 necessitates a second angle adjustment. The measurement of 2 gives rise to the condition that CB must pass through the position of B already established (side adjustment). The measurement of 1 necessitates a third angle adjustment. In all there are then three independent angle equations and one side equation to be satisfied by the measurements.

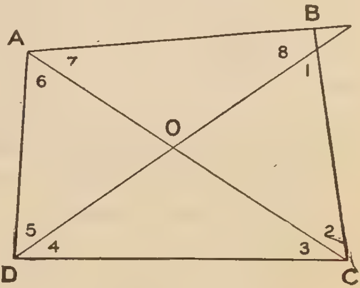

FIG. B-1.

Angle Adjustments. — The corrections to the angles to remove discrepancies in the closure of the triangles are computed by the following equations:

$$v_1 = v_2 = \tfrac{1}{4}\,l_1 - \tfrac{1}{4}\,(l_2 - \tfrac{1}{2}\,l_3 - \tfrac{1}{2}\,l_1)$$
$$v_3 = v_4 = \tfrac{1}{4}\,l_1 + \tfrac{1}{4}\,(l_2 - \tfrac{1}{2}\,l_3 - \tfrac{1}{2}\,l_1)$$
$$v_5 = v_6 = \tfrac{1}{4}\,l_3 + \tfrac{1}{4}\,(l_2 - \tfrac{1}{2}\,l_3 - \tfrac{1}{2}\,l_1)$$
$$v_7 = v_8 = \tfrac{1}{4}\,l_3 - \tfrac{1}{4}\,(l_2 - \tfrac{1}{2}\,l_3 - \tfrac{1}{2}\,l_1)$$

in which v_1, v_2, etc., are corrections to be found for the angles 1, 2, etc., and l_1, l_2, and l_3 are the errors of closure of the triangles BCD, CDA, and DAB, respectively. Stated in words, these equations become the following rule.

(1) Write the angles in order of azimuth in two sets of four each, the first set being the angles of BCD, the second those of DAB.

* For the explanation of the theory of this method see "Adjustment of Observations," by Wright and Hayford, from which this method is taken.

(2) Adjust the angles of each set by one-fourth the difference between the sum and 180° plus the spherical excess, arranging the adjusted angles in two columns, so that the first column will show angles of *ABC* and the second those of *CDA*.

(3) Adjust the first column by one-fourth the difference between the sum and 180° plus the spherical excess, and apply the same correction, with sign changed, to the second column.

Side Adjustment. — After the triangles are adjusted so as to close perfectly, the figure may still be imperfect because the lines *AB*, *DB*, and *CB* may not intersect in a common point. Hence a further adjustment of the angles is necessary to remove this inconsistency. In Fig. B-1 the solution of the triangles about *O* gives the following relation:

$$\frac{OA}{OB} = \frac{\sin 8}{\sin 7}; \quad \frac{OB}{OC} = \frac{\sin 2}{\sin 1}; \quad \frac{OC}{OD} = \frac{\sin 4}{\sin 3}; \quad \frac{OD}{OA} = \frac{\sin 6}{\sin 5}.$$

Multiplying these equations together the result is

$$\frac{\sin 2 \ \sin 4 \ \sin 6 \ \sin 8}{\sin 1 \ \sin 3 \ \sin 5 \ \sin 7} = 1$$

Taking logarithms of both members,

$$(\log \sin 2 + \log \sin 4 + \ldots) - (\log \sin 1 + \log \sin 3 + \ldots) = 0$$

This shows the relation which should exist among the log sines of the angles. If it is found that the log sines of the angles of the quadrilateral are not consistent with this equation, then the angles should be corrected by an amount sufficient to make the above equation hold true. By transforming this equation it may be shown that the corrections to the angles are alternately + and − and in a regular quadrilateral will be approximately equal, so that the equations for this approximate adjustment are

$$v_1' = v_3' = v_5' = v_7' = +\tfrac{1}{8}\,l_4$$
$$v_2' = v_4' = v_6' = v_8' = -\tfrac{1}{8}\,l_4$$

where l_4 is a quantity derived from the log sines as follows. Take the sum of the log sines of the angles 2, 4, 6, and 8 and subtract from this the sum of the log sines of the angles 1, 3, 5, and 7; divide the result by the average value of the tabular differences

for 1″, all being expressed in units of the seventh decimal place. The result is the value of l_4. One-eighth of l_4 is the correction to be applied to each angle, the algebraic signs being as shown in the equations.

EXAMPLE OF ANGLE ADJUSTMENT

	(1) Measured Angles	(2)		(3) Adjusted Angles
1.	61° 07′ 52″.00	52″.99		53″.87
2.	38 28 34 .90	35 .89		36 .77
3.	38 22 19 .10		20.09	19 .21
4.	42 01 12 .15		13.14	12 .26
	179 59 58 .15			2″.11 check
	180 00 02 .11			
	4)3″.96			
	0 .99			
5.	29° 14′ 32″.85		32″.86	31″.98
6.	70 21 59 .20		59 .20	58 .32
7.	49 26 21 .85	21.86		22 .74
8.	30 57 07 .10	07.10		07 .98
	180 00 01 .00	57.84		1″.02 check
	180 00 01 .02	1.36		
	4)0″.02	4)3.52		
	.005	0.88		

EXAMPLE OF SIDE ADJUSTMENT

	Log Sines	Diff. 1″		Log Sines	Diff. 1″
1.	9.942 3708	11.6	2.	9.793 9291	26.5
3.	9.792 9271	26.5	4.	9.825 6798	23.4
5.	9.688 8669	37.6	6.	9.973 9860	7.5
7.	9.880 6545	18.1	8.	9.711 2360	35.1
	9.304 8193			9.304 8309	
				8193	

Mean diff. 1″ = 23.3) 116(4″.98 = l_4

932

2280

2097

1830

1864

8)4.98

0″.62

$$\therefore v_1 = v_3 = v_5 = v_7 = +0''.62$$
$$v_2 = v_4 = v_6 = v_8 = -0''.62$$

The final angles are

1.	61°	07′	54″.49	5.	29°	14′	32″.60
2.	38	28	36 .15	6.	70	21	57 .70
3.	38	22	19 .83	7.	49	26	23 .36
4.	42	01	11 .64	8.	30	57	07 .36

In the above example the angles are first written in column (1), the angles of *BCD* being grouped together and also those of *DAB*. The error of closure of each of these triangles is found by taking the difference between the sum of the observed angles and the theoretical sum, or 180° + e″. In the triangle *BCD* the error of closure is 3″.96 and for *DAB* it is 0″.02. These quantities are each divided by 4 and the resulting correction applied to each angle in the triangle in such a way as to make the sum equal to 180° + e″. The resulting angles (the seconds only) are written in column (2) in such a way that the angles of *ABC* are at the left and those of *CDA* at the right. These two columns, headed column (2), are tested and corrected as before, except that the corrections for the right-hand set in column (2) are the same as those on the left with the sign reversed. It will be seen that the adjusted angles are checked in column (3) by noting if the sums are 180° + e″ as they should be.

In making the side adjustment the log sines to seven places are written so that those of the odd numbered angles are in one column and those of the even numbered angles in another. The tabular differences for 1″ are taken out at the same time. The sums of these two columns differ by 116 units in the seventh figure. This difference divided by 23.3, the mean difference for 1″, gives 4″.98 as the constant l_4. One eighth of this, or 0″.62, is the correction to be applied to each angle from column (3) in the angle adjustment in order to give the final angles. The algebraic sign of this last correction is determined by the rule given above.

After the final angles are obtained the correctness of the work may be tested by seeing if each triangle has an error of closure

equal to zero, and if the length of any side computed from another side by different routes has the same value in all cases. This latter test may be made by repeating the test on the sums of the log sines, using the final angles when entering the tables.

When the quadrilateral is quite irregular, the preceding solution may not be sufficiently accurate. If the differences for 1″ of the log sines are denoted by a_1, a_2, etc., and the corrections by v_1, v_2, etc., it may be shown that

$$a_1v_1 + a_2v_2 + a_3v_3 + a_4v_4 + a_5v_5 + a_6v_6 + a_7v_7 + a_8v_8 = l_4.$$

Solving by the method of Least Squares, the result is

$$v_1 = \frac{a_1}{\Sigma a^2} \times l_4$$

$$v_2 = \frac{a_2}{\Sigma a^2} \times l_4$$

. . . .

In order to find these corrections, the value of Σa^2 is first obtained as follows:

$a_1 = 11.6$	$a_1^2 = 134.56$
$a_2 = 26.5$	$a_2^2 = 702.25$
$a_3 = 26.5$	$a_3^2 = 702.25$
$a_4 = 23.4$	$a_4^2 = 547.56$
$a_5 = 37.6$	$a_5^2 = 1413.76$
$a_6 = 7.5$	$a_6^2 = 56.25$
$a_7 = 18.1$	$a_7^2 = 327.61$
$a_8 = 35.1$	$a_8^2 = 1232.01$
	$\Sigma a^2 = 5116.25$

Solving for the corrections,

$$v_1 = \frac{11.6}{5116} \times 116 = +0.26,$$ and similarly, $v_2 = -0.60$, $v_3 = +0.60$, $v_4 = -0.53$, $v_5 = +0.85$, $v_6 = -0.17$, $v_7 = +0.41$, $v_8 = -0.80$.

Adding these to the angles which result from the angle adjustment, the result is.

(1)	61°	07′	54″.13
(2)	38	28	36 .17
(3)	38	22	19 .81
(4)	42	01	11 .73
(5)	29	14	32 .83
(6)	70	21	58 .15
(7)	49	26	23 .15
(8)	30	57	07 .18

If the angle equations are again tested, it will be found that these no longer hold true, since this side adjustment has disturbed the previous angle adjustment, but they are now more nearly correct than when first adjusted. If a closer adjustment is desired, the whole process must be repeated, the result of such a second approximation being given below.

(1)	61°	07′	54″.38
(2)	38	28	36 .42
(3)	38	22	19 .70
(4)	42	01	11 .62
(5)	29	14	32 .58
(6)	70	21	57 .90
(7)	49	26	23 .26
(8)	30	57	07 .29

A second adjustment for side equation gives the following for the final angles:

(1)	61°	07′	54″.41
(2)	38	28	36 .35
(3)	38	22	19 .77
(4)	42	01	11 .56
(5)	29	14	32 .67
(6)	70	21	57 .88
(7)	49	26	23 .31
(8)	30	57	07 .20

A third test of the angle equations shows such small errors that no further adjustment is necessary.

The quadrilateral may be adjusted in a more direct manner by the following method, taken from Wright and Hayford's *Adjustment of Observations*. The angle adjustments are first made as explained on page 519. If the further corrections required to satisfy the side equation are denoted by $v_1', v_2', \ldots v_8'$, these may be broken into two parts as shown by the following equations.

$$\begin{aligned} v_1' &= +v + v' & \qquad v_5' &= +v + v''' \\ v_2' &= +v - v' & v_6' &= +v - v''' \\ v_3' &= -v + v'' & v_7' &= -v + v'''' \\ v_4' &= -v - v'' & v_8' &= -v - v'''' \end{aligned}$$

We next find the value of a multiplier, C, from the equation

$$\begin{aligned} C = l_4' \div [&\tfrac{1}{4}(a_1 + a_2 + a_5 + a_6 - a_3 - a_4 - a_7 - a_8)^2 \\ &+ (a_1 - a_2)^2 + (a_3 - a_4)^2 + (a_5 - a_6)^2 + (a_7 - a_8)^2] \end{aligned}$$

Then the corrections are found by

$$\begin{aligned} v &= \frac{C}{4}(a_1 + a_2 + a_5 + a_6 - a_3 - a_4 - a_7 - a_8) \\ v' &= C(a_1 - a_2) \\ v'' &= C(a_3 - a_4) \\ v''' &= C(a_5 - a_6) \\ v'''' &= C(a_7 - a_8) \end{aligned}$$

The entire computation may be conveniently worked in tabular form as follows. Separate the log sines so that the + values are in one column and the − values in the other. The log diffs. for 1″ are also in two columns, and arranged in such a manner as to give the coefficient of C in the equation for v. In the column marked "sums" are the coefficients of C in the equations for v', v'', etc. In the column of squares are the terms of the divisor in the equation for C.

Mean Angles	Angle Adj.	Adj. Secs.	log sines		log diff.		Sums	Squares
61° 07′ 52″.00	52″.99	53″.87	9.942 3708		+11.6			
38 28 34 .90	35 .89	36 .77		9.793 9291		+26.5	38.1	1452
38 22 19 .10	20″.09	19 .21	9.792 9271			+26.5		
42 01 12 .15	13 .14	12 .26		9.825 6798	+23.4		49.9	2490
58 .15		2 .11						
02 .11								
4)3 .96								
0 .99								
29° 14′ 32″.85	32 .86	31 .98	9.688 8669		+37.6		45.1	2034
70 21 59 .20	59 .20	58 .32		9.973 9860		+ 7.5		
49 26 21 .85	21 .86	22 .74	9.880 6545			+18.1		
30 57 07 .10	07 .10	07 .98		9.711 2360	+35.1		53.2	2830
01 .00	57 .84	1 .02	193	309	107.7	78.6		
01 .02	01 .36			193	78.6			212
4)0 .02	4)3 .52		l_4' =	116	4) 29.1			9018
0 .005	0 .88				7.27			

$$C = \frac{116}{9018} = .0129$$

$$\begin{aligned}
v &= .0129 \times 7.27 = .0938 \\
v' &= .0129 \times 38.1 = .4915 \\
v'' &= .0129 \times 49.9 = .6437 \\
v''' &= .0129 \times 45.1 = .5818 \\
v'''' &= .0129 \times 53.2 = .6863 \\
v_1' &= +.094 + .492 = +.586 \\
v_2' &= +.094 - .492 = -.398 \\
v_3' &= -.094 + .644 = +.550 \\
v_4' &= -.094 - .644 = -.738 \\
v_5' &= +.094 + .582 = +.676 \\
v_6' &= +.094 - .582 = -.488 \\
v_7' &= -.094 + .686 = +.592 \\
v_8' &= -.094 - .686 = -.780
\end{aligned}$$

Final Angles

$$\begin{aligned}
&61^\circ\ 07'\ 53''.87 + 0''.59 = 54''.46 \\
&38\ \ 28\ \ 36\ .77 - 0\ .40 = 36\ .37 \\
&38\ \ 22\ \ 19\ .21 + 0\ .55 = 19\ .76 \\
&42\ \ 01\ \ 12\ .26 - 0\ .74 = 11\ .52 \\
&29\ \ 14\ \ 31\ .98 + 0\ .68 = 32\ .66 \\
&70\ \ 21\ \ 58\ .32 - 0\ .49 = 57\ .83 \\
&49\ \ 26\ \ 22\ .74 + 0\ .59 = 23\ .33 \\
&30\ \ 57\ \ 07\ .98 - 0\ .78 = 07\ .20
\end{aligned}$$

Appendix C.

ADJUSTMENT OF LEVEL CIRCUITS.

Whenever the elevations of bench-marks are determined by a number of different lines of levels it becomes important to distribute the errors of closure so as to give a single (adopted) elevation for each point and leave a consistent set of elevations. The exact solution of this problem can be effected by the Method of Least Squares, but for adjusting levels of secondary importance there are simpler methods which will give satisfactory results.

The simplest case that can arise is that of a duplicate line, i.e., one in which the line of levels runs from B.M.$_1$ to B.M.$_2$ and back again by the same route. The difference in elevation is taken as the mean of the two determinations because the distances are equal and the two determinations have the same weight. In determining the most probable elevation of B.M.$_2$ to which levels have been run by three different routes from B.M.$_1$, say 2 miles, 3 miles, and 4 miles respectively in length, the line having the greatest length is given the least weight and the shortest line is given the greatest weight in calculating the result. It may be shown that the errors of leveling tend to increase as the square root of the distance run, and hence that the weights of different determinations are inversely as the lengths of the lines. The weights in the above case would be as $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. Since weights are relative values only we may reduce them to convenient numbers by dividing some number, say 12 in this case, by the different lengths of line, obtaining 6:4:3. The most probable value for the elevation of B.M.$_2$ would be taken as the "weighted mean" of the three values, i.e., each difference in elevation would be multiplied by its weight and the sum of these three products divided by the sum of the weights. For example, suppose that in leveling from B.M.$_1$ to B.M.$_2$ by routes 2 mi., 3 mi., and 4 mi. in length the results are 41.16 ft., 41.20 ft., and 41.12 ft. respectively. The weighted mean is then

$$\frac{\frac{1}{2} \times 41.16 + \frac{1}{3} \times 41.20 + \frac{1}{4} \times 41.12}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} = 41.163 \text{ ft.}$$

When a level net composed of several circuits is to be adjusted and it is not desired to employ the exact (Least-Square) solution, the following method will be found convenient. Select the circuit having the largest error of closure and correct the differences between successive bench-marks by distributing the error in direct proportion to the distance. Then select the circuit having the next largest error and adjust as before, not however changing any of the differences previously adjusted. Continue this process until all of the lines have been adjusted.

For adjusting a complicated network of levels,* an iterative method of successive approximations may be used, similar to the Dell Method explained in Appendix A. In the method described below a single junction point is selected and its elevation adjusted by all circuits leading to it from known points, the result being regarded as only a preliminary value, however, if there are other closed circuits whose subsequent adjustment will further change this value. The adjustment is carried through the net until a preliminary value is obtained for each junction point, after which the adjustments of the intermediate junction points are repeated to obtain the final adjusted values. Suppose that the following circuits have been run, elevation of point C being known (Fig. C-1).

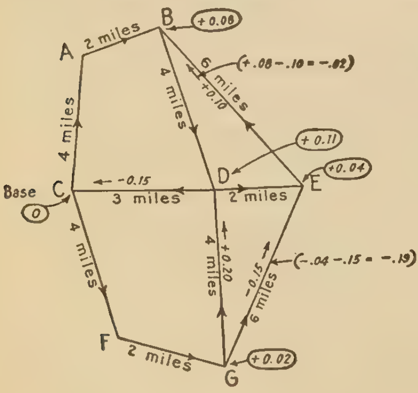

Fig. C-1.

1. $C = 200.55$, $A = 216.26$, $B = 195.21$, $D = 185.17$, $C = 200.40$
2. $C = 200.55$, $F = 169.81$, $G = 162.90$, $D = 185.37$
3. $D = 185.17$, $E = 201.07$, $B = 195.31$
4. $G = 162.90$, E 200.92

The directions in which the lines were run are shown by arrow heads on the lines. The errors of closure as found from the notes

* See "Manual of Leveling Computation and Adjustment," Special Publication No. 240, U. S. Coast and Geodetic Survey.

are indicated on the diagram by arrows accompanied by the amount of the error and its proper sign.

Considering that the elevation of *D* has been determined by three different routes, as shown in the following table, we first find the weighted mean value for the elevation of *D*. The first column in the table shows the route traversed, the second column the distance, the third the weight, based upon the distance run; the fourth column is the correction obtained directly from the error of closure of the circuit as follows. The elevation obtained from the first route is assumed to have a correction = 0; the correction for the second route equals the error of closure for the line *CD* (with sign reversed) as compared with the elevation of *D* found from the line *CABD*; the error in the third route is the error of closure on *D* as found by the line *CFGD*. If the error on either one of the other routes had been assumed = 0 the final result would have been the same.

Preliminary Computation of *D*.

Route	Dist.	Wt.	Corr.	Wt. × Corr.
CABD	10	1	0.00	0.00
CD	3	$3\frac{1}{3}$	+ 0.15	+ 0.50
CFGD	10	1	+ 0.20	+ 0.20
		$5\frac{1}{3}$		$5\frac{1}{3}$) + 0.70
				+ 0.13

Preliminary elevation for *D* = 185.17 + 0.13 = 185.30.

Whenever an elevation is found by working backward along a line (that is, contrary to the direction in which the levels were run), the sign of the error of closure must be reversed. This has been done in determining *D* from *C*, *G* from *D*, and *E* from *B*.

The corrections to the elevations at junction points are entered on the diagram (in red ink) and enclosed by a circle. In Fig. C-1 only the final values of the corrections are shown.

In order to adjust point *E*, preliminary values for *B* and *G* will be needed and these are found by taking proportional parts of the error of closure of the circuits, based upon actual distance run. The preliminary correction for *B* from *C* and *D* = $\frac{6}{10} \times 0.13 = +0.08$, giving elevation 195.29. For point *G* from *C*

and D the preliminary correction is $\frac{6}{10} \times -0.07 = -0.04$, giving elevation 162.86. (The $-.07$ is the difference between the corrected elevation of D, 185.30, and the value 185.37, or the difference between the corrections $+0.13$ and -0.20.)

To adjust E we form the following table:

Route	Dist.	Wt.	Corr.	Wt. × Corr.
BE	6	1	$(+.08 - .10) = -.02$	-0.02
DE	2	3	$+0.13$	$+0.39$
GE	6	1	$(-.04 - .15) = -.19$	-0.19
		5		$5)\underline{+0.18}$
				$+0.04$

The adjusted elevation of E is therefore $201.07 + 0.04 = 201.11$. The correction -0.02 is the sum of the preliminary correction to B and the error of closure from B to E (EB reversed); the correction $+0.13$ is the preliminary correction to D carried to E; the correction -0.19 is the preliminary correction to G plus the error of closure of the line GE.

Before readjusting the elevation of D it is necessary to obtain *second preliminary values* for B and G which shall include the effect of runs from E as well as from C and D. These are given in the next two tables.

Route	Dist.	Wt.	Corr.	Wt. × Corr.
CB	6	4	0.00	0.00
DB	4	6	$+0.13$	$+0.78$
EB	6	4	$(+0.10 + 0.04) = +0.14$	$+0.56$
				$14)\underline{+1.34}$
				$+0.09$

The second preliminary correction for B is therefore $+0.09$.

Route	Dist.	Wt.	Corr.	Wt. × Corr.
CG	6	4	0.00	0.00
DG	4	6	$(+0.13 - 0.20) = -0.07$	-0.42
EG	6	4	$(+0.04 + 0.15) = +0.19$	$+0.76$
				$14)\underline{+0.34}$
				$+0.03$

The second preliminary correction for G is therefore $+0.03$.

The final correction for *D* may now be computed from *C*, *B*, *G*, and *E*, as follows:

Route	Dist.	Wt.	Corr.	Wt. × Corr.
CD	3	4	+ 0.15	+ 0.60
BD	4	3	+ 0.09	+ 0.27
GD	4	3	(+ 0.03 + 0.20) = + 0.23	+ 0.69
ED	2	6	+ 0.04	+ 0.24
				16) + 1.80
			Final correction =	+ 0.11

The final elevation for *D* = 185.17 + 0.11 = 185.28.

This change in *D* would result in final corrections for *B* and *G* as follows:

Route	Dist.	Wt.	Corr.	Wt. × Corr.
CB	6	4	0.00	0.00
DB	4	6	+ 0.11	+ 0.66
EB	6	4	+ 0.14	+ 0.56
				14) + 1.22
			Final correction at *B*	+ 0.08

The final elevation of *B* is 195.21 + 0.08 = 195.29.

Route	Dist.	Wt.	Corr.	Wt. × Corr.
CG	6	4	0.00	0.00
DG	4	6	− 0.09	− 0.54
EG	6	4	+ 0.19	+ 0.76
				14) + 0.22
			Final correction at *G* =	+ 0.02

The final elevation at *G* is therefore 162.90 + 0.02 = 162.92.

After the junction points have all been adjusted the elevations of such intermediate points as *A* and *F* are corrected by giving to each a correction to be interpolated between those corrections assigned to adjacent junction points affecting the particular line. The corrected elevation at *A* is 216.31 and of *F* is 169.82.

APPENDIX D.

CLASSIFICATION AND STANDARDS OF ACCURACY OF GEODETIC CONTROL SURVEYS *

Introduction. Control surveys are of two classes, horizontal and vertical. Horizontal control surveys establish latitude and longitude positions and provide the basis for rectangular coordinates, including State coordinate systems. Vertical control surveys determine elevations referred to mean sea level.

Horizontal control surveys are carried out by triangulation (a procedure of determining the lengths of the sides of a system of joined or overlapping triangles by measuring occasional side lengths upon the ground and computing the others from angles measured at the vertices), and by transit and tape traverses. The lengths of triangle sides or of traverse distances may also be measured by electronic instruments, which measure the travel of time of a beam of light or radio pulse. Recent progress in the development of such instruments indicates increasing use of such procedures.

Vertical control surveys are carried on by precise leveling. The instruments used are of higher precision than those used in ordinary spirit leveling for surveys of small areas, and the computations and final adjustment refer the resultant elevations to mean sea level.

The accompanying tables group control surveys into orders and classes, in accordance with certain standards of accuracy. The recommended spacing or distance between survey stations is also indicated. These standards are primarily intended for guidance of Federal agencies in performing and classifying their control survey operations. They should also be useful to State and local governments, and to private corporations and individuals.

A basic program for establishing geodetic control described in these classifications is in progress to provide adequate spacing as well as sufficient strength and accuracy to meet the needs and satisfy the requirements of engineers and scientists engaged in the development and conservation of the resources of the United States.

The horizontal control network of the United States consists of a framework of arcs of triangulation extending north to south and east to west and

* Approved by the Bureau of the Budget and referred to in Bureau of the Budget Circular A-16, Exhibit C, dated Oct. 10, 1958.

crisscrossing each other at intervals of about 60 miles. The areas between the arcs are subdivided with networks of single triangles, supplemental arcs, or traverses.

The basic program for the ultimate development of the vertical control net of the United States is to form loops of first-order lines spaced at 60-mile intervals, divided by lines of second-order leveling spaced at 25- to 35-mile intervals. In areas where the interest and need for leveling require closer spacing the first-order spacing may be less than 60 miles. In areas where conditions require it, a spacing of second-order lines at 6-mile intervals may be established. The reference datum shall be mean sea level.

Horizontal Control. — Generally, the density of permanently marked control points should be in direct ratio to land values. In metropolitan areas and along interstate highway systems a spacing at 1 or 2 mile intervals may be required and in rural areas of high land value a spacing of 3 to 4 miles may be desirable. Although wider spacing may suffice for Federal topographic mapping, closer spacing may be needed for property surveys, highway programs, transmission lines, reclamation projects, and numerous other engineering activities. The more closely spaced stations should be so situated that they are readily available to local engineers.

Triangulation. — Economic, engineering, and scientific progress has brought an increasing number of requests for higher accuracies in basic first-order triangulation. The range of accuracies is so great that it is necessary to divide first-order into three classes so that satisfactory standards of accuracy can be established.

First-order, Class I: The high value of land in urban areas, the study of small systematic movements in the earth's crust in areas subject to earthquakes, and the testing of military equipment for the National defense require that the triangulation used by engineers and scientists in these varied activities should have an accuracy of at least 1 part in 100,000. Extensive surveys of this nature should make adequate connections with the arcs that make up the National triangulation network. Surveys of such accuracy are designated as Class I of First-order.

First-order, Class II: The basic National horizontal control network consists of arcs of triangulation spaced about 60 miles apart in each direction, forming areas between the arcs which are approximately square. The arcs are planned as chains of quadrilaterals or central point figures, so that the lengths of the sides may be computed through two different chains of triangles. The program for the completion of the network in the United States includes establishing area networks of triangulation within these squares or loops formed by the arcs. To maintain satisfactory mathematical consistency within the area networks, these basic arcs should be measured

with an accuracy of at least 1 part in 50,000. Most of these primary arcs have closures in length and position which are of the order of 1 part in 75,000 or 1 part in 100,000. Triangulation of this standard of accuracy is designated as Class II of First-order.

First-order, Class III: There are many additional demands for first-order triangulation within this National framework, and in some cases even independent of the National net. State, county, and private engineering organizations as well as branches of the Federal Government have need for horizontal control that would have a minimum accuracy of 1 part in 25,000. Surveys of this accuracy have long been recognized both Nationally and internationally as first-order and have attained the status of a widely accepted standard.

In the adjustment of the first-order National network, the surveys of Class I will have precedence and should not be distorted to adjust them to surveys executed under the specifications of Class II. When the surveys of Class III are rigidly adjusted to the basic network, their accuracy should be improved.

The placing of first- or second-order control points within the loops of the basic network requires the extension of area networks, cross arcs, or traverses. These specifications list two classes of second-order triangulation.

Second-order, Class I: This class includes the networks covering the areas within the arcs of the basic network and, if area nets are not feasible, it includes the cross arcs which would be used to subdivide the area. The internal closures of this class of survey should indicate an average accuracy of 1 part in 25,000, with no portion less than 1 in 20,000.

Second-order, Class II: This class of triangulation is used to establish control for hydrographic surveys along the coastline and inland waterways. It may also be used for further breakdown of control within any of the higher classes of triangulation. This class of survey or any of the higher classes may be used by engineers for controlling extensive property surveys. The minimum accuracy to be allowable in Class II of Second-order is 1 part in 10,000.

Third-order triangulation: Triangulation of this order should be supplemental to triangulation of a higher order for the control of topographic or hydrographic surveys, or for such other purposes for which it may be suitable. Although it will usually be established as needed for a specific project, third-order triangulation should be permanently marked, and azimuths should be observed to visible prominent objects, so that the work may be available for future projects and miscellaneous uses in the area. Points located by third-order triangulation may be expected to have an absolute position determination within 10 feet or less in relation to the

adopted datum defined by higher-order positions in the area. The work should be performed with sufficient accuracy to satisfy the standards listed in Table I, p. 13.

Standards for surveys below third-order are not included in these classifications.

Bases. — Bases for the control of the lengths of lines in the triangulation should be measured by appropriate methods and instruments, so that the standards in Table I are satisfied. Recent developments in electronics indicate that accuracies comparable to those obtained with invar tapes may be expected from the Bergstrand geodimeter or similar instruments. The intervals between bases should be such that the standards regarding strength of figure (ΣR_1) also are satisfied.

Traverse. — Traverses are used to supplement all orders and classes of triangulation, and to provide closer and more adequate spacing of horizontal control points. A triangulation net in an urban area provides a frame work for a complete traverse network of first- and second-order accuracies. It is neither economical or feasible to use triangulation for this closer spacing. There are some sections of the United States in addition to these urban areas where traverse can be used efficiently to subdivide the basic network and provide the fundamental spacing of control specified in the National program.

First-order traverses should preferably be connected to first-order triangulation stations of Class I or Class II. If they are connected to Class III of first-order they might be used and given some weight in the adjustment of this class of triangulation. The minimum requirement of accuracy for a first-order traverse is 1 part in 25,000, yet first-order traverse networks, properly executed, will average about 1 part in 40,000. This value is expected and desired. Detailed standards are listed in Table II, p. 14.

Traverses of second- and third-order accuracy are tied to triangulation or traverse of the same or higher order. They are used extensively for cadastral or property surveys and mapping. For property surveys, the value of the property should, in general, determine the accuracy to be used. For map control, the scale of the map and the positional accuracy required usually govern. Details of these orders of traverse are also listed in Table II.

Vertical Control — Leveling. — One of the most important items in the development of a control level net is establishing marks that will remain stable. Releveling has shown that there is considerable vertical movement of bench marks. In some sections of the country there are many factors contributing to vertical change, such as removal of underground water, removal of underground gas and oil, frost action, settling of the soil due to increased moisture content during the rainy seasons, changes in the

underground water table, fault lines, earthquakes, etc. Some of these are so deep-seated that in some areas it is impossible to establish a mark that will remain stable. However, some of these vertical changes can be overcome by installing " super " or " basic " marks at intervals along the line of leveling. The usual practice is to establish a concrete-post type mark at one-mile intervals along a line of first- or second-order leveling, with a " basic " mark at 5-mile intervals. Releveling has shown so many vertical changes that it is advisable to consider releveling first-order lines at least at 25-year intervals, and, in areas where the vertical change is rapid, releveling at least at 5-year intervals. Where vertical change has reached a rate of one foot per year, releveling every two years may be advisable. In addition to the determination of the elevations of regular bench marks, which are installed along the routes of precise level lines, supplementary elevations should be determined at points such as road intersections, railroad crossings, etc., which can be readily identified in aerial photographs.

In first-order leveling the requirement is for a forward and backward running to agree within 4 mm. times the square root of the length of the section in kilometers. If second-order leveling is run with the same equipment as first-order, it can be single run, with loop closures within the criterion 8.4 mm. times the square root of the distance around the loop. In remote areas where a second-order line is longer than 25 miles due to the fact that routes are unavailable for an additional network development, the line should be double-run. This is defined as Class I of Second-order. The single-run area leveling is defined as Class II of Second-order. Summaries of these classifications are listed in Table III, p. 14.

Third-order leveling should be used to subdivide the area surrounded by first- and second-order leveling and should be performed so that the standards in Table III are satisfied. Trigonometric leveling may be considered as fourth-order leveling, and the elevations thus determined are listed with the triangulation data.

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